HARVEY'S ESSENTIALS OF ARITHMETIC

SECOND BOOK

AMERICAN BOOK COMPANY

HARVEY'S ESSENTIALS OF ARITHMETIC

With Everyday Problems Relating to Agriculture, Commerce, and Other Vocations

SECOND BOOK

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AMERICAN BOOK COMPANY

NEW YORK CINCINNATI CHICAGO

HARVEY'S ESSEN. AR. — SECOND BOOK.

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PREFACE

THIS book is intended to cover, in particular, the work of the fifth, sixth, seventh, and eighth years. It begins, however, with a comprehensive review of notation and numeration, and the four fundamental operations, which makes it a book complete in itself. A thorough treatment is given of common and decimal fractions, denominate numbers, percentage and its applications, and other subjects usually taught in the upper grades. Great care has been taken to eliminate useless and obsolete subjects and to fill their places with matters relating to the present-day interests of the pupils and to modern business problems.

The main purpose of the book is to secure accuracy and speed in the handling of numbers, and sureness in the interpretation of such problems as come within the range of the pupils' experience. There is thorough drill in abstract number processes. In their concrete application an effort has been made to substitute for unreal or unstimulating conditions many problems of a prevocational character relating directly to the pupils' experience, such as problems in manual training and household arts, commerce, agriculture, etc.

Many problems suggested by recent national legislation have also been included, such as problems relating to the United States Parcel Post, Postal Savings Banks, the Federal Income Tax, the Tariff of 1913, and the new Federal Banking law.

The reviews throughout the book are cumulative and thorough. At various points in the book thorough drills

are given on all the fundamental operations as well as on common fractions, decimals, and other topics, as far as they have been taught. This insures the pupil against forgetting what he has learned, while his energies are concentrated on the mastery of a new process; and it combines what is best in the topical and spiral methods.

Attention is also called to the following features:

1. The work is carefully graded. While the exercises increase gradually in difficulty, and while they are strong enough to test the pupils' power, they are not so hard as to cause discouragement or fatigue.

2. Great emphasis is placed on correct interpretation of problems and on analysis of the reasoning involved. Pupils are, furthermore, encouraged to study problems with the idea of choosing the most economical method of solution, and are advised to make mental estimates, whenever practicable, to be compared with the results of their written work.

3. The chapter on the "Use of Symbols" tests the pupils' power of reasoning and serves as an introduction to the study of algebra.

4. Suggestions to teachers are scattered throughout the book wherever they may serve useful purposes.

5. The importance of self-activity is recognized in asking the pupils to frame problems for themselves and to draw figures that help toward the comprehension of number relations. The "Problems without Numbers" encourage the pupils to construct their own formulas for the solution of various types of problems. They serve as a further aid in the development of the power of reasoning and give the pupils practice in generalization, which will prove invaluable in later mathematical work.

SUGGESTIONS

BEFORE beginning any recitation, the teacher should bear in mind the following four fundamental propositions:

(1) Each lesson should have a definite purpose.

(2) The teacher must clearly realize what must be known and done to accomplish this purpose.

(3) The teacher must consider how much of this material the pupil has already mastered.

(4) The teacher must then determine what the pupil has still to learn and how the known may best be related to the unknown.

The teacher should be sure that the meaning of new and unusual words and expressions is understood by the pupils before they undertake the mastery of the lesson.

A new topic is frequently introduced by what is called the "Study Recitation." The matter under this heading should be mastered by the pupils under the immediate direction and stimulation of the teacher. The teacher should always thoroughly master the treatment of the subject matter under the "Study Recitation" in advance, and should be able to give the illustrations and applications without reference to the book, and to extend or modify them as may be found necessary. These exercises are given where special preparation for further work by the pupils is deemed necessary. The time required for such preparatory work is time saved, even if an entire recitation period is necessary, because the pupils will do the following assigned work more

intelligently, with greater interest, and more rapidly than would be possible without such preparation.

At the close of the most important subjects, a set of review questions is given. As the pupils proceed in their work upon a topic it is an excellent plan for the teacher to assign as a part of the lesson such of these questions as have been covered by the work already done. In this way pupils will be required to prepare upon not more than one or two questions a day, and will need but little time, when the subject is finished, to prepare for a complete summary of it.

Training in the statement of arithmetical facts, conditions, operations, and relations, correct in matter and form, should be carried on throughout the study of the subject, the requirements in this respect being adapted to the pupils' capacity. Definitions are statements of fact, and rules are statements of processes and of operations involving these processes. Unless the pupil can improve upon the form of these statements, he should learn them and make them a part of his arithmetical vocabulary.

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SECOND BOOK

NOTATION AND NUMERATION

A **unit** is one thing ; as, one dollar, one pupil, one.

A **number** is a unit or a collection of units of the same kind.

Numbers like 1, 2, 3, 4, 5, 6, 20, 35, etc., are called **whole numbers, integers**, or **integral numbers.**

Notation is a method of writing numbers by means of figures or letters.

Numeration is a method of reading numbers expressed by figures or letters.

In arithmetic, the **Arabic** or **decimal** system of notation is used. In this system ten figures, or **digits**, are used in writing numbers. These are :

1	2	3	4	5	6	7	8	9	0
one	two	three	four	five	six	seven	eight	nine	zero

The first nine of these figures are called **significant figures,** because they signify or express value. The last figure is called **zero, cipher,** or **naught.** It expresses no value ; but is used with the other figures in expressing numbers.

Another system of notation, called the **Roman system**, is sometimes used in numbering chapters of books, headings, and in a few other cases. It is explained on page 14.

ARABIC NOTATION AND NUMERATION

STUDY RECITATION

1
10
100
1000
10000
100000
1000000
1111111

1. Read the numbers written at the left.

2. Read their sum.

3. Beginning with 1, what is the value of each number as compared with the number following it?

4. Beginning with 1,000,000, what is the value of each number as compared with the one before it?

5. In the sum of the several numbers, where is the significant figure in each of the numbers added to be found?

6. In the number 1,111,111, give the number represented by each figure.

In any whole number, the places occupied by the figures are called **orders**. The place at the right is of the first order, the second place is of the second order, and so on.

7. In 1,111,111, what is the value of the unit of each order? In 2,222,222 what is the value of the unit of each order?

8. The value of a unit in any order bears what relation to its value in the next higher order? in the next lower order?

9. In whole numbers the first three orders form a **period**, and each three succeeding orders to the left, another period.

Names of periods.	TRILLIONS			BILLIONS			MILLIONS			THOUSANDS			UNITS		
Numbers of orders.	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
Names of orders.	hundred-trillions.	ten-trillions	trillions	hundred-billions	ten-billions	billions	hundred-millions	ten-millions	millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	units

10. Beginning with the units, give the names of the orders in each period; as, *units*, *tens*, *hundreds*, *thousands*, etc.

11. Beginning at the right, give the names of the periods; as, *units*, *thousands*, *millions*, *billions*, *trillions*.

12. Give the names of the periods, in succession, beginning at the right, and the names of the orders beginning at the right. Thus,

The name of the first period is units. The name of the first order in this period is units; of the second order, tens; of the third order, hundreds.

The name of the second period is thousands. The name of the first order in this period is thousands; of the second order, ten-thousands; of the third order, hundred-thousands.

NOTE. Notice that the name of the first order in each period is the same as the name of the period; that the name of the second order in each period after the first is "ten," with the name of the period added; that the name of the third order in each period after the first is "hundred," with the name of the period added.

RULE FOR NUMERATION. *Beginning at units' order, point off the number into periods of three orders each.*

Beginning at the left, read each period as if it stood alone and add the name of the period.

Read the following numbers from the book:

13. 203,417,256.

The third period, standing alone, is read, two hundred three; on adding the name of the period, it is read, two hundred three million. The second period, standing alone, is read, four hundred seventeen; on adding the name of the period, it is read, four hundred seventeen thousand. The first period, standing alone, is read, two hundred fifty-six. In reading a number, the name of the units' period is omitted. The whole number is read two hundred three million, four hundred seventeen thousand, two hundred fifty-six.

14. 56,400,321,256

15. 127,370,209,127

16. 7,048,563,047

17. 397,256,007

18. 400,320,000

19. 900,708,516

20. 100,400,300,260

21. 9,123,405,000

Before attempting to read the entire numbers, read the numbers in each period without adding the name of the period. Whenever, in reading a number, you reach a period of ciphers, omit it and read the next period. Commas are often used, as above, with numbers of more than four figures, to separate the periods.

To the Teacher. Put other numbers on the board and require pupils to read them at sight, until they can read without hesitation.

To write units of any period requires one figure in that period; to write tens or tens and units of any period requires two figures in that period; to write hundreds, hundreds and tens, hundreds and units, or hundreds, tens, and units, of any period requires three figures in that period. If there are vacant places, they should be filled with zeros.

Write the names of the first three periods in order; under these names write the following numbers in their proper places:

22. Five.

23. Fifty.

24. Five hundred.

25. Five thousand.

26. Fifty thousand.

27. Five hundred thousand.

28. Five million.

29. Fifty million.

30. Five hundred million.

31. Four hundred six.

32. Two thousand four hundred six.

33. Twenty-thousand three hundred.

34. Forty-five thousand seven hundred sixty.

35. Write from dictation the numbers in examples 14 to 21.

36. Twenty-five thousand, four hundred six.

37. Two hundred thousand, four hundred six.

38. Two million, twenty-two thousand, four hundred six.

39. Twenty-two million, two thousand, four hundred six.

40. Two hundred twenty million, twenty thousand, four hundred six.

41. Fifteen thousand, nine hundred forty-eight.

42. Two hundred thousand.

43. Four million, four thousand, four hundred.

44. Four hundred thousand six.

45. Nine hundred eight million, fourteen thousand, five hundred.

46. Thirteen thousand eighteen.

47. Eighteen thousand two.

48. Five hundred seventeen million, three hundred sixty-eight thousand, seven hundred ninety-nine.

49. Three hundred million, eight hundred seventeen thousand, eight hundred seven.

50. Nine hundred ninety-nine million, nine hundred ninety-nine thousand, ninety-nine.

TO THE TEACHER. Read the following numbers to pupils, requiring the numbers to be written as read. When the numbers have been written, require pupils to read them.

51.	40,062	785,020	2,469,304	5,860,000
52.	27,009	634,009	15,000,270	7,200,000
53.	146,350	780,350	27,039,009	2,794,352
54.	107,009	609,095	65,078,027	4,000,000

RULE FOR NOTATION. *Begin at the left hand and write the hundreds, tens, and units of each period in succession, filling vacant orders with zeros.*

ROMAN NOTATION

The seven letters used in the Roman system are:

Letters:	I	V	X	L	C	D	M
Values:	1	5	10	50	100	500	1000

The following principles govern the use of these letters in expressing numbers:

1. *Repeating a letter repeats its value.* XX = 20. II = 2.

2. *When a letter is placed after one of greater value, the two express the sum of their values.* XV = 15. CL = 150.

3. *When a letter is placed before one of greater value, the two express the difference of their values.* IX = 9. IV = 4.

4. *When a letter is placed between two letters of greater value, its value is to be taken from the sum of the other two.* XIX = 19. XIV = 14.

5. *Placing a dash over a letter multiplies its value by one thousand.* $\overline{\text{X}}$ = 10,000. $\overline{\text{M}}$ = 1,000,000.

The following table further illustrates the system:

I, 1	VIII, 8	XVI, 16	LXXX, 80	DCC, 700
II, 2	IX, 9	XX, 20	XC, 90	DCCC, 800
III, 3	X, 10	XXX, 30	C, 100	CM, 900
IV, 4	XI, 11	XL, 40	CCC, 300	M, 1000
V, 5	XII, 12	L, 50	CD, 400	MCM, 1900
VI, 6	XIV, 14	LX, 60	D, 500	$\overline{\text{V}}$, 5000
VII, 7	XV, 15	LXX, 70	DC, 600	$\overline{\text{M}}$, 1,000,000

6. Express the following numbers by the Arabic notations: XIX, XXVIII, XXIX, XXXIX, CIX, XCI, CC, DCL, $\overline{\text{M}}$, MC, $\overline{\text{C}}$, MM, $\overline{\text{CMII}}$, MDCCCCV.

7. Express by Roman characters all numbers from 1 to 50; from 51 to 100.

8. Express in the Roman notation: 106, 254, 372, 409, 698, 785, 843, 967, 1914, 10,000.

UNITED STATES MONEY

10 mills = 1 cent	**10 dimes = 1 dollar**
10 cents = 1 dime	**10 dollars = 1 eagle**

The mill is not a coin, but is used for convenience in some calculations.

The sign for dollar or dollars is \$, written before the number. Thus, \$25 is read twenty-five dollars.

When dollars and cents are written as one number, a period, called the **decimal point**, is placed between dollars and cents. Thus, \$25.50 is read twenty-five dollars and fifty cents.

The word *and* is read between dollars and cents.

When cents are written alone, they are written as follows: \$.01 or 1¢, \$.02 or 2¢, \$.03 or 3¢, \$.10 or 10¢, \$.25 or 25¢, etc.

Mills are written thus: \$.001 (1 mill), \$.045 (4 cents 5 mills), \$.345 (34 cents 5 mills).

Read:

	a.	*b.*	*c.*	*d.*
1.	\$.05	\$9.05	\$.004	\$50.054
2.	\$.07	\$7.45	\$.003	\$75.031
3.	\$.47	\$26.67	\$.851	\$24.009
4.	\$.65	\$23.08	\$.807	\$215.003
5.	\$.08	\$175.19	\$.083	\$425.05
6.	\$.75	\$109.05	\$.954	\$403.021
7.	\$.98	\$246.75	\$.454	\$702.505

Write with the dollar sign :

	a.	*b.*
8.	45 cents	354 dollars 4 cents
9.	76 cents	560 dollars 3 cents
10.	20 cents	654 dollars 14 cents 3 mills
11.	73 cents 5 mills	325 dollars 25 cents
12.	15 dollars 6 cents	498 dollars 16 cents
13.	50 dollars 5 cents	500 dollars 2 cents 1 mill
14.	45 dollars 6 cents	700 dollars 23 cents 1 mill

United States money consists of coin and paper money.

The gold coins are the double eagle ($20), eagle ($10), and half-eagle ($5).

The silver coins are the dollar, half-dollar, quarter-dollar, and dime.

The five-cent piece is made of copper and nickel ; the one-cent piece is made of bronze.

Note. There are 23.22 grains of pure gold in a gold dollar and nearly 16 times as many grains of pure silver, 371.25 grains, in a silver dollar. This is what is meant by "the ratio of 16 to 1."

Paper money is issued in denominations of $1, $2, $5, $10, $20, $50, $100, $500, and $1000.

In business, the seller usually regards any part of a cent as an additional cent; but in making computations in interest, etc., anything less than five mills in the final result is disregarded, and anything between five mills and one cent is regarded as an additional cent. Thus, $2.574 is usually regarded as $2.57, and $2.575 is regarded as $2.58.

FUNDAMENTAL OPERATIONS

ADDITION

Addition is the process of uniting two or more numbers into one number.

The numbers added are called **addends**.

The **sum** is the number obtained by addition.

The **sign** of addition is +, read plus.

$$\left.\begin{array}{r}2\\1\\5\end{array}\right\}\textbf{Addends}$$
$$\begin{array}{r}\hline 8\end{array}\ \textbf{Sum}$$

Only **like numbers**, that is, numbers having the same unit, can be added.

In rapid addition, a knowledge of the thirty-six combinations given in the addition table below is necessary. The nine combinations with 1 are not given.

Addition Table

Name the sums rapidly.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**
9.	2+2	3+3	4+4	5+5	6+6	7+7	8+8	9+9
10.	2+3	3+4	4+5	5+6	6+7	7+8	8+9	
11.	2+4	3+5	4+6	5+7	6+8	7+9		
12.	2+5	3+6	4+7	5+8	6+9			
13.	2+6	3+7	4+8	5+9				
14.	2+7	3+8	4+9					
15.	2+8	3+9						
16.	2+9							

To the Teacher. — Drill on these 36 combinations of numbers until they are recognized at sight. For rapid drill, place the table on the board; point to different combinations, requiring individual pupils to give sum of each combination as it is pointed out.

To the Teacher. Put the following table on the board or on a large sheet of manila paper, so that it may be ready for daily use:

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
1	2	3	4	5	6	7	8	9
71	72	73	74	75	76	77	78	79
51	52	53	54	55	56	57	58	59
91	92	93	94	95	96	97	98	99
21	22	23	24	25	26	27	28	29
41	42	43	44	45	46	47	48	49
81	82	83	84	85	86	87	88	89
61	62	63	64	65	66	67	68	69
31	32	33	34	35	36	37	38	39
11	12	13	14	15	16	17	18	19

1. Require pupils to give the sum of 5 and each of the numbers under *A*; as, 6, 76, 56, 96, etc. Continue the drill until 5 has been added to every number in the table; then 4, 3, 6, 7, 2, 8, 9.

Study Recitation

1. Add:

$$\begin{array}{r} 538 \\ 969 \\ \hline 1507 \end{array}$$

The sum of the units is 17 units, or 1 ten and 7 units. Write 7 in units' column and carry 1 to tens' column. The sum of the tens (with 1 carried) is 10 tens, or 1 hundred and 0 tens. Write 0 in tens' column and carry 1 to hundreds' column.

The sum of the hundreds (with 1 carried) is 15 hundreds, or 1 thousand and five hundreds. Write 5 in hundreds' column and 1 in thousands' column.

Say: 17, 10, 15. **Test** by adding in the opposite direction.

Written

Add, testing your work, and timing yourself:

2.	3.	4.	5.	6.	7.
246	366	476	759	9786	74076
136	427	244	346	7976	27345

8.	9.	10.	11.	12.	13.
246	218	619	720	1642	85007
357	965	314	909	2321	96006
800	756	854	802	3414	77005

14.	15.	16.	17.	18.	19.
987	379	777	888	1098	51002
465	248	345	789	3762	63976
230	617	666	466	4519	54689
			132	6030	72123

	20.	21.	22.	23.	24.
a.	986	358	497	273	497
b.	437	449	359	598	235
c.	592	575	648	467	643
d.	785	892	223	523	857
e.	649	261	578	709	923
f.	427	135	421	854	237

	25.	26.	27.	28.	29.
a.	2475	9468	6047	6438	3649
b.	8434	7275	2793	4729	5784
c.	9568	4683	8729	5643	2709
d.	4793	9748	4649	7285	4658
e.	5784	6572	5782	4697	2734
f.	7286	5648	2341	4789	3251

	30.	31.	32.	33.	34.
a.	6184	6347	2784	56409	72003
b.	2321	8459	1347	7348	15564
c.	4598	6572	9106	28407	7005
d.	8727	8137	3490	10456	758
e.	2635	1549	7029	20017	26405
f.	5248	1672	8462	90008	1509

35–37. Add line *a* of examples 20 to 24; of 25 to 29; of 30 to 34.

38–52. Add line *b* of the same examples; then lines *c*; *d*; *e*; *f*.

53–57. Add lines *a* and *b* of examples 20 to 22. Thus, 986 + 437 + 358 + 449 + 497 + 359; lines *a* and *b* of examples 23 to 25; 26 to 28; 29 to 31; 32 to 34.

58–112. In the same examples, that is, 20 to 22, 23 to 25, etc., add lines *a* and *c*; *a* and *d*; *a* and *e*; *b* and *c*; *b* and *d*; *b* and *e*; *b* and *f*; *c* and *d*; *c* and *e*; *c* and *f*; *e* and *f*.

To the Teacher. These drills may be extended indefinitely by different combinations of lines and examples; as *a*, *c*, and *f*, 20 to 24, etc.

	113.	114.	115.	116.	117.
a.	$54.29	$87.62	$33.08	$50.05	$75.07
b.	17.69	58.77	66.93	75.43	98.47
c.	45.67	65.88	56.78	94.05	30.58
d.	47.89	44.99	76.47	58.94	49.62
	$165.54				

	118.	119.	120.	121.	122.
e.	$27.50	$13.42	$13.04	$27.04	$35.06
f.	15.13	12.15	27.00	15.12	27.98
g.	26.25	9.22	10.10	9.09	19.01
h.	42.13	10.20	20.08	.75	27.50
i.	85.75	13.45	13.75	1.28	46.90
j.	23.25	25.42	12.82	.07	82.07
k.	42.51	18.95	26.98	5.70	19.36

123–124. In examples 113 to 114, add lines *a* and *b*; lines *c* and *d*.

125–126. In examples 115 to 117, add lines *a* and *b*; lines *c* and *d*.

127–128. In examples 118 to 119, add lines *efg*; lines *hijk*.

129–130. In examples 120 to 122, add lines *efg*; lines *hijk*.

131–132. In examples 118 to 120, add lines *efg*; lines *fghk*.

133–134. In examples 119 to 121, add lines *egi*; lines *fghk*.

135–136. In examples 120 to 122, add lines *fik*; lines *efhj*.

To the Teacher. This drill may be extended indefinitely by different combinations of lines and examples.

PROBLEMS

Oral

1. Two trolley cars start from the same place and travel in opposite directions. One travels 10 miles and the other 15 miles. How far apart are they?

Statement. Since they travel in opposite directions, one 10 miles and the other 15 miles, they are as far apart as the sum of 10 miles and 15 miles, or —— miles.

2. How many days are there in April and June? in September and November? in February and April? Make and answer similar questions with other months.

Statement. April has —— days; June has —— days; both months have as many days as the sum of —— days and —— days.

3. A dealer buys corn at 76 ¢ a bushel and sells it so as to gain 9 ¢ a bushel. At what price does he sell it?

Statement. If he sells the corn so as to gain 9 ¢ a bushel, he sells it for 9 ¢ more than 76 ¢, or —— ¢.

4. In the sewing class there are 35 pupils; in the paper-cutting class there are 29 pupils. How many pupils are there in both classes?

5. Mary buys ribbon for 20 ¢, tape for 15 ¢, and pins for 10 ¢. How much does she pay for all?

Statement. She pays for all the sum of — ¢, — ¢, — , or — ¢.

6. John pays 35¢ for a drafting board, 25¢ for a try square, $1 for a bit brace, and $1 for a saw. How much does he pay for all?

STATEMENT. He pays for all the sum of —¢, —¢, —¢, —¢, or —¢.

7. A farmer has two cows. One gives 10 quarts of milk a day and the other 9 quarts. How many quarts do they both give in two days? Analyze.

8. If you add 8 to 16 and then 9 and 4 to this sum, what will the result be?

9. Frank made a pencil case 7 inches wide and 10 inches long. Find the distance around it.

10. The top of my desk is 30 inches long and 20 inches wide. What is the distance from one corner to the corner farthest from it, measuring around the desk? What is the entire distance around it?

11. At present prices, what is the value of 1 dozen eggs, 1 pound of sugar, and 2 pounds of tea?

STATEMENT. At present prices 1 dozen eggs cost —— ¢; 1 pound of sugar costs —— ¢; 2 pounds of tea cost —— ¢. All are worth the sum of —— ¢, —— ¢, —— ¢, or —— ¢.

12. John is 9 years old, and 27 years younger than his father. How old is his father?

STATEMENT. If John is 9 years old, and 27 years younger than his father, his father's age is 27 years + 9 years, or —— years.

13. Alfred made 7 articles of paper and cardboard, and James made 8 more than Alfred. How many did both make? Analyze.

14. Edith spent her allowance for a yard of ribbon at $.25, a pair of gloves for $1.50, and a collar for $.25. She had $2 left. How much was her allowance?

15. How many yards of tape does Charles need to bind the top edge of a tray 12 inches long and 5 inches wide?

16. July has 31 days; August, 31; September, 30; October, 31; November, 30; December, 31. How many days are there in the second half of the year? in the first half?

Cooking and Food Problems

17. Find the cost of an Irish stew made from the following recipe:

3 pounds of beef for $.45	4 potatoes for $.05
4 onions for $.05	3 pints of hot water

18. Find the total cost of the following recipe for fruit cup:

1 grapefruit for $.10
2 oranges for $.07
½ can skinned white grapes for $.10
¼ cup sugar for $.01

NOTE. The total costs are here given, not the price of each orange, etc.

19. Find the total cost of the following breakfast for four people: fruit, $.10; cereal and cream, $.16; hot buttered toast, $.09; cocoa, $.10.

20. Find the cost of the following dinner for four people: vegetable soup, $.10; lamb chops, $.32; peas, $.15; potatoes, $.05; creamed carrots, $.07; bread pudding, $.16.

21. Find the cost of the following supper for four people: milk, $.09; whole wheat bread, $.06; salad, $.14; prunes, $.05; cookies, $.08.

22. If a breakfast costs $.45; dinner, $.85; and supper, $.42, what is the cost of the food for the day?

Sewing Problems

23. Ruth trimmed a hat for herself, paying $.40 for the frame, $1.50 for velvet, $.10 for lining, and $.50 for a feather. How much did the hat cost her? How much would it have cost if she had paid $1.50 extra for having it trimmed?

24. Find the cost of a collar protector for which you use one yard of lace for the outside at $.50, one yard of lace for lining at $.25, and a pair of tassels for $.15.

25. How wide a kitchen apron can be made from two widths of 30-inch gingham, if 2 inches is allowed for seams and hems?

26. Helen's apron is 27 inches long and 16 inches wide, and the strings are 3 inches wide. How much finishing braid does she need to go around the apron and across the ends of the strings?

27. Find the length and the width of the linen for a table cover to cover a table 50 inches long and 50 inches wide, if the edges are to hang over 9 inches and 2 inches are allowed for each hem.

Written

Marketing Problems

PRICES OF GROCERIES		PRICES OF MEATS	
Apples per bushel	$2.25	Sirloin steak per pound	$.30
Butter per pound	.39	Chickens per pound	.28
Eggs per dozen	.40	Roast beef per pound	.32
Flour per barrel	5.80	Loin of pork per pound	.31
Sugar per 19 pounds	1.00	Ham per pound	.25
Potatoes per bushel	.50	Leg of lamb per pound	.26

NOTE. The sign @ means at so much a unit. Thus, "5 pounds steak @ 26¢" means "at 26¢ per *pound*."

Using the above prices, find the cost of:

28. 1 bushel apples, 1 barrel flour, and 1 dozen eggs.

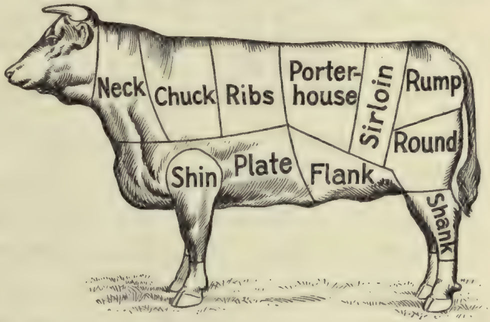
29. 1 pound each of sirloin steak, ham, and lamb.

30. 1 pound butter, 19 pounds sugar, 1 bushel potatoes.

31. 1 pound each of all the meats in the second column.

32. 1 unit each of all the groceries in the first column, except the sugar and the eggs.

33. Florence purchases in a grocery store the following articles: sugar, \$1.50; coffee, \$.29; flour, \$1.05. Find the total cost.



34. This picture shows the general method of cutting up a side of beef. Find the total value of 1 pound of each part of beef bought at retail as follows:

Porterhouse	@ 26¢	Shank	@ 16¢
Sirloin	@ 24¢	Shin	@ 16¢
Round	@ 24¢	Neck and chuck	@ 16¢
Rump	@ 18¢	Flank	@ 15¢
Ribs	@ 20¢	Plate	@ 14¢

35. Find the total number of pounds in a side of beef containing the following number of pounds:

	Pounds		Pounds
Porterhouse	36	Shank	9
Sirloin	18	Shin	7
Round	51	Neck and chuck	120
Rump	14	Flank	15
Ribs	35	Plate	90

Household Problems

Mr. Ames bought furniture as follows for several rooms in his house. Find the cost of furnishing each room and the cost for the six rooms.

36. Dining Room

Table	$25.
Chairs	50.65
Sideboard	35.50
Rug	45.

37. Library

Bookcase	$24.75
Chairs	9.65
Table	35.40
Rug	50.

38. Bedroom

Bed	$20.50
Dresser	16.75
Chairs	9.50
Couch	10.75

39. Kitchen

Table	$10.50
Chairs	6.25
Cabinet	12.75
Range	24.50

40. Parlor

Piano	$375.
Chairs	56.20
Rug	65.
Curtains	15.

41. Nursery

Bed	$13.75
Curtains	3.50
Pictures	4.75
Chairs	5.63

42. A man's expenses for one week are: rent, $7.50; household, $6.50; fuel, $.60; light, $.35; miscellaneous, $8.50. Find his total cost of living for one week.

43. A girl employed in a department store pays $4.50 a week for board and lodging, $1.08 a week for lunches and carfare, $.50 a week for laundry, and $1.50 a week for clothes. What are her weekly expenses?

Farm Problems

44. Mr. Smith paid $34 for a cow, $145 for a horse, and $87.50 for a carriage. Find the entire amount paid.

45. A farmer has 125 bushels of wheat in one bin, 184 bushels in another, and 85 bushels in each of two other bins. How many bushels has he in all?

46. A study of the cost of maintaining a farm work-horse gave the following results. Find the total cost for each year, and the total cost of each item for the entire period of four years.

Items	Year			
	1911	1912	1913	1914
Interest on investment . .	$3.25	$3.75	$4.25	$4.75
Depreciation	15.48	.98	1.45	4.35
Harness depreciation . . .	1.47	1.89	3.64	1.39
Shoeing	1.11	1.55	1.54	1.46
Feed	63.49	51.91	63.54	75.03
Labor	11.77	9.65	11.11	15.01
Miscellaneous	.55	.44	.34	.39

47. During one week a dairyman took to a creamery 420 pounds of milk on Monday, 215 pounds on Tuesday, 230 pounds on Wednesday, 216 pounds on Thursday, 235 pounds on Friday, and 240 pounds on Saturday. How many pounds did he take for the week?

48. The following table is taken from a factory's milk sheet for six days. Find the total number of pounds of milk for the six days for each patron, and the total number of pounds received on each date from the five patrons.

Patron's No.	1	2	3	4	5
Date	Pounds	Pounds	Pounds	Pounds	Pounds
June 1	240	300	150	800	1200
June 2	245	300	155	805	1240
June 3	245	312	150	808	1230
June 4	250	314	155	810	1225
June 5	250	315	152	808	1220
June 6	245	313	158	802	1215

Mercantile Problems

49. A merchant had $87.75 in his cash drawer at the opening of business. He took in during the day, $32.25; $2.87; $39.70; $9.85; $.75; $2.40; $27.09; $9.87; $25; $.50; $3.20. How much cash had he on hand at the close of business?

50. The following is a list of sales of five articles in a dry goods store. Find the total sales of each article for the year; of all the articles for each month; of all the articles for the year.

	Dress Goods	Suits and Notions	Gowns	Lace	Ribbon
Jan. . . .	$25,416.94	$20,860.05	$4,345.62	$10,032.21	$5,494.32
Feb. . . .	17,436.82	15,951.19	3,789.94	11,543.63	6,385.10
March . .	18,573.81	30,740.08	5,516.67	12,710.00	7,276.23
April . . .	26,215.75	25,632.92	2,345.89	13,810.05	8,167.45
May . . .	15,671.94	16,423.76	6,742.61	14,912.17	9,858.67
June . . .	10,143.20	15,514.07	3,501.09	9,316.15	8,749.89
July . . .	2,415.16	9,395.10	4,704.18	5,432.17	7,631.07
August . .	1,313.25	2,280.15	2,304.45	8,719.14	6,522.28
Sept. . . .	17,417.18	8,163.09	8,040.06	10,075.63	8,413.39
Oct.. . . .	25,916.75	25,977.04	9,075.73	15,013.00	9,394.47
Nov. . . .	20,617.18	20,888.79	9,654.21	16,543.20	10,285.58
Dec. . . .	25,715.75	15,779.86	9,821.00	19,617.18	12,176.64

REVIEW

1. Define addition; addends; sum.

2. How should numbers be written for addition? Why?

$$\begin{array}{r} 467 \\ 985 \\ \underline{386} \\ 1838 \end{array}$$

What is the sum of the numbers in the first column? What is the denomination of the left-hand figure in this sum? Why is the 1 added to the next column? What is the sum of 1 and the numbers in the second column? What is the denomination of the right-hand figure in this sum? of the left-hand figure? Why is the 2 added to the third column?

SUBTRACTION

Subtraction is the process of finding the difference between two numbers.

The **minuend** is the number from which we subtract.

The **subtrahend** is the number subtracted.

The **difference**, or **remainder**, is the result obtained by the subtraction.

The **sign** of subtraction is $-$, and is read *minus*.

Minuend		**Subtrahend**		**Difference** or **Remainder**.
27	$-$	9	$=$	18

The minuend equals the sum of the subtrahend and the difference.

To the Teacher. Drill thoroughly on the primary facts in subtraction corresponding to the 36 combinations in addition. (See p. 17.)

Study Recitation

1. From 507 take 398.

$$\begin{array}{r} 507 \\ \underline{398} \\ 109 \end{array}$$

As you cannot take 8 units from 7 units, and there are no tens, take 1 hundred from 5 hundreds, leaving 4 hundreds; 1 hundred, or 10 tens, $+$ 0 tens $=$ 10 tens. Take 1 ten from 10 tens, leaving 9 tens; 1 ten, or 10 units, $+$ 7 units, $=$ 17 units. 8 units from 17 units $=$ 9 units. Write 9 in units' column. 9 tens from 9 tens $=$ 0 tens. Write 0 in tens' column. 3 hundreds from 4 hundreds $=$ 1 hundred. Write 1 in hundreds' column.

Say: 8 from 17, 9; 9 from 9, 0; 3 from 4, 1.

Test. $109 + 398 = 507$.

To the Teacher. The teacher who prefers may teach subtraction by adding 1 to the next order of the subtrahend, instead of subtracting 1 from the next order of the minuend.

$$\begin{array}{r} 507 \\ \underline{398} \\ 109 \end{array}$$

Adding a number to the *minuend* and an equal number to the *subtrahend* does not change the *difference*. 10 must be added to 7 to give 17 before 8 can be subtracted. 8 from 17 leaves 9. 1 added to 9 gives 10; 10 added to 0 gives 10; 10 from 10 leaves 0. 1 added to 3 gives 4; 4 from 5 leaves 1.

Written

Find the differences and test them by adding them to the subtrahends. Time your work :

	2.	3.	4.	5.	6.
a.	9457	8856	9947	7984	8697
b.	6529	7938	8953	7897	8499
	7.	**8.**	**9.**	**10.**	**11.**
c.	35609	36016	37031	39400	36040
d.	26437	10453	16927	19726	12731
	12.	**13.**	**14.**	**15.**	**16.**
e.	59408	49015	49050	51109	50034
f.	42309	44999	40634	48299	44946
	17.	**18.**	**19.**	**20.**	**21.**
g.	96059	96024	94012	93507	94215
h.	75078	85699	73874	82498	83104

22–171. From each number in line *h*, subtract each smaller minuend and subtrahend in the other lines.

172–271. From each number in line *f*, subtract each smaller minuend and subtrahend in the other lines.

272–321. From each number in line *d*, subtract each smaller minuend and subtrahend in the other lines.

322–471. From each number in line *g*, subtract each number in lines *a*, *b*, *c*, *d*, *e*, and *f*.

Find the remainders, and test them by adding them to the subtrahends :

472.	473.	474.	475.	476.
\$563.15	\$529.24	\$206.25	\$495.85	\$385.93
128.78	139.87	178.25	294.87	197.98

477.	478.	479.	480.	481.
$887.65	$904.50	$762.39	$654.25	$829.82
694.29	709.87	573.68	429.89	708.95

482.	483.	484.	485.	486.
$863.41	$657.48	$841.00	$905.06	$753.02
521.98	563.89	671.09	754.78	457.69

487.	488.	489.	490.	491.
$976.82	$843.51	$861.02	$643.00	$908.00
899.99	788.88	677.65	543.29	497.68

To the Teacher. If further drill is required on subtraction of United States money, add dollar signs and decimal points to exercises 2 to 21 on p. 30.

PROBLEMS

Oral

1. If a boy earns 65¢ on Monday and 50¢ on Tuesday, how much more does he earn on Monday than on Tuesday?

Statement. The amount he earns Monday is more than the amount he earns on Tuesday by the difference between 65¢ and 50¢, or —— ¢.

2. If I buy goods for $18 and sell them for $21, how much do I gain?

Statement. I gain the difference between $ —— and $ ——, or $ ——.

3. Make a problem giving the cost and the selling price of goods, showing a case in which the loss is required. Give the loss.

4. A man travels at the rate of 7 miles an hour. Another man starts with him and travels, in the same direction, at the rate of 5 miles an hour. How far apart are they at the end of one hour?

5. John travels at the rate of —— miles an hour; Henry, in the same direction, —— miles an hour. If they start together, how far apart are they at the end of 2 hours?

Fill the blanks and solve.

6. Mary buys a thimble for 10¢, thread for 5¢, and a tape measure for 3¢. How much should she pay for all? She gives the storekeeper a quarter. How much change should she get?

7. A man sells a cow for \$50, a sheep for \$7, and a hog for \$10. The buyer gives him a \$100 bill. How much has he overpaid him?

Notice that two questions are to be answered in solving the seventh problem. The answer to the first forms a part of the second.

First question: A man sells a cow for \$50, a sheep for \$7, and a hog for \$10. How much should he receive for all? *Ans.* \$67.

Second question: If a buyer owes a man \$67 for a cow, sheep, and a hog, and gives the man a \$100 bill, how much has he overpaid him? *Ans.* \$33.

8. A man has 125 bushels of corn, wheat, and oats. He has 25 bushels of wheat and 25 bushels of corn. How many bushels of oats has he?

What is the first question to be answered? Answer it. State the second question and answer it.

9. Make and solve a problem like the eighth, using different numbers.

10. If there are 42 pupils in the paper and cardboard cutting class and 29 pupils in the printing class, how many more pupils are there in the first class than in the second?

11. A boy goes to a store with one dollar in his pocket. He buys a knife for half a dollar, a book for a quarter of a dollar, a pad for a dime, and five pencils for a nickel. How much money has he left after paying for his purchases?

12. James makes 12 objects out of wood during the term, while John makes only 8 objects. How many more objects does James make than John?

13. If a tray is 12 inches long and 5 inches wide, how much more tape is needed to bind the two long edges than the two short edges?

14. If a box of soil when wet weighed 54 ounces and when dry 36 ounces, how much water did it lose in drying?

Sewing Problems

15. If a pillow cover is made 30 inches long and 16 inches wide, what is the difference between the length and the width?

16. If an apron is made from 30-inch gingham and 2 inches is allowed for hems and 3 inches is cut off for a band, how wide will the apron be when finished?

17. If a table cover is made 48 inches long and 36 inches wide, what is the difference between the length and the width?

Cooking and Food Problems

18. What is the difference between the cost of the following breakfast and the dinner?

Breakfast		Dinner		Supper	
Eggs	$ .25	Broiled chicken	$1.00	Omelet	$ .15
Buttered toast	.13	Potatoes	.06	Biscuits	.10
Coffee	.03	Salad	.25	Butter	.03
Milk	.02	Bread and butter	.08	Tea and sugar	.05
Sugar	.01	Rice pudding	.12	Stewed pears	.06

19. How much more does the dinner cost than the supper?

20. How much less does the supper cost than the breakfast?

21. How much more does the chicken cost than all the other items for dinner?

22. A recipe for peanut brittle requires 15¢ worth of peanuts and 6¢ worth of sugar. How much more do the peanuts cost than the sugar?

23. How much can I save by buying a barrel of flour at \$4.25 a barrel instead of in small quantities at \$6.25 a barrel?

Written

Household Problems

24. Turn to p. 26 and find the difference between the cost of furnishing the nursery and the bedroom.

25. How much more did the furniture for the dining room cost than that for the kitchen? the bedroom? the nursery?

26. How much more did the furniture for the parlor cost than that for the dining room? the kitchen? the library? the bedroom? the nursery?

27. How much more did the furniture for the library cost than that for the kitchen? the bedroom? the nursery? Which room cost least for furnishing?

28. A man had 145 tons of coal and sold 97 tons. How many tons had he left?

29. A man who had 145 tons of coal sold a number of tons, and then had 48 tons left. How many tons did he sell?

30. A man sold 97 tons of coal and then had 48 tons left. How many tons had he at first?

Notice how this problem differs from problems 28 and 29.

Buying and Selling Problems

31. If Mr. Jones bought a farm for $8765 and sold it at a loss of $297, how much did he receive for it?

STATEMENT. He received for it ——. (Indicate the operation.)

32. If Mr. Jones bought a farm for $8765, and sold it at a gain of $297, how much did he receive for it?

33. A boy bought a book for $1.75 and sold it for $1.28. How much did he lose?

34. A man bought a lot for $775 and built a house on it costing $2375. He sold both for $5000. Find his profit.

35. Henry bought a saw for $1.25 on which the dealer gained $.38. How much did it cost the dealer?

36. A man sold a house and lot for $2250 and lost $150 by so doing. How much did the house and lot cost him?

37. Make a problem showing a gain of $1.15 on something bought and sold. Solve it. Indicate the solution, using signs.

38. Make a problem giving selling price and loss, or gain, to find the cost. Solve it.

39. A man bought a horse for $125, a cow for $95, and a sheep for $23. If he sold the three animals for $225, did he gain or lose, and how much? For how much should he sell them to gain $45? to lose $30?

40. A man sold a grocer eggs for $1.50, cherries for $2.25, butter for $7.80. He bought from the grocer sugar for $3.50, vinegar for $.40, coffee for $1.20, oil for $.50, and tea for $1.12. In settlement which owed the other, and how much?

Separate into three problems and solve.

41. A newsboy buys papers for —— ¢; he sells them for —— ¢. How many cents does he gain?

Fill the blanks with the proper numbers and solve.

Earning and Spending Problems

42. A man earns $5657.75. How much does he save if he spends $2950.25?

43. A family earns $900 a year and spends $140 for rent, $105.50 for meat, $210.75 for groceries, and $240 for clothing and other expenses. How much does this family save during the year?

44. Make and solve a similar problem.

45. A boy earns $5.65 in one week and $7.80 the second week. He buys a suit of clothes for $10.50. How much has he left?

46. In one year a man earns $350. The next year he earns $375. His expenses for the two years are $460. How many dollars does he save in two years?

47. A man divides $2650 of his earnings among his wife and two children. The wife receives $1500, and the son $900 less. How much does the daughter receive?

48. A man earns $5000 a year. His expenses are $2340, and taxes $10. How much does he save?

REVIEW

1. Define difference or remainder; subtrahend; minuend.

2. If in subtracting, you add 10 to the number in any order of the minuend, what must you do to the number in the next order of the minuend? Explain why.

3. How can you prove the correctness of the result of subtraction?

4. The remainder subtracted from the minuend equals the ——.

MAKING CHANGE

Business men make change by adding. Thus, if a purchase of $1.65 is made and $5.00 is given in payment, the clerk will probably say, $1.75 (handing you a dime), $2.00 (handing you a quarter), $3.00 (handing you $1), and $5.00 (handing you $2.00). Usually the largest coins possible are selected; but many different combinations *may*, of course, be made.

Make change for the following purchases:

	Amount of Purchase	Amount Paid		Amount of Purchase	Amount Paid
1.	$ .07	$ .10	**25.**	$ 6.95	$10.00
2.	$ .12	$ .15	**26.**	$12.15	$15.00
3.	$ .18	$ .25	**27.**	$10.50	$11.00
4.	$ .38	$ .50	**28.**	$ 8.75	$ 9.00
5.	$ .26	$ .50	**29.**	$11.16	$12.00
6.	$ .78	$1.00	**30.**	$ 5.18	$ 6.00
7.	$ .56	$1.00	**31.**	$12.42	$13.00
8.	$1.35	$1.50	**32.**	$ 4.45	$ 5.00
9.	$1.67	$2.00	**33.**	$ 6.94	$ 7.00
10.	$1.89	$2.00	**34.**	$ 3.29	$ 4.00
11.	$2.44	$3.00	**35.**	$ 8.17	$10.00
12.	$3.75	$5.00	**36.**	$10.18	$11.00
13.	$4.16	$5.00	**37.**	$ 9.84	$13.00
14.	$3.18	$3.50	**38.**	$10.46	$12.00
15.	$3.59	$3.75	**39.**	$10.43	$15.00
16.	$3.73	$4.00	**40.**	$ 8.15	$10.00
17.	$5.60	$5.75	**41.**	$ 3.48	$10.00
18.	$7.14	$8.00	**42.**	$16.19	$17.00
19.	$4.35	$4.50	**43.**	$17.25	$18.00
20.	$5.16	$5.50	**44.**	$14.13	$15.00
21.	$5.50	$6.00	**45.**	$ 4.56	$10.00
22.	$7.10	$7.50	**46.**	$ 7.85	$10.00
23.	$2.59	$2.75	**47.**	$ 9.57	$20.00
24.	$3.61	$3.75	**48.**	$11.42	$20.00

MULTIPLICATION

Multiplication is the process of taking one number as many times as there are units in another.

It is a short process of finding the sum of several equal numbers.

The sign of multiplication is $\times$, and is read *times* or *multiplied by.* 15×8 is read 15 times 8, or 15 multiplied by 8. 6×5 is the same as $5+5+5+5+5+5$, or 5 taken 6 times; or it may be regarded as $6+6+6+6+6$, or 6 taken 5 times.

The **multiplicand** is the number multiplied.

The **multiplier** is the number by which the multiplicand is multiplied.

The **product** is the result of the multiplication.

The multiplier and multiplicand are **factors** of the product.

The product divided by either the multiplicand or the multiplier gives the other factor.

$$\begin{array}{l} \left.\begin{array}{rl} 62 & \textbf{Multiplicand} \\ \underline{\quad 3} & \textbf{Multiplier} \end{array}\right\} \textbf{Factors} \\ \begin{array}{rl} 186 & \textbf{Product} \end{array} \end{array}$$

A **power** of a number is the product arising from using the number two or more times as a factor.

Thus, $10 \times 10 = 100$. 100 is the second power, or the **square** of 10. 10×10 is also written 10^2 and is read 10 square, or the second power of 10. $10 \times 10 \times 10 = 1000$, or 10^3. 10^3 is the third power, or the **cube** of 10.

An **abstract number** is a number used without reference to any particular thing; as, 1, 5, 17.

A **concrete number** is one used with reference to some thing; as, 5 dollars, 4 pints, 6 men.

To the Teacher. Strictly speaking, all *numbers* are abstract, but the above distinction is convenient in explaining problems.

The multiplicand may represent a number of abstract or concrete units; the multiplier is always an abstract number. In the actual process of multiplication both factors are used as abstract numbers.

The product always represents a number of units of the same kind as are represented by the multiplicand. 3 times \$10 = \$30. 5 times 8 horses = 40 horses.

The product is not changed by changing the order of its factors in multiplying. If 1 ton of coal costs \$8, how much will 24 tons cost? *Ans.* 24 times \$8. In finding the product it is more convenient to use the smaller number as the multiplier. Thus, $8 \times 24 = 192$. *Ans.* \$192.

Multiplication Table

To the Teacher. The table gives all the products found by the multiplication of any two numbers, to and including 12×12. Pupils should learn all these products of three figures and their factors; then the products between 90 and 100 and their factors; then between 80 and 90 and their factors; and so on. No factor greater than 12 is given.

$144 = 12 \times 12$
$132 = 12 \times 11$
$121 = 11 \times 11$
$120 = 12 \times 10$
$110 = 11 \times 10$
$108 = 12 \times 9$
$100 = 10 \times 10$

$99 = 11 \times 9$
$96 = 12 \times 8$
$90 = 10 \times 9$

$88 = 11 \times 8$
$84 = 12 \times 7$
$81 = 9 \times 9$
$80 = 10 \times 8$

$77 = 11 \times 7$
$72 = \left\{\begin{array}{l} 12 \times 6 \\ 9 \times 8 \end{array}\right.$
$70 = 10 \times 7$

$66 = 11 \times 6$
$64 = 8 \times 8$
$63 = 9 \times 7$
$60 = \left\{\begin{array}{l} 10 \times 6 \\ 12 \times 5 \end{array}\right.$

$56 = 8 \times 7$
$55 = 11 \times 5$
$54 = 9 \times 6$
$50 = 10 \times 5$

$$49 = 7 \times 7$$
$$48 = \begin{cases} 8 \times 6 \\ 12 \times 4 \end{cases}$$
$$45 = 9 \times 5$$
$$44 = 11 \times 4$$
$$42 = 7 \times 6$$
$$40 = \begin{cases} 10 \times 4 \\ 8 \times 5 \end{cases}$$
$$36 = \begin{cases} 12 \times 3 \\ 9 \times 4 \\ 6 \times 6 \end{cases}$$
$$35 = 7 \times 5$$
$$33 = 11 \times 3$$
$$32 = 8 \times 4$$
$$30 = \begin{cases} 10 \times 3 \\ 6 \times 5 \end{cases}$$
$$28 = 7 \times 4$$
$$25 = 5 \times 5$$

$$24 = \begin{cases} 12 \times 2 \\ 8 \times 3 \\ 6 \times 4 \end{cases}$$
$$22 = 11 \times 2$$
$$20 = \begin{cases} 10 \times 2 \\ 5 \times 4 \end{cases}$$
$$18 = \begin{cases} 9 \times 2 \\ 6 \times 3 \end{cases}$$
$$16 = \begin{cases} 8 \times 2 \\ 4 \times 4 \end{cases}$$
$$15 = 5 \times 3$$
$$14 = 7 \times 2$$
$$12 = \begin{cases} 6 \times 2 \\ 4 \times 3 \end{cases}$$
$$10 = 5 \times 2$$
$$9 = 3 \times 3$$
$$8 = 4 \times 2$$
$$6 = 3 \times 2$$
$$4 = 2 \times 2$$

Multiplication of any number by another of several digits.

The product of one number by another number may be found by multiplying the first number by any part of the second number, and then by the other part or parts of the second number and finding the sum of the products.

STUDY RECITATION

1. Multiply 15 by 9.

$$\begin{array}{rcrcr} 15 & & 15 & & 15 \\ 9 & & 4 & & 5 \\ \hline 135 & = & 60 & + & 75 \end{array} \qquad 9 = 4 + 5$$

$23 = 3 + 20$; $123 = 3 + 20 + 100$; $1123 = 3 + 20 + 100 + 1000$, etc.

2. Multiply 256 by 123.

	A.	*B.*	
Multiplicand	256	256	
Multiplier	123	123	
1st partial product	768	768 =	3 × 256
2d partial product	512	5120 =	20 × 256
3d partial product	256	25600 =	100 × 256
Complete product	31488	31488	123 × 256

In *A* the 0's at the right of the second and third partial products are omitted.

In *B* each partial product is written out in full.

3. Multiply $2.56 by 123.

```
 $2.56
   123
  7 68
 51 2
256
$314.88
```

Multiply as before and mark off in the product two places at the right for cents.

Written

Multiply as under *A* above:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
4.	13 21	907 23	308 29	24301 21	897 86	493 587
5.	578 42	2341 12	599 68	258 495	956 89	246 35
6.	234 222	485 987	426 728	678 496	14232 122	856 629

The product of a number by 10 *or any power of ten is found by annexing the zeros of the multiplier to the multiplicand.*

Multiply the following numbers by 10; by 100; by 1000.

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
7.	15	46	568	972	649	321
8.	25	36	744	857	431	432

9. Multiply:

a.	*b.*	*c.*
49	5123	\$50.40
430	403	9008
1470	15369	403 20
196	20492	45360
21070	2064569	\$454003.20

To the Teacher. Be sure that the pupil understands and can explain these processes.

Multiply:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
10.	567	785	434	687	3006	\$16.78
	20	30	40	50	800	230
11.	547	309	542	6321	\$54.39	\$70.04
	608	103	904	4008	9006	6009

$25 = 100 \div 4$; $125 = 1000 \div 8$. Hence,

To multiply a number by 25, *annex two zeros and divide the product by* 4. *To multiply a number by* 125, *annex three zeros and divide the product by* 8.

Thus, $25 \times 652 = 65200 \div 4 = 16300$. $125 \times 652 = 652000 \div 8 = 81500$.

Multiply first by 25; then by 125.

12. 480 568 792 496 576 928 672

The product of one number by another may be found by multiplying the first number by one of the two factors of the other number, and the product by the other factor.

Thus, $15 \times 21 = 3 \times 5 \times 21$; $3 \times 21 = 63$; $5 \times 63 = 315$.

Multiply in the same way:

	a.	*b.*	*c.*	*d.*
13.	43 by 48	165 by 25	457 by 35	658 by 56
14.	54 by 72	409 by 36	612 by 24	597 by 86
15.	59 by 54	213 by 32	229 by 49	314 by 21
16.	63 by 72	315 by 28	314 by 63	816 by 55

Multiplicands		Multipliers		Multiplicands		Multipliers	
1.	349	*a.*	45	*11.*	$ 4.55	*k.*	34
2.	862	*b.*	67	*12.*	6.74	*l.*	217
3.	557	*c.*	233	*13.*	26.98	*m.*	359
4.	784	*d.*	409	*14.*	154.40	*n.*	415
5.	2105	*e.*	500	*15.*	203.05	*o.*	605
6.	3009	*f.*	708	*16.*	607.82	*p.*	500
7.	6500	*g.*	890	*17.*	915.40	*q.*	718
8.	7050	*h.*	600	*18.*	432.18	*r.*	209
9.	7509	*i.*	544	*19.*	500.09	*s.*	3104
10.	8000	*j.*	612	*20.*	607.05	*t.*	1006

17–116. Multiply each number from *1* to *10* by each multiplier from *a* to *j*. Thus, *1 a*, 349 × 45; *1 g*, 349 × 890; *9 i*, 7509 × 544.

117–216. Multiply each number from *11* to *20* by each multiplier from *k* to *t*.

PROBLEMS

Oral

Cooking Problems

1. Allowing 15 minutes for each pound, how long will it take to roast a 10-pound turkey?

2. At what time must a 5-pound chicken be put into the oven, in order to serve it at 1 o'clock, if each pound requires 15 minutes for roasting?

3. How much will it cost to make a custard of 3 eggs at 48¢ a dozen, 2 cups of milk at 2¢ a cup, and $\frac{1}{4}$ cup of sugar at 4¢ a cup?

4. If a class of 26 girls each use 6 tablespoonfuls of butter for baking, how much is used by the class?

5. If it requires 4 eggs and 4 tablespoonfuls of milk to make an omelet, find the cost of an omelet 4 times as large, when eggs are 24¢ a dozen and milk 4¢ a pint (4 tablespoonfuls = $\frac{1}{8}$ pint).

Sewing Problems

6. Find the cost of making a shirtwaist requiring 3 yards of lawn @ 18¢, a pattern for 15¢, and a spool of thread for 5¢.

7. Find the cost of making a petticoat requiring 4 yards of lawn @ 15¢ and 10 yards of lace @ 7¢.

8. Edith made a dress of 6 yards of gingham @ 15¢, 4 yards of lace @ 10¢, and buttons for 15¢. Find the cost of the dress.

9. Find the cost of supplying a class of 8 girls each with 2 yards of cambric at 12¢ a yard.

10. If each girl in a class of 20 needs 1 thimble at 12¢, and 1 package of needles at 4¢, find the total cost of these articles for the class.

Shopping Problems

11. Find the total cost of 3 pairs of gloves @ \$1, 5 yards of muslin @ 10¢, and 4 yards of ribbon @ 25¢.

12. Find the cost of 2 pairs of shoes @ \$2, 3 pairs of hose @ 30¢, and 4 collars @ 25¢.

13. Find the cost of 5 yards of ruching @ 12¢, 2 dozen buttons @ 25¢, and 2 spools of sewing silk at 9¢.

14. Find the cost of 5 yards of lace @ 10¢ and 2 yards of velvet @ $1.25.

15. How much change shall I receive from a $1 bill if I purchase 5 yards of lawn @ 12¢ and 2 yards of ribbon @ 10¢?

Household Problems

Find the cost of:

16. 8 chairs @ $2.50.

17. 12 plates @ $.15.

18. 2 pairs curtains @ $3.50.

19. 5 tables @ $5.20.

20. 15 glasses @ $.12.

21. 4 brooms @ $.28.

22. 5 rugs @ $25.

23. 4 table covers @ $1.25.

24. 15 knives @ $.60.

25. 12 spoons @ $.24.

Marketing Problems

26. Find the cost of 5 bushels of potatoes @ 75¢ and 5 dozen egg plants @ 75¢.

NOTE. Since $5 \times 75¢ + 5 \times 75¢ = 10 \times 75¢$, what is the shortest way of solving this problem?

27. How much change should I receive from $5 for a 2-pound chicken @ 25¢ and a 10-pound turkey @ 30¢?

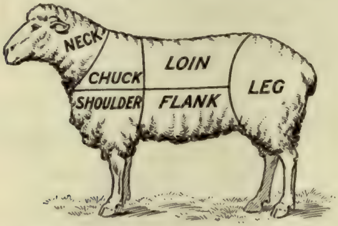
28. How much change should be given from $2 for 5 pounds of butter @ 25¢ and 2 dozen eggs @ 25¢?

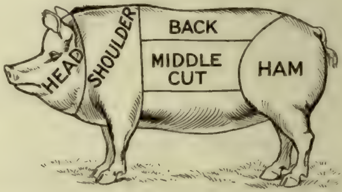
29. Find the cost of 2 barrels of flour @ $4.50 and 2 bushels of potatoes @ 50¢.

30. Find the cost of 6 pounds of roast beef @ $.25 and 6 pounds of lamb chops @ $.25.

31. Find the cost of a forequarter of lamb consisting of neck, chuck, shoulder, and flank, 8 lb. @ $12\frac{1}{2}$ ¢.

32. Find the cost of a hindquarter of lamb consisting of loin and leg, 11 lb. @ 20 ¢.





The picture at the right represents the general method of cutting up a side of pork. Find the value of the pork bought at retail as follows:

33. 9 lb. ham @ 20 ¢.

34. 10 lb. back (ribs and loin) @ 20 ¢.

35. 5 lb. shoulder @ $15\frac{1}{2}$ ¢.

36. 8 lb. middle cut @ 16 ¢.

37. 7 lb. head @ 10 ¢.

38. The part below the middle cut is salted or pickled or made into sausages. Find the value of 7 lb. of this part @ 14 ¢.

39. A schoolgirl planned and bought the following Thanksgiving dinner for a family of five. Find the total cost of the dinner.

Turkey	$2.50	Mince pie	$.50
Potatoes	.10	Nuts and raisins	.40
Vegetables	.50	Fruit	.50
Celery	.10	Ice cream	.40
Cranberries	.15	Cakes	.20

40. Find the cost of the number of pounds of each cut of beef in example 35, p. 25, at the price per pound in example 34, p. 25.

Woodworking and Cardboard Problems

41. Find the cost of 12 drawing boards @ 35¢.

42. Find the cost of one try square for each pupil in a class of 18 at 20¢ apiece.

43. How much will it cost to furnish a class of 20 pupils with one bit brace each at 85¢ apiece?

44. At $1.25 apiece, how much will it cost to supply 12 pupils each with one saw?

45. How many square inches of cardboard will be required to make 12 cards each 4 inches by 6 inches?

Farm Problems

46. Mr. Bell raised 125 chickens, Mr. Grant twice as many, and Mr. Burt 50 more than the other two farmers. How many did they all raise?

47. If a cow gives 12 quarts of milk a day, how many quarts will a farmer get in 5 days from 10 cows?

48. A farmer sells 5 loads of wheat of 60 bushels each at $1 a bushel. How much does he get for the wheat?

49. How much will a farmer receive for 3 hogs at $10 each and 50 chickens at 30¢ each?

50. How much is gained by spraying an apple tree, if on account of being sprayed it bears 6 bushels more apples at $1 a bushel? The spraying costs $.25.

51. A farmer sowed 14 pounds of red clover seed per acre on 20 acres. How many pounds of clover seed did he use?

United States Postal Rates

Domestic Postage Rates

First-class. Letters and sealed matter: 2¢ for each ounce or fraction. Postal cards and post cards: 1¢ each.

Second-class. Newspapers and periodicals: 1¢ for each four ounces or fraction.

Third-class. Miscellaneous printed matter: 1¢ for each two ounces or fraction.

Fourth-class. All matter not included in first three classes: 1¢ for each ounce or fraction. For fourth-class matter weighing more than 4 ounces, see Parcel Postage, p. 49.

Special Delivery: A special-delivery stamp (or 10¢ in ordinary stamps when the letter or parcel is marked "special delivery"), in addition to postage, secures immediate delivery within the carrier delivery limit of city free delivery and within one mile of any other United States post office.

Registered Mail: 10¢ extra for each package.

Foreign Postage Rates

Letters and Sealed Matter: 5¢ for the first ounce, and 3¢ for each additional ounce or fraction.

Postal Cards: Single, 2¢ each; double, 4¢ each.

Commercial Papers: 1¢ for each two ounces or fraction, but not less than 5¢ on each packet.

Printed Matter: 1¢ for each two ounces or fraction.

Samples of Merchandise: 1¢ for each two ounces or fraction, but not less than 2¢ on each packet.

Find the postage on the following articles:

	Domestic Article	Weight		Foreign Article	Weight
52.	Letter	2 oz.	**58.**	Letter	2 oz.
53.	Letter	$2\frac{1}{2}$ oz.	**59.**	Letter	$2\frac{1}{2}$ oz.
54.	Newspaper	16 oz.	**60.**	Newspaper	18 oz.
55.	Calendar	6 oz.	**61.**	Calendar	6 oz.
56.	Magazine	12 oz.	**62.**	Magazine	12 oz.
57.	Registered letter	2 oz.	**63.**	4 double post cards	

Note. The abbreviation oz. means ounce or ounces; lb., pound or pounds; mi., mile or miles.

United States Parcel Post

Fourth-class mail matter embraces all other matter, including farm and factory products, not now embraced by law in either the first, second, or third class, which is not greater in size than 72 inches in length and girth combined, nor in form or kind likely to injure the person of any postal employee, or damage the mail equipment or other mail matter, and not of a character perishable within a period reasonably required for transportation and delivery.

The limit of weight for parcels of fourth-class matter for local delivery and for delivery at other post offices within the *first and second* zones is 50 pounds. The limit of weight for parcels intended for delivery in *other than the first and second zones* is 20 pounds.

Parcels weighing 4 *ounces or less* are mailable at the rate of 1 ¢ for each ounce or fraction of an ounce, regardless of distance. Parcels weighing more than 4 ounces are mailable at the pound rates shown in the following table, a fraction of a pound being considered a full pound.

Books weighing 8 ounces or less are mailable at 1 ¢ for each 2 ounces. On books weighing more than 8 ounces the pound rates shown in the table apply.

The sender of a C.O.D. parcel on which the postage is prepaid may have the price (up to $100) collected of the addressee on payment of a fee of 10 ¢ offered in stamps. This fee includes *insurance* up to $50.

For the purpose of carrying this law into effect, the United States is divided into zones with different rates of postage applicable to each, as follows:

TABLE OF RATES

Weight	1st zone		2d zone, 50 to 150 miles	3d zone, 150 to 300 miles	4th zone, 300 to 600 miles	5th zone, 600 to 1000 miles	6th zone, 1000 to 1400 miles	7th zone, 1400 to 1800 miles	8th zone. all over 1800 miles
	Local Rate	Zone rate, 50 miles							
1 pound	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2 pounds	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 pounds	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 pounds	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 pounds	.07	.09	.09	.14	.23	.32	.41	.51	.60
6 pounds	.08	.10	.10	.16	.27	.38	.49	.61	.72
7 pounds	.08	.11	.11	.18	.31	.44	.57	.71	.84
8 pounds	.09	.12	.12	.20	.35	.50	.65	.81	.96
9 pounds	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10 pounds	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11 pounds	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
20 pounds	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
50 pounds	.30	.54	.54						

NOTE. For packages weighing from 20 to 50 pounds add 1 ¢ for each pound or 2 pounds local rate, and 1 ¢ for each pound in the first and second zones.

Find the postage necessary for each of the following parcels.

	Weight	Distance		Weight	Distance
64.	4 oz.	50 mi.	**72.**	3 lb.	150 mi.
65.	1 lb.	150 mi.	**73.**	4 lb.	300 mi.
66.	2 lb.	1800 mi.	**74.**	6 lb.	600 mi.
67.	5 lb.	2900 mi.	**75.**	7 lb.	1000 mi.
68.	11 lb.	300 mi.	**76.**	10 lb.	1400 mi.
69.	12 lb.	1000 mi.	**77.**	9 lb.	1800 mi.
70.	20 lb.	600 mi.	**78.**	10 lb.	2000 mi.
71.	50 lb.	100 mi.	**79.**	5 lb.	3000 mi.

The express rates between Indianapolis and the following points are:

	Miles	5 lb.	10 lb.	20 lb.
New York, N.Y. . .	800	$.30	$.41	$.62
Chicago, Ill.	180	.24	.28	.36
St. Louis, Mo. . . .	240	.25	.30	.41
Denver, Col.	1100	.40	.60	1.00
Butte, Mont.	1700	.53	.86	1.53
San Francisco, Cal. .	2400	.66	1.12	2.04

Consult the table of parcel post rates and tell which is the cheaper method of sending the following packages from Indianapolis.

5 lb. to	10 lb. to	20 lb. to
80. New York	**86.** New York	**92.** New York
81. Chicago	**87.** Chicago	**93.** Chicago
82. St. Louis	**88.** St. Louis	**94.** St. Louis
83. Denver	**89.** Denver	**95.** Denver
84. Butte	**90.** Butte	**96.** Butte
85. San Francisco	**91.** San Francisco	**97.** San Francisco

Written

98. Find the cost of 18 tons of coal at $ 5.75 a ton.

99. Find the cost of 32 yards of silk at $ 2.25 a yard.

100. How many minutes equal an hour? a day? a week?

101. There are 40 tablespoonfuls of cornstarch in a $.15 package. When eggs are $.36 a dozen, how much may be saved on 20 puddings requiring 4 eggs each, by substituting 4 tablespoonfuls of cornstarch for 4 eggs?

102. If a railway train runs 35 miles an hour, how far can it go in 4 days?

103. A fruit dealer bought 12 bushels of cranberries at $ 2.25 a bushel and sold them at 10 cents a quart. How much did he gain?

104. From the product of 2987 and 78 take their sum.

105. I paid $ 36 an acre for 50 acres of woodland. I sold the wood for $ 1576 and the land for $ 17 an acre. Did I gain or lose, and how much? Solve in two ways.

106. 15 bushels of apples were purchased at $1.25 per bushel. If they were sold at 45 cents a peck, was there a gain or a loss by the sale, and how much? Solve in two ways.

107. I sell to a dealer 14 cords of hard maple at $ 5.75 per cord and buy of him 8 tons of hard coal at $ 6.50 a ton. What sum is due me?

108. What change should be received from $ 50, in payment for 16 dozen buttons at 36 ¢ a dozen, 6 yards of velvet at $ 4.75 a yard, and 15 yards of lining at 18 ¢ a yard?

109. If a mill weaves 76,080 yards of cloth in a day, how many yards will it weave in a year of 313 working days?

110. Two express trains leave St. Louis at the same time. One travels east at the rate of 50 miles an hour, the other west at the rate of 45 miles an hour. How far apart will they be in 12 hours?

111. If the daily pay of a railroad conductor is $3.45, what is his salary for a year of 340 working days? If he spends $12.75 a week, how much can he save in a year of 52 weeks?

Farm Problems

The following is a statement of the receipts of sheep in St. Paul, on Nov. 4, 1913.

Spring Lambs	
No.	Price
54	$6.75
15	6.75
2	6.75
12	6.75
28	6.75
48	5.50
23	5.25
20	5.25
20	5.25
55	5.25
11	5.25

Ewes	
No.	Price
75	$4.00
10	4.00
47	4.00
17	4.00
8	4.00
9	4.00
5	4.00
15	3.00
3	2.50
Bucks	
1	3.00
1	3.00

112. Find the total value of the lambs.

113. Find the total value of the ewes.

114. Find the total value of the bucks. Find the total value of the receipts for the day.

115. Two farmers each have 3 acres of potatoes. One farmer sprays his and gets 212 bushels per acre; the other farmer fails to spray his and gets 183 bushels per acre.

The first farmer gets 56 cents per bushel, but, because of poorer potatoes, the second farmer gets only 48 cents per bushel. What is the difference in the value of the two crops?

116. The following ration is suggested as a daily ration for a 1300-pound farm draft horse: 10 pounds of alfalfa hay, 7 pounds of shredded corn stover, and 14 pounds of cracked corn. How many pounds of each would be fed the horse during the months of October and November?

117. If each of a flock of 110 hens lays 150 eggs a year, what are the returns from the flock at 27 ¢ per dozen for eggs?

The following data cover the cost of erecting a silo on the Experiment Farm at Crookston, Minn. Find the cost of (*a*) the foundation, (*b*) the reënforcement, (*c*) the body, (*d*) the roof, doors, and chute.

118. Cost of Foundation:

30 hours' labor @ 15 ¢
22 sacks cement @ 50 ¢
5 yards gravel @ $ 1.10
3 days' board @ 50 ¢

119. Cost of Reënforcement:

401 pounds iron rods @ 3 ¢

120. Cost of Body:

3487 clay blocks	$ 160.67
Hauling blocks	8.00
6 barrels lime @ $ 1.50	
38 sacks cement @ 50 ¢	
110 hours' labor, bricklayer, @ 35 ¢	
171 hours' labor, helper, @ 15 ¢	
28 days' board @ 50 ¢	

121. Cost of Roof, Doors, and Chute :

Lumber	$ 32.00
Hardware	1.50
Labor	30.00
10 days' board @ 50 ¢	

122. Find the total cost of the silo.

123. A farmer sowed a field of 30 acres to oats. The following were his main items of expense : A man and three horses plowed 3 acres per day, at a cost of $ 3.50 per day. What was the cost of plowing the 30 acres?

124. To put the field in condition for seeding required the work of a man and team for 12 days. The seeding and harrowing required 4 more days of labor on the part of a man and team. Find the total cost at $ 3.00 per day.

125. The oats were seeded at the rate of 3 bushels per acre, at 60 ¢ per bushel. Find the cost of seed for 30 acres.

126. Find the cost of harvesting the 30 acres, at an average of $ 1.25 per acre.

Note. At an "average" means that although some acres may cost a little more and some a little less than $1.25, the 30 acres together cost 30 × $1.25.

127. The average yield for the 30 acres was 50 bushels per acre. The crop was threshed from the shock at a cost of 6 ¢ per bushel. Find the total cost of the threshing.

128. The cost of marketing was 2 cents per bushel, and the oats sold at 43 cents per bushel. What was the net price received for 1500 bushels of oats?

129. If $ 105 is paid for rent of the field, $ 35 for plowing, $ 48 for seeding and harrowing, $ 54 for seed, $ 37.50 for harvesting, and $ 90 for threshing, how much profit is there on 1500 bushels of oats that sell for 41 cents net per bushel?

REVIEW

1. Define multiplication; multiplicand; multiplier; product.

2. What are partial products?

3. In the process of multiplication, where is the first figure in each partial product written?

4. Multiply 24 by 32. What is the first partial multiplier? What is the first partial product? What is the second partial multiplier? What is the second partial product? What is the complete product? Show that the sum of the two partial multipliers equals the complete multiplier. Show that the second partial product, if written in full, equals $24 \times 3 \times 10$. The complete product, 768, is the sum of what two products? Why is this sum the complete product?

Problems without Numbers

5. Having given the product and the multiplier, how can you find the multiplicand?

6. Having given the product and the multiplicand, how can you find the multiplier? Illustrate each case.

7. Having given the number of tons of any article and the cost per ton, how can you find the entire cost? Which number is the multiplicand? What is the denomination of the product? Illustrate.

8. Having given the cost of a quantity of coal and the number of tons, how can you find the cost of one ton? Illustrate.

9. Having given the cost of a quantity of coal and the cost of one ton, how can you find the number of tons? Illustrate.

BILLS

1. INDIANAPOLIS, IND., Feb. 1, 1914.

MR. CHARLES R. BROWN,

Bought of R. H. HOLD & COMPANY

DEALERS IN STAPLE AND FANCY GROCERIES, 34 GRAND AVE.

Jan. 2	19 lb. Granulated Sugar		$1.00
Jan. 2	12 lb. California Prunes	@ $.14	1.68
Jan. 7	15 lb. Carolina Rice	@ .08	1.20
Jan. 9	5 doz. Eggs	@ .17	.85
			$4.73

Received payment,
R. H. HOLD & CO.
Per H. HANSON.

A **bill** is a written statement in detail of indebtedness for goods sold, or services performed.

The **price** is the sum paid for 1 pound, 1 yard, etc., of an article, while the **cost** is the sum paid for the entire quantity.

Every bill should contain the **place** and the **date** of the sale, the names of the **buyer** and the **seller**, the **name**, the **price**, and the **quantity** of each article sold, the **cost** of the **entire quantity** of each article sold, and the **total amount** of all the articles sold.

ABBREVIATIONS OFTEN USED IN BILLS

@	*at* a given price per pound, yard, etc.
acct.	account
amt.	amount
bal.	balance
bu.	bushel or bushels
C. O. D.	cash on delivery
doz.	dozen
ft.	foot or feet
gal.	gallon
in.	inch or inches
lb.	pound or pounds
mdse.	merchandise
no.	number
payt.	payment
pk.	peck or pecks

pt.	pint or pints	rec'd.	received
qt.	quart or quarts	yd.	yard or yards

The sign ″ is often used for inch or inches, and ′ for foot or feet.

To **foot** a bill is to find the total amount of all the articles.

A **debtor** (Dr.) is one who owes a debt.

A **creditor** (Cr.) is one to whom a debt is owed.

A **receipt** is a written statement that a bill has been paid.

A bill is receipted when the creditor or seller or his agent writes "Paid" or "Received payment" and adds his **signature.** This is usually done at the bottom of the bill. When was the bill on p. 56 made out? Who was the debtor? the creditor? Who bought the goods? Who sold the goods? Who received payment for the goods? Who made out the bill? Who receipted it? After the payment is made, who should have the bill? Of what use will it be to him? What does "Per H. Hanson" mean?

The following is a receipted bill for services:

2. WILLIAM G. QUICK,

FORT WAYNE, INDIANA,

MR. J. G. HURD, Jan. 2, 1914.

FORT WAYNE, INDIANA

Dec.	15	To repairing basement bell. Time	1	50
		Received payment, WILLIAM G. QUICK		

3. Get a billhead from your grocer and make out a bill for several groceries. Receipt it.

4. Make out a doctor's or a dentist's bill for services.

Make out, foot, and receipt bills for the following items:

5.	7 yd. apron gingham	@	$.09
	10 yd. flannelette	@	.11
	13 yd. cretonne	@	.23

6.	3 packages crewel needles	@	$ .10
	6 skeins cotton thread	@	.04
	1 ball D. M. C.	@	.17
	1 embroidery hoop	@	.15
	1 stiletto	@	.10
7.	7 yd. embroidery insertion	@	$ .35
	3 yd. embroidery edging	@	.45
	$2\frac{1}{2}$ in. all-over embroidery	@	1.35
	11 yd. seam binding	@	$.12\frac{1}{2}$
8.	Misses' serge suit	@	$17.50
	Misses' blouse	@	1.75
	Altering suit coat		1.50
	Tan shoes	@	3.50
	Tan hose, 3 pairs	@	.35
	Tan rubbers	@	1.00
9.	12 yd. homespun	@	$ 1.59
	7 yd. silesia	@	.18
	7 yd. velvet	@	1.75
	4 yd. brocade silk	@	2.39
10.	3 patterns	@	$ .15
	$7\frac{1}{2}$ yd. percale	@	$.12\frac{1}{2}$
	12 yd. galatea	@	$.16\frac{1}{2}$
	6 yd. chambray	@	.25
11.	15 yd. outing flannel	@	$ $.12\frac{1}{2}$
	5 yd. denim	@	.18
	20 yd. curtain scrim	@	.14
	2 pairs curtain rods	@	.35
12.	$2\frac{1}{4}$ yd. double damask	@	$ 1.50
	1 doz. 18 in. napkins	@	3.25
	2 tray cloths	@	1.23
	1 luncheon cloth	@	1.49

13.

25 yd. linen toweling	@	$.18
20 yd. glass toweling	@	.15
1 doz. bath towels	@	.35
$\frac{1}{2}$ doz. wash cloths	@	.07

14. Make out, foot, and receipt bills for the following material used in the making of a footstool:

$1\frac{1}{2}$ ft. oak (for legs)	@	$.10
$2\frac{1}{4}$ ft. oak (for rails)	@	.08
$2\frac{1}{2}$ sq. ft. leather	@	.20
$\frac{1}{4}$ pt. stain	@	.40

15. Find the cost of the following material used in making a dress:

5 yd. voile	@	$.75
7 yd. lace	@	.30
5 yd. ribbon	@	.18
1 card hooks and eyes	@	.05
1 doz. buttons	@	.10
1 spool thread	@	.05

16. Foot and receipt the following bill:

Chicago, Ill., Sept. 10, 1913.

School Supply Co.

Sold to Board of Education

Terms Cash

8 lb. rug yarn	@	$.60
10 lb. plain chenille yarn	@	.60
20 lb. wool warp	@	.52
12 spools cotton carpet warp	@	.18
2 doz. weaving needles	@	.45

17. Make, foot, and receipt a bill for the following material:

5 lb. No. 1, reed	@	$.85
6 lb. No. 2, reed	@	.55
3 lb. No. 4, reed	@	.45
2 lb. raffia	@	.20
2 lb. raffia, colored	@	.52

18. Make, foot, and receipt a bill for notebook covers requiring the following material:

2 bundles cloth braid	@	$1.10
1 bundle straw board	@	1.00
20 yd. black binder's cloth	@	.16
5 yd. brown binder's cloth	@	.18
5 quires marbled paper	@	.30
20 lb. heavy cover paper	@	.12
10 lb. dry paste	@	.08
4 qt. glue	@	.70
2 boxes eyelets	@	.20

19. Make and receipt two bills, choosing your own articles and prices.

ACCOUNTS

An **account** is a collection of related debits and credits, arranged under an appropriate heading, as, "Cash," "John T. Gray," etc.

Every account has two sides. On the left side are entered all **debits** and on the right side all **credits**.

1. For forms of accounts see pages 62 and 63. The following is a statement of an account with a dry-goods store:

B. ALTMAN & CO.
FIFTH AVENUE, 34 AND 35 STREETS,
NEW YORK, June 1, 1914.

MRS. A. ARNOLD,
32 W. 65 STREET, NEW YORK.

Settlements required the first part of each month.

				Dr.		*Cr.*	
May	1	Account rendered		100	25		
	5	10 yd. serge	1.25	12	50		
	16	1 pair shoes		4			
	25	1 coat		35			
		Cr.					
	6	**10 yd. serge**	**1.25**			**12**	**50**
	26	**1 coat**				**35**	
				151	75	47	50
		Balance due				104	25

NOTE. The dollar sign is often omitted, as above. The black type, representing credits, is printed in red on the statements.

2. Notice how the following form differs from example 1.

L. S. AYRES & COMPANY
WASHINGTON STREET, INDIANAPOLIS, IND.,
Dec. 31, 1913.

MRS. FRANK GAYNOR,
1856 NORTH PENNSYLVANIA STREET.

Goods are sold at cash rates and payment required early part of following month.

1913				DAILY TOTAL	CREDIT
Dec.	9	1 piece ribbon	.27		
		1 jacket	12.00	12.27	
	13	**1 piece ribbon**			**.27**
	18	1 bowl		7.00	
	19	1 doll		5.50	
	20	**1 bowl**			**7.00**
		Balance due			17.50
				24.77	24.77

A **cash account** is an account of the money that is received and paid by a person, firm, etc.

It is like an account between yourself and your cash. Cash is the debtor for all money that you receive and the creditor for all money that you pay out. The receipts are therefore written on the left, or *debit side* and the expenditures on the right, or *credit side.*

The "Balance, Cash on hand," added to the payments, must equal the sum of the receipts.

3. The following is a cash account of a boy for May, 1914, kept on two pages of a cash book :

1914		RECEIPTS			1914		PAYMENTS		
May	1	Cash on hand	5	49	May	2	Saw	1	25
	9	Selling papers		35		5	Ruler		10
	15	Carrying bags		25		10	Carfare		10
	15	Monthly allowance	2	00		19	Book	1	15
	20	Helping Mr. Hall		75		31	Balance	6	24
			8	84				8	84
June	1	Balance	6	24					

4. The above account may also be kept as follows on one page of an account book.

Dr. CASH *Cr.*

1914					1914				
May	1	Cash on hand	5	49	May	2	Saw	1	25
	9	Selling papers		35		5	Ruler		10
	15	Carrying bags		25		10	Carfare		10
	15	Monthly allowance	2	00		19	Book	1	15
	20	Helping Mr. Hall		75		31	Balance	6	24
			8	84				8	84
June	1	Balance	6	24					

Cash is the boy's **debtor** for the five receipts of money and his **creditor** for the four payments. At the end of the month cash is his debtor for the balance of cash on hand, viz. $6.24.

5. Mrs. Hunt bought of Marshall, Field & Co., Oct. 1, 3 yd. of lace @ $.25; Oct. 5, 1 pair of shoes @ $3.50; Oct. 9, 1 suit @ $25.16; Oct. 15, 1 rug @ $55.40; Oct. 25, 5 yd. of voile @ $1.50. She returned for credit, Oct. 6, 1 pair of shoes @ $3.50, and on Oct. 16, 1 rug @ $55.40. Balance the account.

6. Alice Hart had on hand, March 1, $67.85. March 2, she received her monthly allowance of $1; March 10, she earned $.25 by running errands; March 15, she earned $.50 by weeding and raking the garden. She spent $.25, March 3, for a thimble; $.15, March 4, for thread and needles; $.75, March 10, for a book. Balance her account.

7. May 3, Charles Gordon spent for lumber for chicken coops, $4.25; June 4, for chicken feed, $.65; June 25, for corn meal, $2.50; Aug. 15, for corn, $3.50. May 31, he received for eggs for the month, $2.75; June 30, $1.95; July 31, $2.25; Aug. 31, $2.00; Sept. 10, for chickens, $8.00; Sept. 31, $7.75. Find his profit Oct. 1.

8. Find the balance of the following cash account:

Dr. CASH *Cr.*

1914					1914				
Sept.	1	Cash on hand	115	25	Sept.	1	Rent	15	00
	3	Cows	175			10	Coat	10	00
	5	Milk	50	60		22	Provisions	25	63
	10	Butter	48	34		30	Coal	12	75
	25	Eggs	15	75					

Write out the following accounts in proper form, with the heading, "Cash," rule the paper as indicated above, choose

articles representing the various receipts and payments, supply dates where omitted, and balance the accounts:

9. Feb. 1 Frank Crane has on hand $33.45. Feb. 4 he receives $5.25; Feb. 6, $.75; Feb. 12, $4.25; Feb. 24, $.64. Feb. 5 he pays out $5.16; Feb. 7, $.64; Feb. 9, $4.75; Feb. 11, $1.60.

10. A merchant had on hand June 1, $2075.16; June 1, he sold $216.75; June 2, he sold $356.44; June 3, $425.16; June 4, $405.17; June 5, $395.18; June 6, $502.45. June 1, he paid $150 for rent; June 4, he paid $125.16 for goods; June 15, $150 for salaries. Balance the account.

11. Receipts: Jan. 3, $5.67; Jan. 4, $16.54; Jan. 16, $156.38; Jan. 21, $204.09; Jan. 28, $500. Payments: Jan. 4, $15.65; Jan. 7, $100.75; Jan. 10, $43.54.

12. Receipts: March 3, $500.75; March 10, $200.40; March 18, $300.45; March 25, $15.94. Payments: March 6, $100.50; March 12, $198; March 20, $17.45.

13. Receipts: $15.75, $60.50, $40.30, $20.79. Expenses: $3.14, $5.00, $6.75, $8.27.

14. Receipts: $100.40, $200.55, $300.54, $600. Expenses: $55, $63.42, $71.19, $83.74.

15. Receipts: $253.64, $129.76, $205.06, $143.80. Expenses: $100.05, $63.18, $104.03, $200.75.

16. Receipts: $375.81, $254.13, $217.84, $116.95. Expenses: $215.63, $97.45, $111.95, $200.

17. Receipts: $491.67, $116.38, $100.09, $25.63. Expenses: $314.44, $89.29, $92.74, $125.50.

18. Make out and balance a cash account of your own for a month.

DIVISION

Division is the process of finding how many times one number is contained in another, or the process of separating a number into equal parts.

The **dividend** is the number divided.

The **divisor** is the number by which the dividend is to be divided.

The **quotient** is the result of division.

The **remainder** is the undivided part of the dividend, when the division is not exact.

Divisor 8)26 **Dividend**
Quotient 3, 2 **Remainder**

The sign of division is ÷, and is read *divided by*. 18 ÷ 9 is read, 18 divided by 9.

To the Teacher. Review the multiplication and division tables.

The process of division gives the number of times the divisor is contained in the dividend. It also divides the dividend into as many equal parts as there are units in the divisor, and gives the value of one of the parts.

Thus, in 24 ÷ 6 = 4, the division shows that 6 is contained in 24, 4 times, or that 24 is divided into six equal parts, the value of each part being 4.

Short division is the process of division when the work is performed mentally, the quotient only being written.

Study Recitation

1. 2)4168
   2084

2 in 4 (thousands), 2 (thousands). Write 2 under thousands.
2 in 1 (hundreds), 0 (hundreds). Write 0 under hundreds. Carry 1.
2 in 16 (tens), 8 (tens). Write 8 under tens.
2 in 8 (units), 4. Write 4 under units.

Say: 2 in 4, 2; 2 in 1, 0; 2 in 16, 8; 2 in 8, 4.

Test the result by multiplying the quotient by the divisor.

Written

Solve the following and test results. In testing, add the remainder to the product.

	a.	*b.*	*c.*	*d.*
2.	2)476	2)8376	3)40890	3)\$890.67
3.	4)1804	4)71084	5)\$2800.75	5)\$724.30
4.	6)34062	7)13062	8)\$1764.32	9)\$8340.66

5. Divide mentally by 10: 240, 2500, 265, 7495.

NOTE. When the figures cut off are not all 0's, the division is inexact. In such cases the figures cut off represent the remainder. Thus, $2500 \div 100 = 25 | 00 = 25$. $7495 \div 100 = 74 | 95$; quotient 74, remainder 95.

6. Divide mentally by 100 : 300, 4000, 3240, 56525.

7. Divide mentally by 1000 : 4000, 9500, 60000, 54634.

To divide by 10, *cut off one figure from the right; by* 100, *two figures; by* 1000, *three figures, etc.*

8. Divide 7800 by 600.

9. Divide 7935 by 600.

6|00)78|00	6|00)79|35
13	$13\frac{135}{600}$

In example 9, cut off the zeros at the right in the divisor, and the same number of figures at the right in the dividend. Divide 35, the number remaining in the dividend, by 6, the number remaining in the divisor; to the remainder 1, annex 35, the figures cut off, and write this number over the divisor, thus, $\frac{135}{600}$, as a part of the quotient.

	a.	*b.*	*c.*
10.	$3600 \div 60$	$2800 \div 700$	$3600 \div 400$
11.	$3550 \div 50$	$32800 \div 800$	$357000 \div 7000$

LONG DIVISION

Study Recitation

To the Teacher. Teach the pupils, when the second figure of the divisor is large, to add 1 to the first figure before attempting to divide. If necessary, review the drills for finding quotient figures given in the author's "First Book."

1. Divide 86523 by 421.

$$\begin{array}{r} 205\frac{218}{421} \\ 421\overline{)86523} \\ \underline{842} \\ 2323 \\ \underline{2105} \\ 218 \end{array}$$

2. Divide 81970 by 630.

$$\begin{array}{r} 130\frac{70}{630} \\ 63|0\overline{)8197|0} \\ \underline{63} \\ 189 \\ \underline{189} \\ 7\ 0 \end{array}$$

To the Teacher. If the pupil does not understand these processes, review carefully the work given in the "First Book."

Steps in Division: (1) Divide; (2) multiply; (3) subtract; (4) annex next figure of dividend. Repeat the four steps as often as necessary.

3. $287.70 ÷ 15.

$$\begin{array}{r} \$19.18 \\ 15\overline{)287.70} \\ \underline{15} \\ 137 \\ \underline{135} \\ 27 \\ \underline{15} \\ 120 \\ \underline{120} \end{array}$$

Place the decimal point in the quotient immediately above the decimal point in the dividend. Then proceed as before.

Solve the following:

4. $78.47 ÷ 19
5. $14.35 ÷ 35
6. $246.12 ÷ 28
7. $8240.32 ÷ 88
8. $1806.75 ÷ 225
9. $676.40 ÷ 95

	Dividends		Divisors		Dividends		Divisors
1.	2534	*a.*	95	*11.*	\$ 1018.76	*k.*	62
2.	3698	*b.*	49	*12.*	1377.28	*l.*	54
3.	42750	*c.*	321	*13.*	488.92	*m.*	349
4.	34225	*d.*	468	*14.*	764.31	*n.*	516
5.	129704	*e.*	500	*15.*	891.19	*o.*	707
6.	602308	*f.*	695	*16.*	2904.05	*p.*	600
7.	507300	*g.*	798	*17.*	8907.61	*q.*	308
8.	916875	*h.*	894	*18.*	27304.13	*r.*	500
9.	844222	*i.*	700	*19.*	14501.12	*s.*	872
10.	756874	*j.*	767	*20.*	16700.09	*t.*	695

10–109. Divide each number from *1* to *10* by each divisor from *a* to *j*. Thus, *1 a*, $2534 \div 95$; *9 i*, $844222 \div 700$. Test the answers.

110–209. Divide each number from *11* to *20* by each divisor from *k* to *t*. Thus, *16 t*, \$$2904.05 \div 695$. Test the answers.

PROBLEMS

Oral

Farm Problems

1. If 10 cows give 600 quarts of milk in 5 days, how many quarts does each cow average per day?

2. If 9000 eggs are laid by a flock of 60 hens in a year, how many eggs does each hen average?

3. At 60¢ a bushel, how many bushels of oats can be bought for $24?

4. At $1.25 per acre, how many acres of wheat can be cut and bound for $125?

5. If a boy feeds his chickens 7 pecks of corn in 14 days, how many quarts does he feed them per day?

6. If 12 acres yield 168 bushels of potatoes, what is the average yield per acre?

7. If a row of peas produces 12 pecks, how many such rows will produce 192 pecks?

8. A dairyman took 1380 pounds of milk to a dairy in a week of six days. How many pounds did he average per day?

9. If a horse eats 2 quarts of oats in a day, how long will 3 pecks last him? (8 quarts = 1 peck.)

10. If a farmer's horses eat 45 bushels of oats in 9 days, how long will 90 bushels last them?

Cooking Problems

11. Jellies are made of equal weights of cooked fruit and sugar. If 2 cups (1 pint) of granulated sugar weigh 1 pound, how many pounds of sugar are required for 16 quarts of fruit juice? (1 pint of fruit juice weighs 1 pound.)

12. A recipe for baked custard calls for 4 cups of milk, 6 eggs, $\frac{1}{2}$ cup sugar, and $\frac{1}{4}$ teaspoonful of salt. If this is sufficient for 6 persons, how much of each article should be used in making custard for 9 persons?

13. If 10 eggs contain 24 ounces of protein, how many eggs will contain 96 ounces, or 6 pounds, of protein?

14. If a 12-pound turkey put into the oven at 4 P.M. is roasted by 7 P.M., how many minutes does it take to roast each pound?

15. How many jelly glasses will be needed for 15 pints of jelly if 10 pints fill 2 dozen glasses?

Sewing Problems

16. If Alice hemstitches 8 inches in an hour, how long will it take her to finish a dresser scarf 50 inches long and 10 inches wide?

17. How many strips 6 inches wide can be cut from gingham 30 inches wide?

18. How many iron holders can be bound with a bolt of tape 12 yards long, if 36 inches are used for each holder?

19. How long does it take Julia to hem a dozen napkins 20 inches long and 20 inches wide, if she does 160 inches of hemming a day?

20. Material for a pincushion top is cut 4 inches by 9 inches. How many can I cut from a yard of cloth 20 inches wide, cutting it most economically?

21. Material for a chambray bag is cut 10 inches by 18 inches. How many can I cut from 2 yards of material, 20 inches wide?

Household and Marketing Problems

22. If a family uses a peck of potatoes in a week, how long will 4 bushels last them? (4 pecks = 1 bushel.)

23. At 30¢ a dozen, how many dozen eggs can I buy for $1.50?

24. How many pounds of tea at 60¢ a pound can I buy for $1.20?

25. If 3 oranges cost 15¢, how much do 3 dozen cost?

26. If 6 pounds of steak cost $1.68, what is the price per pound?

27. Mr. Ford bought 12 chairs and 1 table for $72. If the table cost $12, what was the price of each chair?

28. My milk bill for one week is $1.26. How many quarts do I use per day, at 9¢ a quart?

Woodworking and Cardboard Problems

29. Each pupil in a class of 7 in paper and cardboard work is supplied with the same material. If the cost for all is $4.62, how much is spent for each pupil?

30. A teacher of a woodworking class had $31 to spend. How many jack planes at $4 each could he buy, and how many try-squares at 20¢ each for the remainder?

31. How many pieces of cardboard for a laundry list 8 inches long and 5 inches wide can be cut from a piece 16 inches long and 10 inches wide?

32. How many blotters 9 inches by 4 inches can be cut from a sheet measuring 27 inches by 16 inches?

33. A boy made a table top. If the lumber for the top 3 feet long and 2 feet wide cost 48¢, what was the price per square foot?

34. If the wood for a drawing board 2 feet long and 1 foot wide costs 18¢, what is the cost per square foot?

Written

35. How many yards of ribbon will it take to run in the tops of 36 bags, if it takes 27 inches for one bag? How much will the ribbon cost at $.12 a yard?

36. If 36 yards of velvet can be bought for \$72, how much will one half of a yard cost? one eighth of a yard? one quarter of a yard?

37. A bushel of corn weighs 56 pounds. How many bushels are there in 244,160 pounds of corn?

38. Charles Williams earns \$71.50 a month. How much does he earn per day, counting 26 working days per month?

39. Rhode Island contains 1250 square miles; Indiana, 36,350 square miles. How many states the size of Rhode Island could be made from Indiana, and how many square miles would be left?

40. The dividend is 203,984; the divisor, 836. Find the quotient.

41. A man with \$2144 bought as many horses as possible for \$165 each, and spent the remainder for sheep at \$4 a head. How many of each did he buy?

42. Find the wages of a laborer who has worked 568 hours at \$1.60 a day of 8 hours each. What is the first question to be answered?

43. The cost of horse labor per hour in a business was determined for a period of eight years with the following results. Determine the average cost for the entire time.

NOTE. The **average** is the sum of the various amounts, divided by the number (in this case by 8).

YEAR	COST	YEAR	COST
1904	\$8.33	1908	\$12.05
1905	8.52	1909	9.64
1906	9.13	1910	8.70
1907	11.02	1911	9.40

44. A man bought 16 cows for \$800 and sold them for \$4 more for each cow than he paid. How much did he pay for each cow, and how much did he get for them all?

45. If 25 pencils cost \$.75, what is the cost of 60 pencils?

46. At a sale of grade Holsteins, 34 cows and heifers sold for \$ 3570. Determine the average sale price.

47. 1800 bushels of wheat are sold for \$ 1620. How much are 1240 bushels worth at the same rate?

48. A grocer paid \$ 67.20 for 240 pounds of coffee, and sold it at \$.30 a pound. Find his gain. Work the problem in two ways.

49. A man earned in 7 weeks the following amounts: \$10.75, \$14.25, \$9.80, \$15.60, \$16.05, \$7.85, \$17.65. What was the average earned per week?

50. The sum of the ages of 15 pupils is 180 years. What is the average age?

51. In a school 8 pupils are 14 years of age; 6 are 13 yr.; 7 are 12 yr.; 2 are 9 yr.; 12 are 8 yr.; 14 are 7yr. What is the sum of their ages? What is their average age?

52. Of what use is a thermometer? During a day in June the thermometer at 6 in the morning stood at 69°; at 7 it showed a temperature of 74°; at 8, 76°; at 9, 80°; and at 10, 86°. Find the average temperature for the five hours.

53. Two men started on their wheels from the same place, at the same time, and traveled in opposite directions, one at the rate of 65 miles a day, the other at the rate of 83 miles a day. At their journey's end they were separated by a distance of 3256 miles. How many days did they travel? Represent the conditions mentioned by means of a diagram. What are the two questions that must be answered?

54. If the daily ration for a horse is 10 pounds of hay and 12 pounds of oats, how many pounds of hay and bushels of oats are required to keep a horse for 365 days? (32 pounds = 1 bushel of oats.)

55. A clerk earns \$950 a year. His board is \$25 per month, and his other expenses are \$11.50 per month. In how many years can he save enough to pay \$1096 for a lot and \$3000 for a house?

56. The divisor is 4, the quotient is 8, and the remainder is 3. What is the dividend?

57. The divisor is 246, the quotient is 384, and the remainder is 157. What is the dividend?

58. Make a practical problem, requiring multiplication and division in its solution. Solve it.

REVIEW

1. Define division; dividend; divisor; quotient; remainder.

2. What is short division? When is it used?

3. What is long division? When is it used?

4. How can you find the dividend when the divisor and the quotient and the remainder are given? Illustrate.

5. Multiplying or dividing both dividend and divisor by the same number has what effect upon the quotient? Show the truth of your answer by examples.

6. Multiplying the dividend has what effect upon the quotient? dividing the dividend? Show by examples what effect multiplying or dividing the dividend has upon the quotient.

7. Show by examples what effect multiplying or dividing the divisor has upon the quotient.

8. How does division of United States money differ from division of other numbers?

COMBINED OPERATIONS

Study Recitation

The parenthesis indicates that the numbers inclosed are to be considered as a single number.

1. $3 \times (10 + 6) = ?$

Solution. $(10 + 6) = 16.$
$3 \times 16 = 48.$ *Ans.*

In examples involving several of the fundamental operations, the operations of division and multiplication must be performed before the operations of addition and subtraction.

2. $18 + 6 \div 2 - 3 \times 4 = ?$

Solution. $6 \div 2 = 3;\ 3 \times 4 = 12.$
$18 + 3 - 12 = 9.$ *Ans.*

If a parenthesis is used, the operations called for by signs within the parenthesis must be performed before the operation indicated by the sign that precedes it is performed.

3. $9 + 12 \div 4 - (10 - 6) = ?$

Solution. $12 \div 4 = 3;\ (10 - 6) = 4.$
$9 + 3 - 4 = 8.$ *Ans.*

Find the value of the following expressions :

4. $7 \times 2 - 8 \div 4.$

5. $3 - (8 - 6) + 10 \times 3.$

6. $5 + 6 \times 4 - 12 \div 6.$

7. $(8 + 3 - 7) \div 2 + 16.$

8. $4 + 5 \times 6 + 10.$

9. $(4 + 5) \times 6 + 10.$

10. $4 + 5 \times (6 + 10).$

11. $(12 + 18 \div 3) \div 9 + 2.$

12. $48 - 3 \times 3 + 5 \times 2 - (30 - 6) \div 6.$

13. $(6 + 10 - 8) \div 4 + (28 - 5 \times 2) \div 9 + 19 - 6.$

GENERAL REVIEW

Written

Time yourself in these exercises. Then work them again and try to beat your record. Test the answers. Add:

	a.	*b.*	*c.*	*d.*	*e.*
1.	26	246	365	4641	$130.41
	38	374	274	2738	227.58
	47	895	381	4254	329.43
	29	408	623	2134	491.37
	43	639	472	6403	542.51
2.	87	375	321	4023	$681.24
	56	946	978	4256	527.56
	93	258	462	3007	434.23
	72	724	357	9842	352.45
	14	397	825	7345	263.27
3.	98	923	275	7856	$827.39
	87	847	149	9427	742.31
	46	282	206	2341	661.45
	23	197	372	7205	520.07
	75	218	425	8006	430.09
	89	643	986	2037	327.48
4.	44	584	724	5002	$246.23
	55	729	375	6343	327.21
	63	134	428	2784	498.71
	98	423	653	1462	562.95
	74	156	527	2275	681.79
	89	649	837	4138	767.84

Write from dictation:

	a.	b.	c.	d.
5.	10452	20851	56766	$156.60
6.	75657	6056	20009	$140.57

Subtract:

7.	89809 71498	65012 53767	80390 37492	$700.18 289.49
8.	72103 46786	51437 39789	40002 28768	$800.10 654.34
9.	90157 25638	61052 31657	51103 45794	$957.61 760.95
10.	85670 27898	60000 39879	70000 56427	$500.53 355.76

Multiply:

11.	349 503	2654 785	4305 704	$715.16 1610
12.	859 689	9032 908	6039 547	$825.98 1503
13.	697 697	8104 104	5308 697	$562.25 8005
14.	708 809	6057 196	7205 348	$427.68 3009

Find the quotients:

	a.	b.	c.
15.	$8845 \div 29$	$10164 \div 231$	$789.14 ÷ 422
16.	$8100 \div 36$	$141435 \div 449$	$2609.67 ÷ 867

	a.	*b.*	*c.*
17.	$1740 \div 87$	$141702 \div 678$	$\$361.92 \div 348$
18.	$2108 \div 68$	$68400 \div 225$	$\$896.67 \div 729$

PROBLEMS

NOTE. Before starting to solve a problem, read it carefully to find what facts are given and what is asked for. Then think of the easiest method of finding the answer from the given conditions. If two or more methods can be used, choose the shortest one. In dealing with large numbers, *estimate* the approximate answer, whenever possible. Then solve the example and compare the answer with your estimate to see whether it is reasonable. Test your answer by proving that it fulfills all the conditions of the problem. In the future apply these methods to the solution of all problems.

TO THE TEACHER. While the above suggestions are made only at this point, it is intended that they shall be followed throughout the book. Insist always on careful *interpretation* of problems, on a choice of the most *economical method of solution*, and wherever practicable, on a *mental estimate*, to precede the written work.

ORAL

1. (*a*) If 4 pads cost 8¢, how much do 5 pads cost?

(*b*) If 3 pads cost 5¢, how much do 6 pads cost?

SOLUTION. (*a*) 1 pad costs 8¢ ÷ 4, or 2¢; 5 pads cost 5 × 2¢, or 10¢.
(*b*) Since 6 pads are 2 × 3 pads, 6 pads cost 2 × 5¢, or 10¢.

Explain why each method is better for the case given.

2. Find by the easiest method the cost of a dozen eggs when 3 cost 10¢.

3. Find the cost of 10 eggs, when 3 cost 9¢.

Find the cost of the following:

4. 12 yd. lace when 3 yd. cost 10¢.

5. 5 caps when 4 caps cost $8.

6. 4 lb. butter when 6 lb. cost $1.80.

7. 2 lb. steak when 4 lb. cost 56¢.

8. 6 yd. cloth when 3 yd. cost $1.

9. 4 doz. buttons when 2 doz. cost 25¢.

10. 9 qt. milk when 3 qt. cost 24¢.

11. A farmer has 14 cows in one field and 6 less in another field. How many cows has he?

SOLUTION. The facts given are the number of cows in one field and the number less in another field. The answer required is the total number of cows. The first step is to find by subtraction the number of cows in the second field. 14 cows − 6 cows = 8 cows. The second step is to add the number of cows in both fields.

14 cows + 8 cows = 22 cows. *Ans.*

12. If you have $25 and your brother has $10 less, how much have you both together?

13. What is your average, if your mark in spelling is 95, in reading 100, in arithmetic 80, in drawing 75, and in history 90?

14. At 8¢ a quart, how much will 2 gallons of milk cost?

15. At 15 minutes per pound how long will it take to roast a 6-pound duck?

16. If you buy 5 yd. of cloth at $1.50 a yard and sell it at $1.75 a yard, how much do you gain?

Find the gain on:

17. 5 yd. lace bought @ 15¢ and sold @ 20¢.

18. 6 lb. meat bought @ 12¢ and sold @ 16¢.

19. 12 doz. buttons bought @ 25¢ and sold @ 28¢.

20. 12 chairs bought @ $2 and sold @ $2.50.

21. 10 pairs shoes bought @ $2 and sold @ $2.75.

Find the reduction in price on each of the following rugs:

	REGULAR PRICE	SALE PRICE		REGULAR PRICE	SALE PRICE
22.	$ 3.50	$ 2.60	**28.**	$18.75	$14.00
23.	$ 5.25	$ 3.75	**29.**	$28.00	$21.00
24.	$11.00	$ 8.45	**30.**	$45.00	$33.75
25.	$14.75	$11.25	**31.**	$50.00	$37.50
26.	$18.50	$14.00	**32.**	$70.00	$50.65
27.	$25.25	$17.25	**33.**	$77.50	$58.00

Written

34. If A has $2563.75, and B has $3421.85, how much more money has B than A, and how much have both together?

35. If A has $2563.75, and B has $1099.85 less than A, how much money have they both together?

36. At $2.56 a yard, how many yards of velvet can you buy for $61.44?

37. Mr. Smith's bank book at the end of March shows the following debits and credits:

1914	*Cr.*			*Dr.*	
March	1, Deposit,	$967.60	March	2, Checks,	$346.78
	5, Deposit,	25.74		4, Checks,	25.96
	9, Deposit,	875.00		5, Checks,	327.30
	11, Deposit,	146.75		12, Checks,	24.93
	16, Deposit,	85.36		30, Checks,	879.40
	26, Deposit,	128.98		31, Checks,	32.48
	30, Deposit,	475.50			
	31, Deposit,	326.84			

Find the balance. What is meant by balance? State how it is found.

38. March 1, 1900, the Goodyear Shoe Factory of Milwaukee sold to S. M. Jones of Fort Wayne, Ind., 36 pairs of calfskin boots at $4; 99 pairs of gaiters at $1.25; 26 pairs of slippers at $.98; 136 pairs of arctics at $1.05.

Make out a bill in due form and properly receipt it.

NOTE. *Estimate* mentally as follows, 3 doz. pairs boots @ $4 cost nearly $150, 99 pairs gaiters @ $1.25 cost nearly 100 × $1.25, or $125; $275; 26 pairs slippers @ $.98 cost nearly 26 × $1, or $26; $301; 136 pairs arctics @ $1.05 cost nearly 136 × $1, or $136; $437.

Compare this result with the exact answer, when you have found it. If there is more than a few dollars' difference, you have made some mistake in your work.

39. A bought a block of land for \$75,350. He paid \$25 for having it divided into 325 lots, spent \$675 in building sidewalks, and then sold the entire block at a profit of \$50 per lot. What was the selling price of a lot?

State the several questions that must be answered to solve the problem.

40. Find the cost of 100 notebook covers each requiring the following materials:

Clothboard, \$.015; binder's cloth, \$.02; marbled paper, \$.005; paste, glue, and eyelets, \$.005.

41. How much would a boy gain by making and selling 100 such covers at \$.15 each? How much would his work yield him per day, if he made 100 covers in 6 days?

42. Mr. Bain has 1560 sheep; Mr. Smith has 6 times as many as Mr. Bain, less 204; Mr. Mason has one half as many as Mr. Bain. How many sheep have the three?

43. What is the cost of 30 oranges at 20 cents a dozen, 132 eggs at 14 cents a dozen, and 2 pecks of apples at 80 cents a bushel?

44. Find the difference between the sum and the product of 756 and 894.

45. A clerk has a salary of \$42 a month, and his expenses are \$27 a month. How many years will it take him to save \$900?

46. A grocer bought 2250 pounds of sugar at \$5 a hundred pounds; he sold 250 pounds at 6 cents, and the remainder at 5 cents a pound. How much did he gain?

47. A merchant finds that during the day he has taken in 208 half dollars, 73 quarters, 110 dimes, 42 nickels, and 13 cents. How much has he received in all?

48. A boy buys from a dealer in canceled stamps 24 German stamps, at 4 for a cent; 39 English stamps, at 3 for a cent; 76 Mexican stamps, at 2 for a cent; 13 French stamps, at 2 cents apiece; 16 Cuban stamps, at 3 cents each. If he hands the dealer $2, what change should the boy receive?

49. How much will my milk bill be for a year of 365 days if I take a quart in the morning and a pint at night, the price being 9 cents a quart? How much will it be during the first 6 months? during the winter months? during the spring months? during the summer months? during the autumn months?

50. If good wheat produces 15 bushels to the acre and makes 45 pounds of flour for every bushel, while poor wheat produces 10 bushels to the acre with 34 pounds to the bushel, how many barrels of flour will be saved by sowing good wheat on 10 acres? (60 pounds = 1 bushel of wheat.)

51. A man has an income of $1825 a year. He spends $\frac{1}{5}$ of it for groceries, $200 for rent, $437 for clothing, and $579 for sundries. How much will he have left at the end of the year? What is his average net income per month? What is the meaning of "net income"? of "sundries"?

52. If a boy spends 25 minutes each day in reading, how many hours does he read in a year? What more do you need to know to find how many books he could read in a year?

53. Mr. Roberts bought a horse for $80, paid $2.50 a week for keeping it, and received $3.75 a week for its work. He kept it a year (52 weeks) and then sold it for $70. How much did he gain?

54. Lord and Taylor advertised a sale of 8500 yards of Scotch flannel reduced from 48¢ a yard to 28¢ a yard. What was their loss on the entire lot?

55. Mrs. Ames bought 14 yards of the flannel before it was reduced. How many more yards could she have bought for the same amount of money at the sale?

56. Find the cost of the following kitchen equipment in a domestic science school:

1 refrigerator	$65	1 gas range @	$ 34
4 tables @	55	1 oven @	140
2 porcelain sinks @	65	18 Bunsen burners @	.25
2 cupboards @	35	6 scales @	5

The following table shows a minimum cost of equipment for various lines of manual training work.

TABLE

	Pupils in Each Class	Cost
Paper and cardboard	18	$ 5.49
Basketry and weaving	18	3.15
Bent iron	18	14.50
Clay	18	6.21
Knife work	18	14.12
Bench woodwork	18	150.43
Hammered metal	9	126.10
Molding	12	69.00
Wood turning	12	650.30
Forging	12	678.00
Machine shop	7	1158.99
Mechanical drawing	18	124.95

If four classes can be accommodated daily, with the full number of pupils indicated in each class, find the number of pupils taking each class of work daily, and find the cost to mills of equipment for each pupil for:

57. Paper and cardboard.

58. Basketry and weaving.

59. Bent iron.

60. Clay.

61. Knife work.

62. Bench woodwork.

63. Hammered metal.

64. Molding.

65. Wood turning.

66. Forging.

67. Machine shop.

68. Mechanical drawing.

69. An Iowa farmer made the following estimate of the cost per acre of raising corn: Find the cost per acre.

Preparation of sod	$1.00
Plowing	1.25
Harrowing before planting	.20
Planting	.25
Harrowing after planting	.20
Four cultivations	1.40
Husking	1.75
Interest on one acre land, at $90	4.50
Taxes	.45

70. He raised 50 bushels per acre. Find the cost per bushel.

71. Find the profit on 40 acres of corn sold at 53 cents per bushel, if marketing costs 3 cents per bushel?

The following figures show the cost of growing a crop of 60 acres of corn on a farm in Illinois:

Plowing at $1.00 per acre	
Breaking stalks on 15 acres at $.20	
Disking on 45 acres of fall plowing at $.40	
Harrowing	$10.50
Seed corn, 9 bushels at $.75	
Planting at $.20 per acre	
Harrowing after planting	10.50
Cultivating three times	78.00
Thinning and weeding	10.00
Husking at $2\frac{1}{2}$ cents per bushel	62.25
Shelling and hauling	62.25
Taxes	20.00
Insurance and repairs	10.00

72. The crop on 60 acres is 2460 bushels. What is the average yield per acre?

73. What is the average price per bushel (to mills) of a crop of 2460 bushels costing \$363.25?

74. What is the average cost per acre of a 60-acre crop costing \$363.25?

75. The Department of Agriculture reported 634,000 milch cows in Indiana and their average value as \$45.70. What was their total value?

76. In the same year the number of milch cows in Wisconsin was 1,504,000, and their total farm value was \$71,740,800. What was their average value?

In the following group of states the average yield of potatoes per acre, and the price on the farm, Dec. 1, 1912, were as follows:

	Av. per Acre	Farm Price per Bushel
Ohio	112 bu.	\$.53
Indiana	114 bu.	.50
Illinois	101 bu.	.60
Michigan	105 bu.	.41
Wisconsin	120 bu.	.34

77. What was the value per acre for each state?

78. What was the average value per acre for the entire group of states?

79. The Kansas Experiment Station compared the gains made by three lots of 10 calves each, fed on skim milk, on whole milk, and nursed by the dam. The first lot was fed 154 days and gained 233 pounds; the second lot was fed 154 days and gained 287 pounds; and the third lot was nursed 140 days and gained 248 pounds. What was the average daily gain on each lot of calves?

80. Construct three problems, each involving the four fundamental operations. Solve them.

Problems without Numbers

NOTE. After answering each question, make and solve an original problem applying your answer.

81. If you know the price of one thing, how can you find the cost of a given number of things?

82. If you know the cost of a given number of things, how can you find the price of one thing?

83. If you know the total cost of two things and the cost of one of them, how can you find the cost of the other?

84. If you know the marks that a pupil has received in each of five different subjects, how can you find his total average?

85. If you know the price of a quart of milk, how can you find the price of any number of pints? of any number of gallons?

86. How can you find the total amount of a bill when you know the price and the amount purchased of each article on the bill?

87. If a certain number of articles cost a given sum, how can you find how much twice as many articles cost?

88. If you know how much a certain article costs and for what price it was sold, how can you find the gain?

89. If you know the cost and the gain on an article, how can you find the selling price?

90. If you know the selling price of an article and the gain, how can you find the cost?

91. If you buy a given number of yards of cloth at a certain price per yard and sell them at a greater price, in what two ways can you find the total gain?

92. If A has a certain sum of money, and B has a certain sum less than A, how can you find how much both have?

FACTORING

A **factor** is one of the makers of the product. 3 and 5 are factors of 15. The multiplicand and multiplier are factors of the product.

Factoring is the process of separating a product into its factors. Thus, $15 = 3 \times 5$.

An **integer** or an **integral number** is a whole number.

A **prime number** is a number that is not exactly divisible by any whole number except itself and 1. 17 is a prime number, because 17 and 1 are its only integral factors.

A **composite number** is one that has other integral factors besides itself and one. 21 is a composite number, because it has the factors 3 and 7.

An **even number** is one that is exactly divisible by 2. All even numbers end in 0, 2, 4, 6, or 8.

An **odd number** is one that is not exactly divisible by 2. Odd numbers end in 1, 3, 5, 7, or 9.

Are the following numbers even or odd? prime or composite? 4, 9, 7, 12, 35, 120, 8, 11, 39, 40, 25, 19, 36?

Tests of divisibility.

Any even number is exactly divisible by 2.

A number is exactly divisible by 4, *if the number represented by its last two figures is exactly divisible by* 4, *or if the last two figures are zeros;* as, 324, 948, 500.

A number is exactly divisible by 8 *if the number represented by its last three figures is exactly divisible by* 8, *or if the last three figures are zeros;* as 3456, 7000.

A number is exactly divisible by 3 *if the sum of its figures is exactly divisible by* 3; as, 147. $(1 + 4 + 7 = 12$; 12 is divisible by 3.)

A number is exactly divisible by 9 *if the sum of its figures is exactly divisible by* 9; as, 342. $(3 + 4 + 2 = 9$; 9 is divisible by 3.)

A number is exactly divisible by 5 *if its last figure is* 0 *or* 5.

A number is exactly divisible by 25 *if the number represented by its last two figures is divisible by* 25, *or if the last two figures are zeros;* as, 575, 550, 6500.

By which of the numbers 2, 3, 4, 5, 8, 9, 25 are the following exactly divisible?

1. 24, 72, 45, 30, 125
2. 145, 152, 144, 196, 920
3. 60, 96, 100, 231, 625
4. 320, 1728, 2250, 3600, 8376

Resolving a number into its prime factors.

STUDY RECITATION

A factor of a number is an **exact divisor** of that number.

TO THE TEACHER. Review the table of factors and products on p. 39.

A **power** of a number is the product arising from using the number two or more times as a factor.

Two or more equal factors may be written as a power of the number.

a. $4 = 2 \times 2 = 2^2$, read, 2 to the second power, or 2 square.
b. $8 = 2 \times 2 \times 2 = 2^3$, read, 2 to the third power, or 2 cube.
c. $16 = 2 \times 2 \times 2 \times 2 = 2^4$, read, 2 to the fourth power.
d. $243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$, read, 3 to the fifth power.

A **prime factor** is a factor that cannot be separated into other integral factors.

The numbers 2, 3, 4, 5, written to the right and a little above the numbers, 2, 3, in *a*, *b*, *c*, and *d*, are **exponents**. An exponent shows how many times the number to which it is attached is used as a factor. Thus, 3^5 shows that 3 is used 5 times as a factor, in $3 \times 3 \times 3 \times 3 \times 3$; 2^3 shows that 2 is used 3 times as a factor in $2 \times 2 \times 2$.

Learn the prime factors of 4, 6, 8, 9, 10, and 12:

$4 = 2^2$	The prime factors of 4 are two 2's.
$6 = 2 \times 3$	The prime factors of 6 are 2 and 3.
$8 = 2^3$	The prime factors of 8 are three 2's.
$9 = 3^2$	The prime factors of 9 are two 3's.
$10 = 2 \times 5$	The prime factors of 10 are 2 and 5.
$12 = 2^2 \times 3$	The prime factors of 12 are two 2's and 3.

The prime numbers from 1 to 20 are: 1, 2, 3, 5, 7, 11, 13, 17, 19. The remaining prime numbers to 100 are: 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

1. The product of two factors is 48, and one of the factors is 8. What is the other factor?

Find the missing factors:

	Factor	Product	Other Factor		Factor	Product	Other Factor
2.	9	72		**6.**	25	2500	
3.	12	108		**7.**	13	390	
4.	11	121		**8.**	7	1400	
5.	19	76		**9.**	17	510	

10. Find the prime factors of 450.

$$\begin{array}{r} 9\,)\underline{450} \\ 5\,)\underline{50} \\ 5\,)\underline{10} \\ 2 \end{array}$$

Divide first by 9; then by 5; then by 5. The factors are 9, 5, 5, 2; but 9 is not a prime number. $9 = 3 \times 3$. The prime factors are $2 \times 3 \times 3 \times 5 \times 5$.

Find the prime factors of:

	a.	*b.*	*c.*	*d.*	*e.*
11.	95	225	144	891	1350
12.	100	168	576	315	2310
13.	108	150	250	540	4550
14.	132	128	118	280	6000
15.	170	140	105	396	4800
16.	180	198	253	405	4480
17.	189	195	348	288	6480
18.	165	336	264	392	2880

REVIEW

1. Define and give an example of : factor; prime factor; prime number; composite number; integer; power; exponent; exact divisor.

2. How can you determine whether a number is even or odd?

3. How can you tell at a glance whether a number is divisible by 2? 3? 4? 5? 8? 9? 10? 25?

4. How can you find the prime factors of a number?

5. Construct and solve one example each in testing divisibility of numbers by 2, 3, 4, 5, 8, 9, 10, and 25.

6. Construct and solve two examples, finding the prime factors of numbers.

FRACTIONS

An **integral unit** is a single thing considered as an undivided whole. 1, the least integer, is the unit in arithmetic.

A **fractional unit** is one of the equal parts into which a unit has been separated; as, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{10}$, $\frac{1}{16}$.

A **fraction** consists of one or more fractional units; as, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{7}$, $\frac{8}{8}$, $\frac{9}{4}$.

A **fraction** may also be defined as a quotient expressed by writing the dividend above a horizontal line and the divisor below. Thus, $2 \div 7 = \frac{2}{7}$; $4 \div 9 = \frac{4}{9}$; $8 \div 2 = \frac{8}{2}$.

The **denominator**, written below the line, shows into how many equal parts the whole has been divided.

The **numerator**, written above the line, shows how many of the equal parts are taken.

The numerator and the denominator are the **terms** of the fraction.

A **proper fraction** is one whose numerator is less than its denominator; as, $\frac{2}{5}$, $\frac{5}{6}$, $\frac{4}{8}$.

An **improper fraction** is one whose numerator is not less than its denominator; as, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{7}{4}$, $\frac{12}{7}$.

A **mixed number** is an integer and a fraction united; as, $2\frac{1}{2}$, $12\frac{3}{4}$, $16\frac{3}{8}$.

Read the following fractions and mixed numbers:

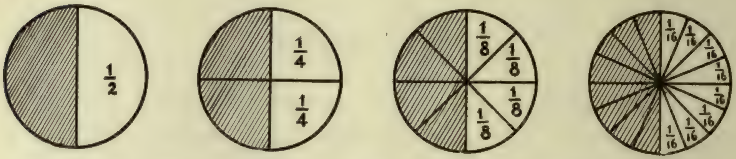
1. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{9}{6}$ $\frac{2}{3}$ $\frac{3}{7}$ $\frac{8}{5}$ $\frac{7}{8}$

2. $\frac{10}{20}$ $\frac{15}{16}$ $\frac{16}{16}$ $\frac{17}{16}$ $\frac{25}{100}$ $\frac{20}{100}$ $\frac{75}{100}$ $\frac{12}{24}$ $\frac{24}{12}$ $\frac{36}{10}$

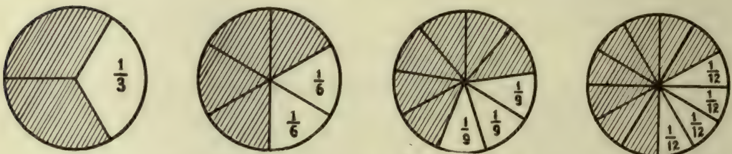
3. $15\frac{2}{3}$ $24\frac{5}{6}$ $30\frac{4}{5}$ $22\frac{7}{8}$ $19\frac{4}{9}$ $40\frac{5}{8}$

4. Select in examples 1 to 3 the proper fractions; the improper fractions; the mixed numbers.

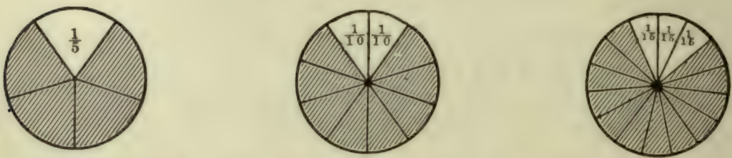
5. Write the fractions in examples 1 to 3 from dictation.



$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$



$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$



$\frac{1}{5} = \frac{2}{10} = \frac{3}{15}$

6. What is $\frac{1}{2}$ of 10? $\frac{1}{3}$ of 15? $\frac{2}{3}$ of 21? $\frac{1}{4}$ of 16? $\frac{3}{4}$ of 24? $\frac{2}{5}$ of 20? $\frac{3}{5}$ of 25? $\frac{4}{5}$ of 30? $\frac{1}{6}$ of 18? $\frac{5}{6}$ of 24? $\frac{3}{7}$ of 28? $\frac{5}{8}$ of 32? $\frac{7}{8}$ of 64? $\frac{4}{9}$ of 36? $\frac{3}{10}$ of 30? $\frac{1}{11}$ of 22? $\frac{5}{12}$ of 60? $\frac{3}{16}$ of 32? $\frac{5}{16}$ of 64?

7. 1 unit $= \frac{?}{2} = \frac{?}{3} = \frac{?}{4} = \frac{?}{5} = \frac{?}{6} = \frac{?}{7} = \frac{?}{8} = \frac{?}{9} = \frac{?}{10} = \frac{?}{11} = \frac{?}{12} = \frac{?}{16}$.

8. Show by drawings or paper cutting that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16}$.

9. Show in the same way that $\frac{1}{3} = \frac{3}{9} = \frac{4}{12}$; $\frac{2}{3} = \frac{6}{9} = \frac{8}{12}$.

10. Show in the same way that $\frac{1}{5} = \frac{2}{10} = \frac{3}{15}$; $\frac{2}{5} = \frac{4}{10} = \frac{6}{15}$.

11. $\frac{1}{2} = \frac{?}{4} = \frac{?}{6} = \frac{?}{8} = \frac{?}{10} = \frac{?}{12} = \frac{?}{14} = \frac{?}{16}$.

12. $\frac{1}{3} = \frac{?}{6} = \frac{?}{9} = \frac{?}{12}$; $\frac{2}{3} = \frac{?}{6} = \frac{?}{9} = \frac{?}{12}$.

13. $\frac{1}{4} = \frac{?}{8} = \frac{?}{12} = \frac{?}{16}$; $\frac{3}{4} = \frac{?}{8} = \frac{?}{12} = \frac{?}{16}$.

14. $\frac{1}{5} = \frac{?}{10} = \frac{?}{15}$; $\frac{2}{5} = \frac{?}{10} = \frac{?}{15}$; $\frac{3}{5} = \frac{?}{10} = \frac{?}{15}$.

15. $\frac{1}{6} = \frac{?}{12}$; $\frac{5}{6} = \frac{?}{12}$; $\frac{1}{7} = \frac{?}{14}$; $\frac{1}{8} = \frac{?}{16}$; $\frac{3}{8} = \frac{?}{16}$.

16. $\frac{9}{18} = \frac{?}{2}$; $\frac{8}{16} = \frac{?}{2}$; $\frac{4}{16} = \frac{?}{4}$; $\frac{3}{9} = \frac{?}{3}$; $\frac{5}{15} = \frac{?}{3}$.

17. $\frac{6}{12} = \frac{?}{2}$; $\frac{5}{10} = \frac{?}{2}$; $\frac{2}{8} = \frac{?}{4}$; $\frac{6}{9} = \frac{?}{3}$; $\frac{6}{8} = \frac{?}{4}$.

18. Express the following as mixed numbers:

$\frac{3}{2}, \frac{6}{4}, \frac{4}{3}, \frac{7}{5}, \frac{9}{6}, \frac{8}{7}, \frac{10}{8}, \frac{12}{9}, \frac{25}{10}, \frac{12}{11}, \frac{13}{12}, \frac{19}{16}$.

19. How much less than 1 is each of the following?

$\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{2}{5}, \frac{3}{5}, \frac{5}{8}, \frac{5}{6}, \frac{7}{8}, \frac{4}{9}, \frac{3}{10}, \frac{5}{12}, \frac{7}{16}, \frac{9}{16}$.

Arrange in the order of their size:

20. $\frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{5}, \frac{3}{4}, \frac{7}{8}, \frac{1}{10}, \frac{1}{16}$.

21. $\frac{5}{4}, \frac{5}{5}, \frac{6}{6}, \frac{6}{5}, \frac{9}{8}, \frac{16}{8}$.

22. $\frac{1}{2} + \frac{1}{2} = ?$ $\frac{1}{2} + \frac{1}{4} = ?$ $\frac{1}{2} - \frac{1}{4} = ?$ $\frac{3}{4} - \frac{1}{2} = ?$ $\frac{1}{5} - \frac{1}{10} = ?$

23. $\frac{3}{4} + \frac{1}{4} = ?$ $\frac{1}{3} + \frac{1}{6} = ?$ $\frac{1}{3} - \frac{1}{6} = ?$ $\frac{5}{6} - \frac{1}{3} = ?$ $\frac{1}{4} - \frac{1}{8} = ?$

Study Recitation

1. Add $\frac{3}{8}$ and $\frac{1}{4}$.

$$\begin{aligned} \tfrac{3}{8} &= \tfrac{3}{8} \\ \tfrac{1}{4} &= \underline{\tfrac{2}{8}} \\ &\ \tfrac{5}{8} \end{aligned}$$

2. Add $2\frac{3}{8}$ and $1\frac{1}{4}$.

$$\begin{aligned} 2\tfrac{3}{8} &= 2\tfrac{3}{8} \\ 1\tfrac{1}{4} &= \underline{1\tfrac{2}{8}} \\ &\ 3\tfrac{5}{8} \end{aligned}$$

The sum of the fractions is $\frac{5}{8}$; of the whole numbers 3. The entire sum is $3\frac{5}{8}$.

3. From $\frac{3}{8}$ take $\frac{1}{4}$.

$$\begin{aligned} \tfrac{3}{8} &= \tfrac{3}{8} \\ \tfrac{1}{4} &= \underline{\tfrac{2}{8}} \\ &\ \tfrac{1}{8} \end{aligned}$$

$\frac{3}{8} - \frac{2}{8} = \frac{1}{8}$

4. From $2\frac{3}{8}$ take $1\frac{1}{4}$.

$$\begin{aligned} 2\tfrac{3}{8} &= 2\tfrac{3}{8} \\ 1\tfrac{1}{4} &= \underline{1\tfrac{2}{8}} \\ &\ 1\tfrac{1}{8} \end{aligned}$$

$\frac{3}{8} - \frac{2}{8} = \frac{1}{8}$

$2 - 1 = 1$

The difference is $1\frac{1}{8}$.

Add:

	a.	*b.*	*c.*	*d.*
5.	$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$	$\frac{1}{4}+\frac{3}{8}$	$\frac{3}{16}+\frac{5}{8}$	$\frac{5}{8}+\frac{3}{16}$
6.	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$	$\frac{1}{5}+\frac{3}{10}$	$\frac{3}{10}+\frac{2}{5}$	$\frac{1}{2}+\frac{2}{3}$
7.	$\frac{3}{8}+\frac{3}{8}+\frac{1}{8}$	$\frac{1}{6}+\frac{5}{12}$	$\frac{5}{12}+\frac{5}{6}$	$\frac{2}{9}+\frac{1}{3}$

Subtract:

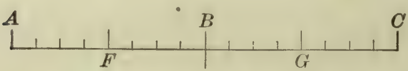
8.	$\frac{3}{4}-\frac{1}{4}$	$\frac{3}{4}-\frac{3}{16}$	$\frac{3}{5}-\frac{1}{10}$	$\frac{1}{2}-\frac{3}{10}$
9.	$\frac{2}{3}-\frac{1}{3}$	$\frac{2}{3}-\frac{1}{9}$	$\frac{7}{16}-\frac{3}{8}$	$\frac{1}{2}-\frac{3}{8}$
10.	$\frac{7}{8}-\frac{2}{8}$	$\frac{7}{8}-\frac{3}{16}$	$\frac{5}{12}-\frac{1}{3}$	$\frac{1}{2}-\frac{5}{12}$

First add; then subtract:

	a.	*b.*	*c.*	*d.*	*e.*
11.	$5\frac{1}{2}$	$6\frac{3}{4}$	$7\frac{1}{3}$	$8\frac{1}{5}$	$7\frac{1}{2}$
	$1\frac{1}{8}$	$3\frac{1}{2}$	$5\frac{1}{6}$	$2\frac{1}{10}$	$3\frac{1}{6}$
12.	$14\frac{5}{8}$	$25\frac{5}{12}$	$16\frac{4}{9}$	$43\frac{7}{10}$	$17\frac{1}{2}$
	$12\frac{1}{4}$	$13\frac{1}{4}$	$9\frac{1}{3}$	$20\frac{1}{5}$	$15\frac{3}{16}$
13.	$24\frac{1}{2}$	$38\frac{3}{4}$	$42\frac{7}{8}$	$29\frac{1}{2}$	$48\frac{2}{3}$
	$13\frac{3}{10}$	$27\frac{3}{16}$	$31\frac{3}{16}$	$16\frac{5}{12}$	$27\frac{2}{9}$

Principles governing Operations in Fractions

Study Recitation



The line AC is divided into 16 equal parts; by the lines at F, B, and G it is divided into 4 equal parts; by the line at B it is divided into 2 equal parts.

1. What part of the line AC is the line AB?
2. What part of the line AC is the line AF?
3. What part of the line AB is the line AF?

4. What is the fractional unit in each of the fractions:

$$\tfrac{3}{4}, \tfrac{4}{8}, \tfrac{2}{3}, \tfrac{4}{5}, \tfrac{5}{6}, \tfrac{2}{9}, \tfrac{3}{7}, \tfrac{7}{10}, \tfrac{5}{12}, \tfrac{3}{16}?$$

5. What is the number of fractional units in each of the above fractions?

6. What figure in the fraction $\frac{3}{4}$ shows the number of parts into which a whole has been divided? What figure shows the number of the parts taken?

7. Draw a line representing the line AC, and under it draw lines to represent $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, of the line AC.

A.	*B.*
$\frac{1}{2 \times 2} =$	$\frac{1}{4}$
$\frac{1}{4 \times 2} =$	$\frac{1}{8}$
$\frac{1}{8 \times 2} =$	$\frac{1}{16}$

8. If the denominator of each fraction under A is multiplied by 2, are the resulting fractions larger or smaller than the fractions whose denominators have been multiplied?

9. Show by means of the line AC above, the value of each fraction under B as compared with the fraction at its left under A.

10. Multiplying the denominator of a fraction produces what effect upon the fraction? Why?

11. If the denominator of each fraction under B is divided by 2, what are the resulting fractions?

12. Dividing the denominator of a fraction has what effect upon the fraction? Why?

13. What does the numerator of a fraction show?

14. If the numerator of a fraction is multiplied by any number, what is the effect upon the fraction? Why? Illustrate.

15. If the numerator of a fraction is divided by any number, what is the effect upon the fraction? Illustrate.

16. How can the fraction $\frac{8}{16}$ be obtained from the fraction $\frac{4}{8}$? What is its value as compared with $\frac{4}{8}$? Illustrate by use of line *AC*. What is the effect of multiplying both terms of a fraction by the same number?

17. How can $\frac{4}{8}$ be obtained from $\frac{8}{16}$? What is the effect of dividing both terms of a fraction by the same number? Illustrate by use of line *AC*.

PRINCIPLES. 1. *Multiplying the numerator of a fraction by any number multiplies the fraction by that number.*

2. *Dividing the numerator of a fraction by any number divides the fraction by that number.*

3. *Multiplying the denominator of a fraction by any number divides the fraction by that number.*

4. *Dividing the denominator of a fraction by any number multiplies the fraction by that number.*

5. *Multiplying or dividing both terms of a fraction by the same number does not alter the value of the fraction.*

18. What is $\frac{2}{4}$ of 24?

With 2 as multiplier and divisor perform upon the fraction $\frac{2}{4}$ the operations indicated in Principles 1, 2, 3, 4, and 5, and find the parts of 24 indicated by the resulting fractions.

A fraction is **changed** or **reduced** to **higher terms** when it is changed to a fraction with a larger denominator, as when $\frac{1}{2}$ is changed to $\frac{2}{4}$, $\frac{4}{8}$, $\frac{8}{16}$, etc.

A fraction is **changed** or **reduced** to **lower terms** when it is changed to a fraction with a smaller denominator, as when $\frac{8}{16}$ is changed to $\frac{4}{8}$, $\frac{2}{4}$, or $\frac{1}{2}$.

A **common factor** or divisor of two or more numbers is a divisor common to all of them. Thus, 4 is a common divisor of 8, 12, and 16.

A fraction is in its **lowest terms** when the terms have no common factor. Thus, the fraction $\frac{8}{16}$ in its lowest terms equals $\frac{1}{2}$; $\frac{3}{9}=\frac{1}{3}$; $\frac{8}{12}=\frac{2}{3}$.

The **greatest common divisor** or the **greatest common factor** of two or more numbers is the greatest factor common to all of them. Its abbreviation is g. c. d. or g. c. f.

Thus, the common divisors of 18 and 24 are 2, 3, and 6, because both 18 and 24 are divisible by each of these numbers. The greatest common divisor, however, is 6.

To the Teacher. It is usually more convenient for the pupil to reduce fractions to lowest terms by dividing successively by factors, seen by inspection, rather than by the g. c. d. If the teacher wishes to teach the method of finding the g. c. d., the following explanation may be given:

To find the g. c. d. of several numbers.

Find the prime factors of the given numbers. The product of such of the prime factors as are common to all the numbers is the g. c. d. of the numbers.

1. Find the g. c. d. of 48, 72, 96.

$$\begin{aligned} 48 &= 2^4 \times 3 \\ 72 &= 2^3 \times 3^2 \\ 96 &= 2^5 \times 3 \\ \hline & 2^3 \times 3 = 24, \text{ g. c. d.} \end{aligned}$$

In 48, 2 is found 4 times as a factor; in 96, 5 times; in 72, only 3 times; hence 2 must be used as a factor 3 times in the g. c. d.

In the same way, it is found that 3 must be used but once as a factor in the g. c. d.

The factors common to these numbers are 2^3 and 3; and their product, 24, is the g. c. d. of 48, 72, and 96.

Find the g. c. d. of:

2. 15, 20, 25	**5.** 26, 52, 78	**8.** 42, 56, 70
3. 16, 20, 24	**6.** 60, 75, 90	**9.** 36, 54, 72
4. 27, 36, 42	**7.** 17, 34, 51	**10.** 19, 76, 95

REDUCTION OF FRACTIONS

To reduce a fraction to lowest terms.

STUDY RECITATION

1. Reduce $\frac{28}{84}$ to its lowest terms.

$$\frac{28}{84} \div \frac{7}{7} = \frac{4}{12}$$
$$\frac{4}{12} \div \frac{4}{4} = \frac{1}{3}$$
Or,
$$\frac{28}{84} \div \frac{28}{28} = \frac{1}{3}$$

You may divide both terms of a fraction by the same number without altering the value of the fraction. You see at once that 7 is a common factor of 28 and 84. Divide both terms by 7. The result is $\frac{4}{12}$. Then divide both terms of $\frac{4}{12}$ by 4. The result is $\frac{1}{3}$.

Or, divide both terms at once by g. c. d., which is 28.

Divide both terms of the fraction by any common divisor; divide the resulting terms by any common divisor, and so continue until the terms have no common divisor. Or,

Divide both terms of the fraction by their g. c. d.

Change:

a.	*b.*	*c.*
2. $\frac{2}{4}$ to halves.	**6.** $\frac{6}{12}$ to halves.	**10.** $\frac{2}{12}$ to sixths.
3. $\frac{3}{6}$ to halves.	**7.** $\frac{6}{10}$ to fifths.	**11.** $\frac{12}{16}$ to fourths.
4. $\frac{4}{8}$ to halves.	**8.** $\frac{4}{12}$ to thirds.	**12.** $\frac{5}{15}$ to thirds.
5. $\frac{3}{9}$ to thirds.	**9.** $\frac{8}{16}$ to halves.	**13.** $\frac{3}{15}$ to fifths.

14. Reduce at sight to lowest terms:

$\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{6}{12}$, $\frac{8}{16}$, $\frac{3}{9}$, $\frac{6}{9}$, $\frac{4}{10}$, $\frac{2}{6}$, $\frac{6}{8}$, $\frac{6}{10}$, $\frac{5}{10}$, $\frac{10}{12}$, $\frac{9}{12}$, $\frac{8}{12}$, $\frac{4}{16}$, $\frac{10}{16}$, $\frac{6}{16}$, $\frac{4}{6}$, $\frac{8}{10}$, $\frac{12}{16}$, $\frac{14}{16}$, $\frac{10}{20}$, $\frac{5}{15}$.

WRITTEN

Reduce to lowest terms:

	a.	*b.*	*c.*	*d.*	*e.*
15.	$\frac{6}{21}$	$\frac{5}{100}$	$\frac{35}{100}$	$\frac{26}{39}$	$\frac{90}{450}$
16.	$\frac{18}{24}$	$\frac{10}{100}$	$\frac{55}{100}$	$\frac{22}{66}$	$\frac{270}{300}$

	a.	*b.*	*c.*	*d.*	*e.*
17.	$\frac{20}{25}$	$\frac{20}{100}$	$\frac{75}{100}$	$\frac{49}{70}$	$\frac{125}{600}$
18.	$\frac{14}{16}$	$\frac{15}{100}$	$\frac{95}{100}$	$\frac{63}{81}$	$\frac{250}{1200}$
19.	$\frac{15}{45}$	$\frac{25}{100}$	$\frac{45}{100}$	$\frac{35}{40}$	$\frac{170}{100}$
20.	$\frac{9}{24}$	$\frac{40}{100}$	$\frac{85}{100}$	$\frac{27}{36}$	$\frac{875}{1000}$
21.	$\frac{21}{27}$	$\frac{60}{100}$	$\frac{105}{100}$	$\frac{12}{84}$	$\frac{25}{800}$
22.	$\frac{25}{30}$	$\frac{80}{100}$	$\frac{15}{87}$	$\frac{45}{60}$	$\frac{65}{100}$
23.	$\frac{36}{42}$	$\frac{90}{100}$	$\frac{17}{34}$	$\frac{19}{38}$	$\frac{375}{1000}$
24.	$\frac{21}{28}$	$\frac{50}{100}$	$\frac{17}{51}$	$\frac{57}{60}$	$\frac{500}{1000}$
25.	$\frac{56}{84}$	$\frac{64}{80}$	$\frac{24}{28}$	$\frac{24}{96}$	$\frac{625}{1000}$
26.	$\frac{32}{36}$	$\frac{32}{40}$	$\frac{13}{39}$	$\frac{25}{125}$	$\frac{1125}{1000}$

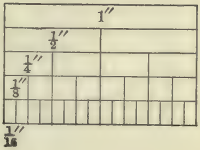
Study Recitation

To reduce a fraction to a fraction of the same value having a given larger denominator.

1. Reduce $\frac{1}{2}$ to 4ths.

$1 = \frac{4}{4}$. $\frac{1}{2}$ of $1 = \frac{1}{2}$ of $\frac{4}{4} = \frac{2}{4}$.

2. Prove your answer by reference to the illustration.



Thus, $\frac{1}{2}$ inch equals $\frac{2}{4}$ of an inch.

3. Change $\frac{1}{2}$ to 8ths; to 16ths.

4. Change $\frac{1}{4}$ to 8ths; to 16ths.

5. Change $\frac{3}{4}$ to 16ths; $\frac{3}{8}$ to 16ths; $\frac{5}{8}$ to 16ths.

6. Reduce $\frac{2}{3}$ to twenty-fourths.

$24 \div 3 = 8$

$\frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$

You may multiply both terms of a fraction by the same number without changing its value. $24 \div 3 = 8$. To change the denominator 3 to 24, it must be multiplied by 8. Hence the numerator must also be multiplied by 8. *Ans.* $\frac{16}{24}$.

NOTE. In practice, merely multiply the numerator and write the result over the required denominator.

Divide the required denominator by the denominator of the given fraction, and multiply the quotient by the numerator. Write the product over the required denominator.

7. Change to eighths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$.

8. Change to tenths: $\frac{1}{2}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

9. Change to sixteenths: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

10. Change to twelfths: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

11. Change to twenty-fourths: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{1}{12}$.

12. Change to twentieths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{5}$, $\frac{1}{10}$, $\frac{3}{4}$, $\frac{3}{5}$.

13. Change $\frac{2}{9}$ to eighteenths; $\frac{3}{7}$ to twenty-firsts.

14. Change $\frac{4}{5}$ to tenths; fifteenths.

15. Change $\frac{3}{4}$ to eighths; twelfths; sixteenths. Change the resulting fractions back to $\frac{3}{4}$. What is the process?

16. Change $\frac{2}{3}$ to sixths; ninths; twelfths; fifteenths. Change the resulting fractions back to $\frac{2}{3}$. What is the process?

17. Change $\frac{3}{5}$ to fractions having the following denominators: 20, 10, 25, 15.

18. Change $\frac{2}{7}$ to fractions having the following denominators: 14, 35, 49, 21.

19. Change $\frac{5}{6}$ to fractions having the following denominators: 12, 36, 24, 18.

WRITTEN

20. Change $\frac{7}{8}$ to fractions having the following denominators: 40, 16, 32, 56.

21. Change $\frac{5}{9}$ to fractions having the following denominators: 45, 18, 99, 54.

22. Change $\frac{7}{10}$ to fractions having the following denominators: 20, 90, 40, 30.

23. Change $\frac{5}{12}$ to fractions having the following denominators: 36, 48, 24, 60.

Reduce:

24. $\frac{3}{4}$ and $\frac{3}{5}$ to 20ths.

25. $\frac{5}{6}$ and $\frac{2}{3}$ to 12ths.

26. $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$ to 16ths.

27. $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{5}{6}$ to 24ths.

28. $\frac{3}{7}$, $\frac{1}{5}$, and $\frac{6}{7}$ to 35ths.

29. $\frac{1}{2}$, $\frac{1}{7}$, and $\frac{5}{6}$ to 42ds.

30. $\frac{3}{8}$, $\frac{4}{7}$, and $\frac{1}{2}$ to 56ths.

31. $\frac{3}{5}$, $\frac{3}{4}$, and $\frac{1}{2}$ to 100ths.

Study Recitation

A **multiple** of a number is a number exactly divisible by the number.

A **common multiple** of two or more numbers is a multiple of each of them. 36 is a common multiple of 2, 3, 4, and 6, because each of the numbers is an exact divisor of 36.

24 is a common multiple of 2, 3, 4, 6, for the same reason.

12 is a common multiple of 2, 3, 4, 6, for the same reason.

The **least common multiple** of two or more numbers is the least number that is a multiple of each of them. 36, 24, and 12 are common multiples of 2, 3, 4, and 6. 12 is the smallest of these multiples, and as it is the smallest number exactly divisible by 2, 3, 4, and 6, it is their least common multiple.

Fractions have a **common denominator** when they have the same denominator. The fractions $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, have a common denominator, 8. Such fractions are called **similar fractions**.

The fractions $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$, have not a common denominator, but may be reduced to other fractions having a common denominator. The *smallest* denominator common to these three fractions is 16. Thus, $\frac{3}{4}=\frac{12}{16}$; $\frac{5}{8}=\frac{10}{16}$; $\frac{7}{16}=\frac{7}{16}$.

The *smallest* denominator common to several fractions is called their **least common denominator**.

The least common denominator of several fractions is the least common multiple of their denominators.

The least common denominator of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{16}$ is 16; of $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{9}$ is 18.

Find by inspection the least common denominator of:

1. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$.

2. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

3. $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{10}$.

4. $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$.

5. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$.

6. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{8}$.

7. $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$.

8. $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{10}$.

9. $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$.

10. $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{12}$.

11. $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$.

12. $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{9}$.

13. $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{12}$.

14. $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{4}$.

NOTE. Before finding the l. c. d., always reduce all fractions to their lowest terms.

The l. c. d. of fractions whose denominators are prime to each other is their product. Thus, the l. c. d. of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ is $3 \times 4 \times 5$, or 60. The l. c. d. of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ is also 60, since 2 is a factor of 4, and any factor of another denominator may be disregarded.

If none of the numbers are prime, try multiples of the largest number until you find one that contains each of the other numbers. Thus, in $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{10}$, try 20, 30, 40, 50, 60. If several, but not all, of the denominators are prime to each other, find the product of those which are prime, and try multiples of this product until you find one that contains the other denominators. Thus, in $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{9}$, $9 \times 5 = 45$. The first multiple of 45 that contains 6 is 90, which is the l. c. d.

TO THE TEACHER. The above method for finding the l. c. d. is easily understood by the pupils and is not confused with the process for finding the g. c. d. The following method may, however, be taught, if preferred.

To find the l. c. d. of two or more fractions.

STUDY RECITATION

Find the prime factors of the denominators of the fractions reduced to their lowest terms. Find the product of all the different prime factors of the given denominators, using each as a factor the greatest number of times it is found in any of them.

1. Find the l. c. d. of $\frac{1}{10}$, $\frac{1}{16}$, and $\frac{1}{24}$.

$$\begin{aligned} 10 &= 2 \times 5 \\ 16 &= 2^4 \\ 24 &= 2^3 \times 3 \\ \text{l. c. d. } & \overline{2^4 \times 3 \times 5} \end{aligned}$$

The different prime factors are 2 and 3 and 5. In 10, 5 is used as a factor once, the greatest number of times it is found in any of the numbers. In 16, 2 is used as a factor 4 times, the greatest number of times it is found in any of the numbers. In 24, 3 is used as a factor once, the greatest number of times it is found in any of the numbers. On bringing these factors together, the l. c. d. is found to be $2^4 \times 3 \times 5$, or 240.

2. Find the l. c. d. of $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{12}$, and $\frac{1}{24}$.

$$\begin{aligned} 10 &= 2 \times 5 \\ 24 &= 2^3 \times 3 \\ \text{l. c. d. } & \overline{2^3 \times 3} \times 5 = 120 \end{aligned}$$

Since 10 is a multiple of 5, any multiple of 10 is also a multiple of 5; hence, 5 may be rejected from the work. Likewise, 12 may be rejected. The l. c. d. of 10 and 24 is found, as in example 1, to be 120.

3. Find the l. c. d. of $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{7}$.

These numbers are prime to each other, hence their product, 105, is their l. c. d.

WRITTEN

4. Reduce $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{3}{16}$ to similar fractions having the l. c. d.

l. c. d. = 48

$$48 \div 8 = 6 \qquad \frac{3 \times 6}{48} = \frac{18}{48}$$

$$48 \div 6 = 8 \qquad \frac{5 \times 8}{48} = \frac{40}{48}$$

$$48 \div 16 = 3 \qquad \frac{3 \times 3}{48} = \frac{9}{48}$$

The l.c.d. is 48. To change $\frac{3}{8}$ to 48ths, write the product of 3×6 over 48: $\frac{18}{48}$. To change $\frac{5}{6}$ to 48ths, write the product of 5×8 over 48: $\frac{40}{48}$. To change $\frac{3}{16}$ to 48ths, write the product of 3×3 over 48: $\frac{9}{48}$.

To the Teacher. Skill in addition and subtraction of fractions depends upon the mastery of this work. When the fractions have been reduced to a common denominator, their addition or subtraction by simple addition or subtraction of the numerators involves no new difficulty.

Change to similar fractions having the l. c. d.:

	a.	*b.*	*c.*	*d.*
5.	$\frac{1}{4}, \frac{1}{6}$	$\frac{1}{6}, \frac{1}{9}$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	$\frac{1}{6}, \frac{1}{12}, \frac{1}{3}$
6.	$\frac{2}{3}, \frac{1}{5}$	$\frac{1}{3}, \frac{1}{10}$	$\frac{1}{2}, \frac{1}{5}, \frac{1}{6}$	$\frac{1}{4}, \frac{1}{8}, \frac{1}{16}$
7.	$\frac{4}{5}, \frac{1}{15}$	$\frac{1}{2}, \frac{2}{15}$	$\frac{1}{3}, \frac{1}{4}, \frac{1}{10}$	$\frac{1}{5}, \frac{1}{6}, \frac{1}{4}$
8.	$\frac{1}{2}, \frac{3}{4}$	$\frac{1}{2}, \frac{5}{8}$	$\frac{1}{3}, \frac{1}{6}, \frac{1}{4}$	$\frac{1}{2}, \frac{1}{5}, \frac{1}{10}$
9.	$\frac{5}{6}, \frac{3}{4}$	$\frac{2}{3}, \frac{3}{7}$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{7}$	$\frac{1}{4}, \frac{1}{5}, \frac{1}{7}$

Change to similar fractions having the l. c. d.:

	a.	*b.*	*c.*
10.	$\frac{1}{2}, \frac{3}{4}, \frac{1}{6}$	$\frac{3}{5}, \frac{3}{4}, \frac{5}{7}$	$\frac{3}{5}, \frac{7}{9}, \frac{29}{30}$
11.	$\frac{2}{3}, \frac{3}{4}, \frac{1}{2}$	$\frac{4}{5}, \frac{7}{9}, \frac{5}{12}$	$\frac{7}{9}, \frac{5}{16}, \frac{7}{24}$
12.	$\frac{1}{2}, \frac{3}{4}, \frac{2}{5}$	$\frac{5}{6}, \frac{7}{9}, \frac{5}{8}$	$\frac{7}{8}, \frac{2}{3}, \frac{5}{6}, \frac{7}{30}$
13.	$\frac{2}{3}, \frac{1}{2}, \frac{4}{5}$	$\frac{7}{12}, \frac{9}{16}, \frac{11}{20}$	$\frac{3}{10}, \frac{4}{15}, \frac{19}{20}, \frac{9}{5}$
14.	$\frac{1}{2}, \frac{1}{6}, \frac{3}{5}$	$\frac{3}{4}, \frac{5}{9}, \frac{13}{14}$	$\frac{3}{4}, \frac{7}{8}, \frac{1}{2}, \frac{7}{60}$
15.	$\frac{2}{3}, \frac{5}{6}, \frac{2}{9}$	$\frac{11}{12}, \frac{7}{20}, \frac{7}{10}$	$\frac{3}{8}, \frac{7}{9}, \frac{3}{5}, \frac{7}{10}$
16.	$\frac{2}{3}, \frac{5}{6}, \frac{4}{5}$	$\frac{7}{10}, \frac{13}{18}, \frac{3}{5}$	$\frac{7}{10}, \frac{4}{25}, \frac{9}{50}, \frac{3}{100}$
17.	$\frac{3}{4}, \frac{2}{5}, \frac{7}{10}$	$\frac{5}{12}, \frac{7}{15}, \frac{9}{50}$	$\frac{9}{20}, \frac{5}{50}, \frac{6}{100}, \frac{9}{25}$
18.	$\frac{7}{30}, \frac{3}{5}, \frac{5}{6}$	$\frac{3}{4}, \frac{7}{32}, \frac{5}{8}$	$\frac{1}{2}, \frac{5}{8}, \frac{7}{16}$
19.	$\frac{5}{18}, \frac{1}{2}, \frac{1}{3}$	$\frac{5}{24}, \frac{5}{8}, \frac{5}{10}$	$\frac{5}{12}, \frac{7}{36}, \frac{2}{3}$

20. Reduce the following fractions to hundredths:

$\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}, \frac{1}{20}, \frac{1}{50}, \frac{3}{4}, \frac{2}{5}, \frac{3}{5}, \frac{3}{10}, \frac{4}{10}, \frac{7}{10}, \frac{9}{10}, \frac{3}{20}, \frac{7}{20}, \frac{9}{20}, \frac{13}{20}, \frac{17}{20}, \frac{19}{20}, \frac{3}{25}, \frac{7}{25}, \frac{9}{25}, \frac{3}{50}, \frac{7}{50}, \frac{9}{50}.$

To the Teacher. The above exercise should be thoroughly mastered because it prepares for work in percentage.

To reduce an improper fraction to a whole or a mixed number.

STUDY RECITATION

1. Reduce $\frac{39}{12}$ to a mixed number.

$\frac{39}{12} = 39 \div 12 = 3\frac{3}{12}$, or $3\frac{1}{4}$.

$\frac{39}{12}$ is an indicated division.

Performing the indicated division gives $3\frac{3}{12}$. Reducing the fractional part, $\frac{3}{12}$, to the lowest terms gives the result $3\frac{1}{4}$.

Divide the numerator by the denominator.

Reduce to whole or to mixed numbers:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
2.	$\frac{15}{4}$	$\frac{17}{8}$	$\frac{19}{6}$	$\frac{25}{3}$	$\frac{48}{9}$	$\frac{63}{9}$
3.	$\frac{27}{8}$	$\frac{46}{12}$	$\frac{29}{7}$	$\frac{45}{5}$	$\frac{59}{7}$	$\frac{96}{12}$
4.	$\frac{52}{8}$	$\frac{96}{12}$	$\frac{82}{10}$	$\frac{75}{11}$	$\frac{66}{12}$	$\frac{84}{11}$
5.	$\frac{28}{8}$	$\frac{19}{4}$	$\frac{18}{7}$	$\frac{21}{6}$	$\frac{22}{7}$	$\frac{36}{12}$
6.	$\frac{72}{5}$	$\frac{96}{6}$	$\frac{87}{11}$	$\frac{45}{16}$	$\frac{79}{8}$	$\frac{63}{10}$
7.	$\frac{98}{8}$	$\frac{126}{9}$	$\frac{376}{12}$	$\frac{560}{9}$	$\frac{185}{7}$	$\frac{231}{8}$

WRITTEN

Reduce to whole or to mixed numbers:

8.	$\frac{285}{27}$	$\frac{786}{95}$	$\frac{278}{19}$	$\frac{567}{84}$	$\frac{575}{25}$	$\frac{584}{73}$
9.	$\frac{100}{20}$	$\frac{75}{15}$	$\frac{125}{25}$	$\frac{200}{10}$	$\frac{400}{20}$	$\frac{625}{25}$
10.	$\frac{175}{25}$	$\frac{164}{10}$	$\frac{320}{16}$	$\frac{325}{16}$	$\frac{525}{15}$	$\frac{320}{20}$

To reduce a whole number to a fraction.

STUDY RECITATION

1. Change 3 to 8ths.

$$\begin{array}{r} 8 \\ 3 \\ \hline 24 \end{array} \quad \frac{24}{8}$$

$1 = \frac{8}{8}$; $3 = 3 \times \frac{8}{8}$, or $\frac{24}{8}$.

Multiply the required denominator, 8, by the whole number, 3, and write the product, 24, over 8: $\frac{24}{8}$. To do the work without writing, think of 8 times 3 over 8, or $\frac{24}{8}$.

Multiply the denominator of the required fraction by the whole number, and write the product over the denominator.

2. Change the following numbers to 7ths; 2, 3, 5, 8, 7, 9, 6, 12, 4, 15, 18.

3. Change the same numbers to 5ths; to 4ths; to 8ths; to 9ths; to 6ths; to 10ths; to 12ths; to 16ths.

To reduce a mixed number to an improper fraction.

STUDY RECITATION

1. Reduce $7\frac{2}{3}$ to an improper fraction.

$$\begin{array}{r} 7\frac{2}{3} \\ 3 \\ \hline 21 \\ 2 \\ \hline \frac{23}{3} \end{array}$$

1 unit = 3 thirds; therefore 7 units = 7 × 3 thirds, or 21 thirds. $\frac{21}{3} + \frac{2}{3} = \frac{23}{3}$.

Multiply the denominator of the fractional part by the whole number; add the numerator to the product, and write the sum over the denominator.

Reduce to improper fractions:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
2.	$1\frac{1}{2}$	$3\frac{1}{4}$	$5\frac{2}{3}$	$3\frac{3}{8}$	$4\frac{1}{9}$	$9\frac{3}{5}$
3.	$2\frac{1}{3}$	$4\frac{1}{2}$	$3\frac{3}{4}$	$8\frac{9}{10}$	$6\frac{2}{3}$	$8\frac{4}{7}$
4.	$3\frac{5}{8}$	$5\frac{5}{6}$	$4\frac{2}{5}$	$3\frac{5}{12}$	$2\frac{8}{9}$	$7\frac{4}{9}$
5.	$5\frac{3}{4}$	$7\frac{1}{2}$	$6\frac{1}{5}$	$5\frac{2}{3}$	$4\frac{5}{9}$	$6\frac{5}{8}$

WRITTEN

6.	$3\frac{1}{16}$	$98\frac{2}{15}$	$275\frac{3}{8}$	$256\frac{1}{2}$	$175\frac{5}{8}$	$162\frac{5}{6}$
7.	$8\frac{5}{12}$	$112\frac{9}{15}$	$125\frac{2}{3}$	$127\frac{5}{9}$	$180\frac{4}{5}$	$154\frac{4}{5}$
8.	$37\frac{7}{8}$	$318\frac{3}{5}$	$145\frac{2}{7}$	$103\frac{7}{8}$	$120\frac{3}{8}$	$201\frac{6}{7}$
9.	$12\frac{2}{3}$	$213\frac{4}{5}$	$135\frac{7}{12}$	$215\frac{3}{5}$	$130\frac{2}{5}$	$205\frac{7}{8}$

ADDITION AND SUBTRACTION OF FRACTIONS

Study Recitation

In fractions, as in whole numbers, only units of the same denomination can be added or subtracted; hence, the fractions to be added or subtracted must be reduced to fractions having a common denominator. In other words, they must be made **similar**. The indicated operation may then be performed and the result reduced to lowest terms, or to a whole number or a mixed number.

When mixed numbers are to be added or subtracted, the fractions and integers may be added or subtracted separately.

1. Find the sum of $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{5}{6}$.

$$\text{l. c. d.} = 24$$
$$\tfrac{2}{3} + \tfrac{7}{8} + \tfrac{5}{6} =$$
$$\tfrac{16}{24} + \tfrac{21}{24} + \tfrac{20}{24} = \tfrac{57}{24} = 2\tfrac{3}{8}$$

The l. c. d. of the fractions is 24.
$\frac{2}{3} = \frac{16}{24}$; $\frac{7}{8} = \frac{21}{24}$; $\frac{5}{6} = \frac{20}{24}$
The sum of these fractions is $\frac{57}{24}$, or $2\frac{9}{24}$, or, in lowest terms, $2\frac{3}{8}$.

2. $\frac{7}{12} + \frac{9}{16} + \frac{5}{8} = ?$

$$\frac{28 + 27 + 30}{48} = \frac{85}{48}$$
$$= 1\tfrac{37}{48}$$

In reducing the fractions to other fractions having a common denominator, it is convenient to write the resulting numerator in each case directly under the fraction from which it is obtained, as in example 2.

3. Add $2\frac{3}{4}$, $6\frac{3}{5}$, and $12\frac{1}{3}$.

$$\begin{aligned} 2\tfrac{3}{4} &= 2\tfrac{45}{60} \\ 6\tfrac{3}{5} &= 6\tfrac{36}{60} \\ 12\tfrac{1}{3} &= 12\tfrac{20}{60} \\ \hline & \, 20\tfrac{101}{60} = 21\tfrac{41}{60} \end{aligned}$$

The sum of the whole numbers, 2, 6, and 12, is 20. The l. c. d. of the fractions $\frac{3}{4}$, $\frac{3}{5}$, and $\frac{1}{3}$ is 60.

Reduce each fraction to 60ths and add. The sum is $1\frac{41}{60}$, which added to 20 equals $21\frac{41}{60}$.

4. From $\frac{5}{8}$ take $\frac{2}{5}$.

$$\begin{aligned} \tfrac{5}{8} &= \tfrac{25}{40} \\ \tfrac{2}{5} &= \tfrac{16}{40} \\ \hline & \, \tfrac{9}{40} \end{aligned}$$

Reduce both fractions to 40ths;
$\frac{5}{8} = \frac{25}{40}$; $\frac{2}{5} = \frac{16}{40}$. $\frac{25}{40} - \frac{16}{40} = \frac{9}{40}$.

5. From $26\frac{2}{5}$ subtract $12\frac{5}{12}$.

$$\begin{array}{rcl} 26\frac{2}{5} & = 26\frac{24}{60} = & 25\frac{84}{60} \\ 12\frac{5}{12} & = 12\frac{25}{60} = & \underline{12\frac{25}{60}} \\ & & 13\frac{59}{60} \end{array}$$

The l. c. d. is 60. As $\frac{25}{60}$ cannot be taken from $\frac{24}{60}$, take 1, or $\frac{60}{60}$, from 26, leaving 25. $\frac{60}{60} + \frac{24}{60} = \frac{84}{60}$; $\frac{84}{60} - \frac{25}{60} = \frac{59}{60}$.
12 from 25 = 13.

To the Teacher. The following method may be taught, if preferred.

$$\begin{array}{rl} 26\frac{2}{5} & = 25 + 1 \ + \frac{2}{5} \\ 12\frac{5}{12} & = \underline{12 + \frac{5}{12} \qquad\quad} \\ & \ \ 13 + \frac{7}{12} + \frac{2}{5} \end{array}$$

$$13 + \frac{35 + 24}{60} = 13\frac{59}{60}$$

$\frac{5}{12}$ cannot be taken from $\frac{2}{5}$, but can be taken from 1. Take 1 from 26, leaving 25. From the 1 take $\frac{5}{12}$, leaving $\frac{7}{12}$. Take 12 from 25, leaving 13. The entire remainder is $13 + \frac{7}{12} + \frac{2}{5}$, which equals $13\frac{59}{60}$.

Note. Always reduce fractions to lowest terms before attempting to find the l. c. d.

Add or subtract as indicated:

	a.	*b.*	*c.*	*d.*
6.	$\frac{1}{2} + \frac{1}{4}$	$\frac{2}{3} + \frac{3}{4}$	$\frac{1}{2} + \frac{2}{3}$	$\frac{1}{12} + \frac{5}{6}$
7.	$\frac{1}{3} - \frac{1}{6}$	$\frac{5}{6} - \frac{5}{12}$	$\frac{4}{5} - \frac{1}{2}$	$\frac{5}{6} - \frac{3}{8}$
8.	$\frac{9}{12} - \frac{1}{2}$	$\frac{9}{24} - \frac{1}{4}$	$\frac{8}{15} - \frac{2}{5}$	$\frac{1}{3} - \frac{7}{30}$
9.	$\frac{1}{4} + \frac{1}{8}$	$\frac{1}{2} - \frac{3}{10}$	$\frac{5}{6} - \frac{3}{4}$	$\frac{5}{9} - \frac{1}{3}$
10.	$\frac{5}{16} - \frac{2}{8}$	$\frac{9}{24} - \frac{1}{8}$	$\frac{7}{9} - \frac{3}{5}$	$\frac{8}{15} - \frac{2}{5}$
11.	$\frac{1}{15} + \frac{1}{5}$	$\frac{7}{10} - \frac{1}{2}$	$\frac{5}{6} + \frac{3}{4}$	$\frac{5}{16} - \frac{1}{4}$
12.	$\frac{2}{3} + \frac{4}{5}$	$\frac{2}{3} + \frac{3}{8}$	$\frac{5}{12} + \frac{3}{4}$	$\frac{7}{8} - \frac{2}{3}$

First add; then subtract:

	a.	*b.*	*c.*	*d.*	*e.*
13.	$5\frac{1}{2}$	$8\frac{5}{6}$	$9\frac{9}{10}$	$7\frac{4}{5}$	$6\frac{3}{16}$
	$3\frac{1}{3}$	$4\frac{5}{12}$	$2\frac{2}{5}$	$4\frac{1}{2}$	$3\frac{1}{2}$
14.	$8\frac{3}{4}$	$9\frac{7}{12}$	$7\frac{3}{4}$	$6\frac{5}{9}$	$5\frac{1}{4}$
	$3\frac{7}{8}$	$4\frac{1}{3}$	$2\frac{5}{6}$	$2\frac{1}{3}$	$4\frac{5}{16}$
15.	$15\frac{1}{15}$	$14\frac{5}{12}$	$13\frac{1}{2}$	$20\frac{3}{4}$	$15\frac{7}{8}$
	$10\frac{1}{5}$	$12\frac{5}{6}$	$11\frac{2}{3}$	$12\frac{5}{6}$	$13\frac{5}{12}$

Written

Add:

	a.	b.	c.
16.	$\frac{1}{3}, \frac{1}{5}, \frac{1}{10}$	$\frac{7}{8}, \frac{4}{5}, \frac{7}{10}$	$\frac{7}{8}, \frac{5}{6}, \frac{4}{5}, \frac{2}{3}$
17.	$\frac{7}{8}, \frac{3}{4}, \frac{1}{5}$	$\frac{1}{8}, \frac{3}{5}, \frac{2}{9}$	$\frac{8}{9}, \frac{6}{7}, \frac{4}{5}, \frac{3}{4}$
18.	$\frac{5}{6}, \frac{2}{3}, \frac{1}{2}$	$\frac{4}{7}, \frac{6}{11}, \frac{2}{5}$	$\frac{8}{9}, \frac{7}{8}, \frac{6}{7}, \frac{5}{12}$
19.	$\frac{5}{6}, \frac{1}{2}, \frac{3}{10}$	$\frac{3}{8}, \frac{3}{10}, \frac{1}{12}$	$\frac{15}{16}, \frac{7}{8}, \frac{6}{7}, \frac{3}{4}$
20.	$\frac{7}{8}, \frac{3}{4}, \frac{2}{3}$	$\frac{3}{4}, \frac{2}{7}, \frac{1}{3}$	$\frac{4}{5}, \frac{3}{4}, \frac{3}{5}, \frac{1}{2}$
21.	$\frac{4}{5}, \frac{1}{2}, \frac{1}{3}$	$\frac{11}{12}, \frac{9}{10}, \frac{4}{5}$	$\frac{19}{20}, \frac{15}{16}, \frac{7}{8}, \frac{4}{5}$

22. In the above examples, find the difference between the fractions in the first and second columns; between those in the second and third columns. Prove your answers. Thus, 16, *a*, $\frac{1}{3} - \frac{1}{5}$, $\frac{1}{5} - \frac{1}{10}$. 18, *b*, $\frac{4}{7} - \frac{6}{11}$; $\frac{6}{11} - \frac{2}{5}$.

23–52. Add the fractions in examples 10 to 19, p. 104.

Add or subtract as indicated:

	a.	b.	c.
53.	$\frac{7}{8} + \frac{9}{10} + \frac{4}{5}$	$\frac{8}{15} + \frac{3}{8} + \frac{1}{4}$	$\frac{13}{15} - \frac{7}{20}$
54.	$\frac{9}{14} + \frac{5}{8} + \frac{4}{5}$	$\frac{15}{22} - \frac{7}{12} + \frac{3}{8}$	$\frac{15}{16} - \frac{7}{24}$
55.	$\frac{11}{12} + \frac{7}{18} + \frac{7}{20}$	$\frac{11}{18} - \frac{5}{12}$	$\frac{9}{20} - \frac{3}{8}$

Add:

	a.	b.	c.
56.	$2\frac{1}{2} + 4\frac{8}{9}$	$8\frac{1}{2} + 9\frac{2}{3} + 4\frac{1}{4}$	$3\frac{7}{8} + 9\frac{4}{5} + 12\frac{3}{4}$
57.	$5\frac{1}{3} + 12\frac{4}{5}$	$13\frac{3}{4} + 7\frac{1}{5} + 8\frac{1}{2}$	$68\frac{1}{2} + 27\frac{5}{16} + 10\frac{1}{3}$
58.	$16\frac{7}{8} + 12\frac{3}{4}$	$21\frac{1}{2} + 3\frac{5}{6} + 4\frac{2}{3}$	$56\frac{3}{8} + 21\frac{4}{9} + 10\frac{1}{3}$
59.	$16\frac{7}{9} + 10\frac{2}{3}$	$12\frac{5}{12} + 5\frac{3}{4} + 2\frac{1}{2}$	$56\frac{3}{4} + 27\frac{3}{10} + 14\frac{1}{6}$
60.	$15\frac{3}{8} + 7\frac{5}{6}$	$10\frac{3}{10} + 5\frac{4}{5} + 2\frac{1}{2}$	$15\frac{3}{8} + 10\frac{3}{16} + 12\frac{3}{4}$

Subtract:

	a.	*b.*	*c.*
61.	$16\frac{2}{3} - 12\frac{1}{2}$	$18\frac{3}{8} - 12\frac{4}{7}$	$125\frac{7}{8} - 114\frac{3}{16}$
62.	$33\frac{1}{3} - 16\frac{2}{3}$	$156\frac{5}{12} - 85\frac{1}{3}$	$223\frac{9}{10} - 116\frac{4}{15}$
63.	$37\frac{1}{2} - 16\frac{2}{3}$	$275\frac{5}{16} - 183\frac{5}{12}$	$100\frac{1}{2} - 50\frac{7}{10}$
64.	$87\frac{1}{2} - 83\frac{1}{3}$	$286\frac{3}{10} - 156\frac{5}{7}$	$200\frac{1}{10} - 16\frac{3}{4}$
65.	$83\frac{1}{3} - 62\frac{1}{2}$	$314\frac{5}{12} - 156\frac{5}{6}$	$119\frac{5}{8} - 100\frac{3}{4}$

First add; then subtract:

	a.	*b.*	*c.*	*d.*	*e.*
66.	$\begin{array}{r}17\frac{2}{3}\\ 8\frac{1}{2}\\ \hline\end{array}$	$\begin{array}{r}23\frac{3}{10}\\ 5\frac{2}{5}\\ \hline\end{array}$	$\begin{array}{r}92\frac{1}{2}\\ 60\frac{5}{12}\\ \hline\end{array}$	$\begin{array}{r}163\frac{3}{4}\\ 109\frac{3}{5}\\ \hline\end{array}$	$\begin{array}{r}320\frac{3}{4}\\ 261\frac{3}{10}\\ \hline\end{array}$
67.	$\begin{array}{r}50\frac{3}{4}\\ 48\frac{5}{8}\\ \hline\end{array}$	$\begin{array}{r}40\frac{1}{4}\\ 15\frac{15}{16}\\ \hline\end{array}$	$\begin{array}{r}75\frac{1}{2}\\ 16\frac{1}{2}\\ \hline\end{array}$	$\begin{array}{r}219\frac{7}{8}\\ 157\frac{6}{7}\\ \hline\end{array}$	$\begin{array}{r}135\frac{1}{5}\\ 122\frac{4}{15}\\ \hline\end{array}$
68.	$\begin{array}{r}80\frac{5}{6}\\ 18\frac{2}{3}\\ \hline\end{array}$	$\begin{array}{r}30\frac{1}{4}\\ 14\frac{13}{16}\\ \hline\end{array}$	$\begin{array}{r}90\frac{7}{12}\\ 67\frac{3}{4}\\ \hline\end{array}$	$\begin{array}{r}300\frac{5}{9}\\ 150\frac{5}{7}\\ \hline\end{array}$	$\begin{array}{r}180\frac{1}{4}\\ 112\frac{5}{24}\\ \hline\end{array}$
69.	$\begin{array}{r}80\frac{3}{7}\\ 19\frac{3}{4}\\ \hline\end{array}$	$\begin{array}{r}92\frac{3}{5}\\ 49\frac{1}{4}\\ \hline\end{array}$	$\begin{array}{r}70\frac{11}{24}\\ 48\frac{5}{8}\\ \hline\end{array}$	$\begin{array}{r}500\frac{11}{12}\\ 250\frac{3}{4}\\ \hline\end{array}$	$\begin{array}{r}700\frac{7}{20}\\ 300\frac{7}{12}\\ \hline\end{array}$
70.	$\begin{array}{r}75\frac{2}{3}\\ 16\frac{7}{12}\\ \hline\end{array}$	$\begin{array}{r}83\frac{5}{9}\\ 68\frac{2}{3}\\ \hline\end{array}$	$\begin{array}{r}95\frac{3}{8}\\ 89\frac{1}{5}\\ \hline\end{array}$	$\begin{array}{r}600\frac{3}{4}\\ 300\frac{2}{3}\\ \hline\end{array}$	$\begin{array}{r}500\frac{5}{7}\\ 400\frac{3}{4}\\ \hline\end{array}$
71.	$\begin{array}{r}94\frac{3}{4}\\ 69\frac{3}{7}\\ \hline\end{array}$	$\begin{array}{r}87\frac{4}{5}\\ 28\frac{8}{9}\\ \hline\end{array}$	$\begin{array}{r}65\frac{5}{6}\\ 54\frac{7}{8}\\ \hline\end{array}$	$\begin{array}{r}408\frac{5}{8}\\ 399\frac{15}{16}\\ \hline\end{array}$	$\begin{array}{r}700\frac{2}{3}\\ 200\frac{11}{12}\\ \hline\end{array}$

Add:

72.	$\begin{array}{r}69\frac{3}{4}\\ 14\frac{1}{5}\\ 54\frac{2}{3}\\ \hline\end{array}$	$\begin{array}{r}84\frac{1}{2}\\ 73\frac{1}{4}\\ 69\frac{2}{3}\\ \hline\end{array}$	$\begin{array}{r}17\frac{2}{5}\\ 96\frac{3}{10}\\ 85\frac{4}{15}\\ \hline\end{array}$	$\begin{array}{r}52\frac{1}{2}\\ 69\frac{2}{3}\\ 78\frac{5}{6}\\ \hline\end{array}$	$\begin{array}{r}43\frac{3}{8}\\ 99\frac{4}{5}\\ 71\frac{7}{16}\\ \hline\end{array}$

73. $83\frac{1}{2}$	$56\frac{3}{4}$	$40\frac{1}{3}$	$69\frac{3}{10}$	$48\frac{2}{3}$
$79\frac{1}{3}$	$48\frac{1}{2}$	$39\frac{5}{12}$	$84\frac{7}{12}$	$57\frac{4}{5}$
$56\frac{1}{4}$	$37\frac{3}{8}$	$27\frac{1}{6}$	$57\frac{3}{20}$	$69\frac{5}{6}$
$42\frac{1}{5}$	$24\frac{3}{16}$	$16\frac{5}{6}$	$68\frac{1}{4}$	$85\frac{7}{8}$

Drill Exercises

To the Teacher. Read the following examples, and require pupils to write results. After each example is read, the teacher should pause a moment and then give the order "Write," and at that instant each pupil should write his result. If he has not secured the result, he should make a dash in place of it. After several of the examples have been read, one pupil should be called upon to read his results in order, and errors should be corrected. The pupils who are slowest in the work should have the drill more frequently than the others.

	a.	*b.*	*c.*	*d.*
1.	$2\frac{1}{2} + 3\frac{1}{4}$	$\frac{5}{6} + \frac{2}{3}$	$\frac{7}{8} - \frac{2}{3}$	$3\frac{2}{3} - 2\frac{1}{2}$
2.	$3\frac{1}{3} + 5\frac{1}{6}$	$\frac{3}{8} + \frac{3}{4}$	$\frac{5}{6} - \frac{2}{3}$	$12\frac{7}{8} - 5\frac{3}{4}$
3.	$4\frac{1}{2} + 3\frac{5}{8}$	$\frac{4}{5} + \frac{2}{3}$	$\frac{3}{4} - \frac{1}{3}$	$9\frac{11}{16} - 4\frac{3}{8}$
4.	$6\frac{1}{4} + 7\frac{2}{3}$	$\frac{5}{6} + \frac{1}{2}$	$\frac{4}{7} - \frac{1}{2}$	$7\frac{5}{9} + 2\frac{2}{3}$
5.	$8\frac{1}{5} + 3\frac{1}{3}$	$\frac{2}{3} + \frac{5}{9}$	$\frac{7}{10} - \frac{3}{5}$	$6\frac{3}{4} - 4\frac{1}{6}$

PROBLEMS

Oral

Indicate operations:

1. A farmer sold $\frac{1}{3}$ of his land at one time, and $\frac{1}{4}$ of it at another time. What part of his land did he sell? What part did he have left?

2. I buy sugar for $\frac{3}{4}$ of a dollar, cloth for $\frac{3}{5}$ of a dollar, and ribbon for $\frac{2}{5}$ of a dollar. How much change do I get if I pay with a $2 bill?

3. A boy walks $2\frac{1}{2}$ mi. the first hour, $3\frac{1}{4}$ mi. the second hour, and $2\frac{3}{4}$ mi. the third hour. How many miles does he walk in the three hours?

4. From a cask containing 30 gallons of water, $10\frac{1}{4}$ gallons are drawn off at one time, and $8\frac{1}{2}$ gallons at another time. How many gallons are left?

5. A pillowcase is to be 27 inches long when finished. How long must it be cut to allow for a hem of $2\frac{1}{2}$ inches and a $\frac{1}{4}$-inch seam at the closed end?

6. A doll's quilt that is 14 inches long and 12 inches wide is bound with tape. Allowing $\frac{3}{8}$ of an inch for each corner, how much tape is required?

7. How much lace do I need for a handkerchief 12 inches square, allowing $1\frac{1}{2}$ inches on each side for fullness and turning corners?

8. How long did it take Ethel to make a cushion cover, if it took her $1\frac{1}{2}$ hours to sew the cover, $2\frac{1}{2}$ hours to embroider the initials, and $2\frac{3}{4}$ hours to crochet a band of lace for it?

9. If I cut $\frac{1}{2}$ a yard of lace from $\frac{7}{8}$ of a yard, how many eighths of a yard are there in each piece?

10. Find the distance around a sheet of paper that is $10\frac{1}{2}$ inches long and $8\frac{1}{4}$ inches wide.

11. Martha made handkerchiefs for her father that were 16 inches square when finished. If the hems were $\frac{1}{2}$ inch wide, how long and wide were the squares of linen before they were hemmed?

Written

12. Frank made a pen tray in $4\frac{5}{6}$ hours, while John made one in $3\frac{2}{3}$ hours. How much longer did it take Frank than John?

13. Find the distance around a room that is $18\frac{7}{8}$ feet long and $13\frac{1}{2}$ feet wide. How much greater is the length than the width?

14. John made $6\frac{3}{4}$ gallons of lemonade for a fair and sold $4\frac{1}{2}$ gallons. How much had he left?

15. I bought $15\frac{5}{8}$ yards of silk and cut it into 2 pieces, one of which contained $7\frac{1}{2}$ yards. How long was the other piece?

16. If a room is $18\frac{3}{4}$ feet long and $12\frac{5}{8}$ feet wide, how much greater is the length than the width?

17. Charles cut $2\frac{1}{4}$ inches from a piece of wood $15\frac{5}{16}$ inches long. How long was the piece he had left?

18. Arthur lives $1\frac{1}{10}$ miles from school. How far does he walk if he goes to school and returns home twice a day?

19. I buy a piece of ribbon 10 yards long and cut from it first $1\frac{5}{8}$ yards and then $5\frac{1}{2}$ yards. How much is left in the piece?

20. A boy studies arithmetic $\frac{1}{2}$ hour, grammar $\frac{2}{3}$ of an hour, and spends $\frac{3}{4}$ of an hour in recitation in each subject daily. How much time each day does he give to arithmetic and grammar together?

21. Mr. Smith sold some wheat and spent $\frac{1}{2}$ of the money received for clothing, and $\frac{1}{5}$ of it for groceries. What part of the money did he spend? What part remained?

22. Anna walked $\frac{3}{4}$ of a mile in the forenoon, $\frac{2}{3}$ of a mile in the afternoon, and $\frac{1}{2}$ of a mile in the evening. How far did she walk that day?

23. A merchant sells $\frac{1}{2}$ dozen eggs to one man, $\frac{3}{4}$ dozen to another, and enough to a third to make $1\frac{2}{3}$ dozen in all. How many does the third man buy?

24. From a $2\frac{3}{4}$-acre lot were sold $\frac{1}{3}$ acre and $\frac{1}{4}$ acre. How much remained?

25. The sum of two numbers is $\frac{7}{8}$; one of the numbers is $\frac{2}{3}$. Find the other number.

26. If the printed part of your arithmetic page is 6 inches long and $3\frac{5}{6}$ inches wide and the margins all around are $\frac{2}{3}$ of an inch, how long and how wide is the whole page?

27. A box is made of cardboard $5\frac{3}{4}$ inches long and $2\frac{3}{8}$ inches wide. How much longer is the length than the width? What is the distance around the top of the box?

28. How large must you cut the sheet for a doll's bed that measures 18 inches by 10 inches if the sheet is to be 6 inches longer and 5 inches wider than the bed, allowing $\frac{3}{4}$ inch for the top hem, $\frac{1}{4}$ inch for the foot hem, and $\frac{1}{8}$ inch for each side hem?

29. A doll's bed is 24 inches by 15 inches. How large must the quilt cover be cut if it is to be 6 inches wider and 6 inches longer than the bed, and $\frac{1}{4}$ inch is allowed all the way around for seams?

30. A farmer has 5 fields containing $26\frac{3}{4}$ acres, $32\frac{2}{3}$ acres, $19\frac{5}{8}$ acres, $38\frac{1}{2}$ acres, and $40\frac{7}{10}$ acres. How many acres has he in all?

31. A man having $39\frac{1}{2}$ cords of wood sold $4\frac{3}{4}$ cords to one man, and $\frac{7}{8}$ cord to another. How much remained?

32. A field is $75\frac{1}{2}$ rods long and $33\frac{7}{8}$ rods wide. What is the distance around the field?

33. A man spends $\frac{2}{9}$ of his salary for clothing, $\frac{2}{5}$ of it for board, and $\frac{1}{6}$ of it for other expenses. What part of his salary has he remaining?

34. A steamer sails $350\frac{5}{8}$ miles the first day, $312\frac{2}{3}$ miles the second day, $338\frac{4}{5}$ miles the third day, and is then $312\frac{1}{2}$ miles from port. When the port is reached, how far will the steamer have sailed on the trip?

35. From a cask containing 50 gallons of sirup, a grocer sold to one customer $16\frac{1}{2}$ gallons and to another, $22\frac{7}{8}$ gallons. How many gallons remained unsold? Solve in two ways.

36. If I do $\frac{3}{10}$ of a piece of work one day and $\frac{1}{5}$ of it the next day, what part of it remains to be done?

37. A man does $\frac{3}{8}$ of a piece of work on Monday, and $\frac{5}{16}$ of it on Tuesday. He finishes it on Wednesday. What part of the work does he do on Wednesday?

38. Ella had $\frac{7}{8}$ of a yard of gingham for dolls' dresses. How much more cloth did Ella have to get if she needed $\frac{5}{6}$ of a yard, $\frac{3}{8}$ of a yard, and $1\frac{1}{4}$ yards for three dresses?

39. Lucy was to make a cover for her dresser which was to hang over the ends 10 in. when finished. The hem was $2\frac{1}{2}$ in. wide. How large a piece of linen did she need if the dresser top was 42 in. by 17 in.?

40. If Lucy were to use 2 in. wide lace instead of a hem in the dresser scarf, how much lace would she have to get, allowing 16 in. for corners?

41. Mary's waist measures 23 in. How long must she cut material for a skirt band, if the band is to lap 1 in., and $\frac{1}{4}$ in. is turned in on each end for finishing?

42. How wide should that band be cut, if it is to be 1 in. wide when finished, and $\frac{1}{4}$ in. is allowed on each edge for turning in?

43. How large must I cut a patch for a hole $2\frac{1}{2}$ in. square, if I wish the patch to extend $\frac{1}{4}$ in. beyond the hole on all sides for strength and if I allow $\frac{1}{8}$ in. both on the edges of the hole and on the edge of the patch for finishing?

44. Make five problems involving addition of fractions or mixed numbers and five problems involving subtraction of fractions or mixed numbers.

CANCELLATION

Study Recitation

Cancellation is the process of rejecting equal factors from dividend and divisor, or from numerator and denominator.

In the following statements, the word *numerator* may be substituted for *dividend*, and *denominator* for *divisor*.

1. $\frac{35}{21}$ indicates that 35 is to be divided by 21, 35 being the dividend and 21 the divisor.

$$21\overline{)35}$$
$$1\tfrac{14}{21} = 1\tfrac{2}{3}$$

If both dividend and divisor are divided by 7, the value of the resulting fraction is the same.

$$\frac{7)35}{7)21} = \frac{5}{3} = 1\tfrac{2}{3}$$

What principle is applied in canceling a factor in the numerator and the denominator of a fraction?

2. Reduce $\frac{35}{21}$ to a mixed number.

By the process of cancellation, or rejecting equal factors from both dividend and divisor, the operation is shortened.

$$\frac{35}{21} = \frac{5 \times \cancel{7}}{3 \times \cancel{7}}$$

$$\frac{\overset{5}{\cancel{35}}}{\underset{3}{\cancel{21}}} = \frac{5}{3} = 1\tfrac{2}{3}$$

Canceling the factor 7 in both dividend and divisor, you have remaining $\frac{5}{3}$, or $1\frac{2}{3}$. Striking out the factor 7 is the same as dividing both dividend and divisor, or numerator and denominator, by 7. The same result may be obtained by dividing both 35 and 21 by 7.

3. Reduce $\frac{210}{78}$ to a mixed number.

$$\frac{210}{78} = \frac{\cancel{2} \times \cancel{3} \times 5 \times 7}{\cancel{2} \times \cancel{3} \times 13} = \frac{5 \times 7}{13} = \frac{35}{13} = 2\tfrac{9}{13}$$

In this case the prime factors of the numerator and the denominator are found, and the common factors are canceled.

$$\frac{\overset{\overset{35}{\cancel{105}}}{\cancel{210}}}{\underset{\underset{13}{\cancel{39}}}{\cancel{78}}} = \frac{35}{13} = 2\tfrac{9}{13}$$

In this case both 210 and 78 are divided by 2, giving $\frac{105}{39}$; both 105 and 39 are then divided by 3, giving $\frac{35}{13}$, which equals $2\frac{9}{13}$.

When all the equal factors have been rejected from both dividend and divisor, the remaining factors in the dividend are multiplied together, and the product is divided by the product of the remaining factors in the divisor. Give this statement, substituting the word *numerator* for *dividend* and *denominator* for *divisor*.

4. $$\frac{\overset{\overset{1}{\cancel{3}}}{\cancel{6}} \times 8 \times \overset{8}{\cancel{24}} \times \overset{8}{\cancel{40}}}{\underset{\cancel{3}}{\cancel{9}} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{2}}} = 512$$

Here the factors 5 and 40 are first divided by 5, giving 1 in the divisor and 8 in the dividend. The factors 2 and 6 are then divided by 2, giving 1 in the divisor and 3 in the dividend. The 3 resulting from this division and the 9 in the divisor are divided by 3, giving 1 in the dividend and 3 in the divisor. The 3 in the divisor resulting from this division and the 24 in the dividend are divided by 3, giving 1 in the divisor and 8 in the dividend. Multiplying $8 \times 8 \times 8 \times 1$, the remaining factors in the dividend, gives the product, 512. As there are no factors other than 1 in the divisor, and as dividing by 1 does not affect the quotient, 512 is the answer.

Written

Solve the following by cancellation:

5. $\dfrac{4 \times 9 \times 16}{8 \times 3 \times 5} = ?$

6. $\dfrac{5 \times 18 \times 27 \times 30}{10 \times 9 \times 6 \times 2} = ?$

7. $\dfrac{120 \times 72 \times 48}{60 \times 9 \times 8} = ?$

8. $\frac{7}{8} \times \frac{12}{15} \times \frac{20}{24} = ?$

9. $\dfrac{6 \times 12 \times 14}{7 \times 3 \times 5} = ?$

10. $\dfrac{27 \times 30 \times 36}{15 \times 18} = ?$

11. $\dfrac{48 \times 30 \times 65}{18 \times 7} = ?$

12. $\frac{9}{12} \times \frac{18}{24} \times \frac{15}{25} = ?$

MULTIPLICATION OF FRACTIONS

Study Recitation

The line *AB* is divided into 8 equal parts, each of which is $\frac{1}{8}$ of the whole line. The line *Ac* is $\frac{2}{8}$ of the whole line; it is also $\frac{1}{4}$ of the whole line. The line *Ae* is $\frac{4}{8}$ of the

whole line; it is also $\frac{2}{4}$, also $\frac{1}{2}$, of the whole line. The line *Ad* is $\frac{6}{8}$ of the whole line; it is also $\frac{3}{4}$ of the whole line. In each of the fractions $\frac{1}{8}$, $\frac{2}{8}$, $\frac{4}{8}$, $\frac{6}{8}$, the denominator shows the number of parts into which the line has been divided or the fractional unit, and the numerator shows the number of the parts taken or the number of the fractional units in each.

2 times $\frac{1}{8}$ of the line equals $\frac{2}{8}$ or $\frac{1}{4}$ of the line, or the line *Ac*. 4 times $\frac{1}{8}$ of the line equals $\frac{4}{8}$ or $\frac{1}{2}$ of the line, or the line *Ae*.

Multiplying the numerator of the fraction $\frac{1}{8}$ by 2, multiplies the fraction by 2, because it multiplies the number of parts or the number of fractional units by 2; the result is $\frac{2}{8}$.

Multiplying the numerator of the fraction $\frac{1}{8}$ by 4, multiplies the fraction by 4, because it multiplies the number of parts or the number of fractional units by 4; the result is $\frac{4}{8}$.

Dividing the denominator of the fraction $\frac{1}{8}$ by 2, multiplies the fraction by 2, because it makes each part, or fractional unit, twice as large; $\frac{1}{4}$ is twice as large as $\frac{1}{8}$.

Dividing the denominator of the fraction $\frac{1}{8}$ by 4, multiplies the fraction by 4, because it makes each part or fractional unit 4 times as large as $\frac{1}{8}$; $\frac{1}{2}$ is 4 times as large as $\frac{1}{8}$.

Multiplying the numerator or dividing the denominator of a fraction by a whole number, multiplies the fraction by that number.

1. Draw a line and divide it into 12 equal parts. Each part is what fraction of the whole line? Show that 3 times $\frac{1}{12}$ of the line equals $\frac{3}{12}$ of the line. What is done to change $\frac{1}{12}$ to $\frac{3}{12}$?

2. Show that 3 times $\frac{1}{12}$ of the line equals $\frac{1}{4}$ of the line. What is done to change $\frac{1}{12}$ to $\frac{1}{4}$?

3. In what ways can $\frac{1}{12}$ be multiplied by 3?

4. In what ways can $\frac{1}{12}$ be multiplied by 6? Show by the divisions of the line.

Find products by multiplying numerators:

	a.	*b.*	*c.*	*d.*
5.	$3 \times \frac{1}{8}$	$3 \times \frac{1}{5}$	$3 \times \frac{1}{7}$	$3 \times \frac{1}{10}$
6.	$2 \times \frac{3}{7}$	$3 \times \frac{3}{10}$	$4 \times \frac{2}{11}$	$4 \times \frac{2}{9}$

Find products and reduce to lowest terms:

7.	$3 \times \frac{2}{10}$	$3 \times \frac{4}{6}$	$4 \times \frac{3}{14}$	$5 \times \frac{2}{12}$

Find products by canceling and multiplying:

8.	$4 \times \frac{\overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}}} = \frac{4}{3} = 1\frac{1}{3}$	$3 \times \frac{4}{10}$	$3 \times \frac{5}{10}$	$5 \times \frac{4}{12}$
9.	$4 \times \frac{3}{8}$	$6 \times \frac{5}{12}$	$3 \times \frac{5}{9}$	$2 \times \frac{7}{16}$

Find products by dividing the denominators:

10.	$3 \times \frac{1}{6}$	$4 \times \frac{1}{8}$	$5 \times \frac{1}{10}$	$2 \times \frac{1}{6}$
11.	$3 \times \frac{2}{9}$	$2 \times \frac{5}{12}$	$4 \times \frac{3}{16}$	$5 \times \frac{3}{20}$

12. Illustrate by multiplying $\frac{3}{8}$ by 2 how a fraction may be multiplied by a whole number.

The expression $\frac{3}{4}$ equals 3 times $\frac{1}{4}$, or $\frac{1}{4}$ of 3, or $3 \div 4$.

$\frac{3}{4}$ of $8 = 3 \times \frac{1}{4}$ of 8, or $\frac{1}{4}$ of 3×8, or $3 \times 8 \div 4$, which equals 6.

$\frac{7}{8} \times \frac{3}{4} = (\frac{7}{8} \times 3) \div 4$, or $(\frac{7}{8} \div 4) \times 3$, or $\frac{21}{32}$.

To the Teacher. Every example in multiplication, when either factor, or neither factor, is a whole number, may be treated as a case of multiplying one fraction by another.

To avoid confusion in the pupil's mind, but one rule is given. When the application of this rule is thoroughly mastered, the pupil may readily be led to see that it involves the processes for the different cases, which may be employed if desired.

Finding a fractional part of a fraction is called **multiplying a fraction by a fraction.**

The word "of" between two fractions signifies multiplication.

Find quickly:

	a.	*b.*	*c.*	*d.*
13.	$\frac{1}{2}$ of 8.	$\frac{1}{2}$ of $\frac{1}{6}$.	$\frac{3}{4}$ of $\frac{1}{8}$.	$\frac{3}{4} \times \frac{1}{4}$.
14.	$\frac{2}{3}$ of 6.	$\frac{1}{3}$ of $\frac{1}{9}$.	$\frac{2}{3}$ of $\frac{3}{5}$.	$\frac{2}{3} \times \frac{2}{4}$.
15.	$\frac{3}{4}$ of 12.	$\frac{1}{4}$ of $\frac{4}{9}$.	$\frac{5}{6}$ of $\frac{1}{3}$.	$\frac{3}{4} \times \frac{1}{2}$.
16.	$\frac{1}{2}$ of $\frac{1}{4}$.	$\frac{1}{2}$ of $\frac{2}{4}$.	$\frac{1}{2}$ of $\frac{3}{4}$.	$\frac{1}{2} \times \frac{3}{5}$.
17.	$\frac{1}{3}$ of $\frac{1}{2}$.	$\frac{1}{3}$ of $\frac{1}{5}$.	$\frac{2}{3}$ of $\frac{3}{4}$.	$\frac{2}{3} \times \frac{3}{8}$.

To multiply a fraction by a fraction.

Study Recitation

1. Find $\frac{3}{4}$ of $\frac{7}{8}$.

$\frac{3}{4}$ of $\frac{7}{8} = \frac{21}{32}$. To find $\frac{1}{4}$ of $\frac{7}{8}$, divide $\frac{7}{8}$ by 4 by multiplying the denominator 8 by 4. The result is $\frac{7}{32}$.

$\frac{3}{4}$ of $\frac{7}{8} = 3$ times this result, or $\frac{21}{32}$.

Multiply the numerators together for the numerator of the product and the denominators for the denominator of the product.

If the product is an improper fraction, reduce it to a mixed number.

2. Find the value of $\frac{7}{8} \times \frac{4}{5} \times \frac{3}{14}$.

$$\frac{7}{8} \times \frac{4}{5} \times \frac{3}{14} = \frac{7 \times 4 \times 3}{8 \times 5 \times 14}\,\frac{\text{Numerator}}{\text{Denominator}} = \frac{84}{560} = \frac{3}{20}.$$

Since dividing both numerator and denominator by the same number does not change the value of the fraction, the operation may be shortened by canceling the equal factors in numerator and denominator.

$$\frac{\overset{1}{\cancel{7}} \times \overset{1}{\cancel{4}} \times 3}{\underset{2}{\cancel{8}} \times 5 \times \underset{2}{\cancel{14}}} = \frac{3}{20}$$

Canceling the equal factors 4 and 7 in numerator and denominator, you have left in the numerator the factors 1, 1, and 3, and in the denominator the factors 2, 5, and 2.

Multiplying together the remaining factors in the numerator for the numerator of the product, and the remaining factors in the denominator for the denominator of the product, gives the result $\frac{3}{20}$.

The product of the sum of two numbers by a third number is the same as the sum of the products of each of the two numbers by the third number.

$$\begin{array}{rrl} \text{Sum} & 12 = & 5 + 7\text{, two numbers.} \\ \text{Third number} & \underline{2} = & \underline{2}\text{, third number.} \\ \text{Product of sum by 2,} & 24 = & 10 + 14 = 24\text{, sum of products.} \end{array}$$

3. Multiply $3\frac{2}{5}$ by 4 without reducing the mixed number to an improper fraction.

$$\begin{array}{rl} 3\frac{2}{5} = 3 + & \frac{2}{5} \\ \underline{4} & \underline{4} \\ 12 + & \frac{8}{5} = 13\frac{3}{5}. \end{array}$$

Multiply, and add the products.

4. Find $2\frac{1}{2} \times 4\frac{2}{5}$.

$$2\frac{1}{2} = \frac{5}{2} \qquad 4\frac{2}{5} = \frac{22}{5} \qquad \frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{\cancel{5}} = 11.$$

Write both factors as improper fractions, and solve as in example 2.

5.

$$
\begin{array}{r}
796 \times 19\frac{2}{3} \\
796 \\
\underline{19\frac{2}{3}} \\
530\frac{2}{3} = 796 \times \frac{2}{3} \\
7164 \\
\underline{796} \\
15654\frac{2}{3}
\end{array}
$$

To the Teacher. Require pupils to read the examples, perform the indicated operations mentally, and give the results orally.

Multiply:

	a.	*b.*	*c.*
6.	$5 \times \frac{8}{10}$	$\frac{5}{8}$ of 3	$8 \times 3\frac{2}{3}$
7.	$3 \times \frac{5}{7}$	$\frac{9}{10}$ of 2	$12 \times 3\frac{3}{4}$
8.	$4 \times \frac{5}{12}$	$\frac{5}{8}$ of 3	$7 \times 8\frac{1}{5}$
9.	$6 \times \frac{7}{18}$	$\frac{8}{9}$ of 7	$7 \times 3\frac{1}{4}$
10.	$7 \times \frac{3}{14}$	$\frac{3}{10}$ of 5	$9 \times 5\frac{5}{8}$
11.	$7 \times \frac{3}{28}$	$6 \times 3\frac{1}{2}$	$9 \times 2\frac{2}{3}$
12.	$4 \times \frac{5}{9}$	$7 \times 4\frac{1}{3}$	$8 \times 12\frac{3}{4}$
13.	$5 \times \frac{3}{15}$	$5 \times 2\frac{3}{8}$	$7 \times 13\frac{3}{5}$
14.	$8 \times \frac{5}{6}$	$4 \times 3\frac{7}{8}$	$9 \times 13\frac{5}{8}$
15.	$5 \times \frac{4}{25}$	$8 \times 2\frac{1}{12}$	$8 \times 12\frac{3}{4}$
16.	$\frac{2}{3} \times \frac{4}{5}$	$\frac{5}{6} \times \frac{7}{8}$	$3\frac{2}{5} \times 2$
17.	$\frac{3}{8} \times \frac{5}{7}$	$\frac{4}{7} \times \frac{3}{5}$	$4\frac{1}{2} \times 3$
18.	$\frac{5}{6} \times \frac{5}{8}$	$\frac{5}{12} \times \frac{1}{3}$	$5\frac{1}{3} \times 4$
19.	$\frac{7}{8}$ of $\frac{3}{4}$	$2\frac{1}{2} \times 2$	$6\frac{2}{3} \times 3$
20.	$\frac{2}{7}$ of $\frac{3}{10}$	$3\frac{1}{3} \times 3$	$7\frac{3}{8} \times 3$
21.	$\frac{5}{9}$ of $\frac{2}{3}$	$4\frac{1}{5} \times 2$	$12\frac{1}{2} \times 2$
22.	$\frac{4}{11}$ of $\frac{2}{5}$	$2\frac{3}{8} \times 2$	$37\frac{1}{2} \times 2$
23.	$\frac{5}{6}$ of $\frac{9}{10}$	$3\frac{3}{5} \times 5$	$66\frac{2}{3} \times 30$

Written

Multiply:

	a.	*b.*	*c.*
24.	48 by $16\frac{2}{3}$	$37\frac{1}{2}$ by 100	425 by $29\frac{3}{8}$
25.	39 by $33\frac{1}{3}$	$66\frac{2}{3}$ by 50	632 by $32\frac{4}{5}$
26.	24 by $87\frac{1}{2}$	$83\frac{1}{3}$ by 25	547 by $20\frac{2}{3}$
27.	64 by $37\frac{1}{2}$	$87\frac{1}{2}$ by 1000	421 by $16\frac{3}{8}$
28.	54 by $83\frac{2}{3}$	$33\frac{1}{3}$ by 300	642 by $17\frac{3}{4}$
29.	$2\frac{1}{3} \times 4\frac{2}{5}$	$\frac{2}{3} \times \frac{4}{5} \times \frac{1}{2}$	$\frac{5}{8} \times \frac{3}{16} \times \frac{8}{9}$
30.	$3\frac{1}{8} \times 2\frac{1}{10}$	$\frac{3}{8} \times \frac{1}{2} \times \frac{3}{5}$	$\frac{9}{10}$ of $\frac{15}{16} \times \frac{18}{25}$ of $\frac{8}{9}$
31.	$5\frac{2}{7} \times 4\frac{2}{3}$	$\frac{4}{5} \times \frac{2}{3} \times \frac{7}{10}$	$\frac{45}{49} \times \frac{28}{15} \times \frac{7}{9}$
32.	$27\frac{2}{3} \times 8\frac{2}{5}$	$\frac{3}{8} \times \frac{3}{4} \times \frac{9}{10}$	$12\frac{1}{2} \times 8\frac{1}{3} \times \frac{6}{50}$
33.	$15\frac{1}{2} \times 8\frac{2}{5}$	$\frac{2}{7} \times \frac{5}{8} \times 3$	$37\frac{1}{2} \times 6\frac{1}{4} \times \frac{7}{8}$
34.	$4\frac{3}{8} \times 2\frac{2}{3}$	$\frac{4}{5} \times \frac{3}{7} \times 2$	$33\frac{1}{3} \times 3\frac{1}{3}$
35.	$8\frac{1}{2} \times 3\frac{1}{9}$	$\frac{3}{8} \times \frac{3}{5} \times \frac{3}{4}$	$16\frac{2}{3} \times 12\frac{1}{2} \times \frac{12}{25}$
36.	$6\frac{2}{3} \times 1\frac{3}{4}$	$2\frac{1}{2} \times 2 \times 3\frac{1}{3}$	$37\frac{1}{2} \times \frac{4}{7}$ of $\frac{21}{50}$
37.	$12\frac{1}{2} \times 5\frac{3}{8}$	$4\frac{1}{3} \times 3 \times 2$	$66\frac{2}{3} \times \frac{19}{20} \times 3\frac{1}{3}$
38.	$33\frac{1}{3} \times \frac{3}{13}$	$2\frac{2}{3} \times 3 \times \frac{1}{2}$	$87\frac{1}{2} \times \frac{5}{6}$ of $\frac{36}{55}$

PROBLEMS

Oral

1. A family uses $\frac{7}{8}$ of a pound of sugar a day. How much will it use in a week?

2. John walks at the rate of $3\frac{3}{4}$ miles per hour. How far can he walk in 6 hours, at the same rate?

3. If a man saws $\frac{7}{8}$ of a cord of wood in a working day, how much can he saw in $\frac{2}{3}$ of a working day?

4. Find the cost of $2\frac{1}{2}$ pounds of rice at $7\frac{1}{2}$ ¢ a pound.

5. Find the cost of $3\frac{3}{4}$ cords of wood at \$6 a cord.

6. Find the cost of $2\frac{5}{8}$ yards of velvet at \$1.60 a yard.

7. Find the cost of $2\frac{1}{2}$ pounds of steak at $12\frac{1}{2}$ cents a pound.

8. I bought $\frac{3}{4}$ of a yard of satin @ \$2.60, and handed the clerk a \$2 bill. How much change did I receive?

9. I sell $\frac{3}{4}$ of 70 bushels of potatoes at \$.50 a bushel. How much do I receive for them?

10. A gardener has a garden containing 48 square rods. He has $\frac{3}{8}$ of it planted with peas, and $\frac{1}{4}$ of it with beans. What part of the garden is planted with peas and beans? how many square yards?

11. Draw a plan of the garden, showing the part planted with peas; with beans.

12. There are 45 pupils in the sewing class, and $\frac{7}{9}$ as many in a woodworking class. How many are there in both classes?

13. Allowing $4\frac{1}{2}$ gallons for each tree, how many gallons of Paris green will be needed to spray 40 trees?

14. Find the cost of $3\frac{3}{16}$ yards of ribbon at 32¢ a yard.

Written

Lace is said to be "fulled on" $\frac{1}{6}$ when 6 inches extra is allowed to the yard, $\frac{1}{3}$ when 12 inches is allowed to the yard, etc.

15. The neck of a nightgown measures 30 inches and each sleeve 24 inches. How much lace is needed for it, if it is fulled on $\frac{1}{6}$?

16. If Esther's skirt is 2 yards wide, and the flounce $1\frac{1}{4}$ times as wide as the skirt, how much lace must she use to trim the flounce if it is fulled on $\frac{1}{3}$?

17. How much embroidery is needed for a flounce on a skirt that is 2 yards wide at the bottom, if $1\frac{1}{2}$ times the width of the skirt is allowed for fullness in the flounce?

18. The following is a recipe for Imperial cake. Find the total cost when butter is 34¢ a pound, sugar 6¢ a pound, flour $2\frac{1}{2}$¢ a pound, and eggs are 24¢ a dozen, allowing 12¢ for lemon juice, soda, raisins, and walnuts.

2 cups butter = 1 lb.
4 cups flour = 1 lb.
2 cups granulated sugar = 1 lb.

Imperial Cake

1 cup butter	1 cup raisins
1 cup sugar	$\frac{1}{2}$ cup walnut meats
5 eggs	2 cups flour
2 teaspoonfuls lemon juice	$\frac{1}{4}$ teaspoonful soda

19. At 10¢ a yard, how much will it cost to furnish a class of 10 girls each with $\frac{3}{4}$ yd. of crash?

20. The bottom of a skirt measures 2 yards. How much embroidery will be required for a flounce, if $\frac{1}{12}$ of the width of the skirt is allowed for fullness?

21. Three girls bought their cambric for school work together. Mary had 4 yards, Ruth had $3\frac{7}{8}$ yards, and Ella had $4\frac{1}{4}$ yards. How much did each of them pay, if cambric cost 16¢ a yard? What was the entire bill?

22. 1 cup of baking powder weighs $\frac{1}{4}$ pound. How many cups are there in a 5-pound can?

23. If a 1-pound can costs 50¢, what is the cost per cup?

24. If the cooking class wore sleeve protectors and caps which required $\frac{1}{3}$ of a yard and $\frac{1}{2}$ of a yard, respectively, how much would it cost to buy material at 12¢ a yard for a class of 19 girls?

25. What quantities does each child measure out if $\frac{1}{3}$ the following recipe for white sauce is used?

1 cup of milk
2 teaspoonfuls of butter
2 teaspoonfuls of flour
$\frac{1}{8}$ teaspoonful of salt

26. There are 16 girls in a sewing class. How much will it cost to furnish each girl with $\frac{2}{3}$ of a yard of cloth at 12¢ per yard?.

27. If $\frac{1}{2}$ cup of flour, $\frac{1}{2}$ cup of milk, 1 cup of sugar, 2 eggs, and $\frac{1}{4}$ cup of butter, is a recipe for a cake for six people, find the recipe for one.

28. There are 30 girls in a cooking class. If each girl uses $\frac{1}{2}$ an orange and 2 eggs, what will be the cost for the class when oranges are 24¢ a dozen and eggs 36¢ a dozen?

29. In an orchard there are 240 trees. $\frac{3}{8}$ of them are apple trees; $\frac{1}{3}$ of them are plum trees; and the rest are pear trees. How many trees of each kind are there?

30. Find the cost of 8 yards of lace at $12\frac{1}{2}$ cents a yard and 10 yards of ribbon at $37\frac{1}{2}$ cents a yard.

31. A farmer put 600 sheep into 4 fields. He put $\frac{1}{5}$ of them into the first field, $\frac{1}{4}$ into the second field, and $\frac{3}{5}$ of the remainder into the third field. How many sheep were there in the fourth field?

Solve the problem by separating it into the different questions that must be answered.

32. Mr. Jones spent $\frac{4}{5}$ of his money, and then found that he had $24 remaining. How much money had he?

SUGGESTION. What part of his money had he left? $24 is $\frac{1}{5}$ of how many dollars?

Written

33. A room is $16\frac{2}{3}$ feet wide and $18\frac{1}{2}$ feet long. What will be the cost of a picture molding extending around the room, at 8¢ a foot?

34. If I sell $12\frac{3}{4}$ tons of coal at $4.40 a ton, $8\frac{1}{2}$ tons at $5.20 a ton, and $16\frac{1}{4}$ tons of soft coal at $2.72 a ton, how much do I receive in all?

35. Ethel drew threads across towels which were to be hemstitched. She drew threads at each end of 6 towels 18 inches wide, 4 towels 14 inches wide, and 4 towels 16 inches wide. At $2\frac{1}{2}$¢ per running foot, how much was she paid for her work?

Make and receipt bills for the following, using any names you wish.

36.	$8\frac{1}{4}$ yards cotton poplin	@	24¢
	$1\frac{1}{2}$ dozen buttons	@	16¢
	$5\frac{3}{4}$ yards braid	@	20¢
37.	$2\frac{1}{2}$ yards French crepe	@	50¢
	$3\frac{1}{4}$ yards lawn	@	24¢
	$5\frac{1}{2}$ yards voile	@	38¢
	6 yards val. lace	@	$12\frac{1}{2}$¢
38.	$6\frac{1}{4}$ yards linen	@	$.72
	$5\frac{1}{2}$ yards cloth	@	$1.80
	2 linen handkerchiefs	@	$12\frac{1}{2}$¢
39.	$2\frac{1}{4}$ yards taffeta	@	64¢
	4 yards silicia	@	$12\frac{1}{2}$¢
	$3\frac{1}{2}$ yards crepe	@	80¢

40. At 24¢ a bushel how much will it cost to thresh 50 A. of wheat averaging $20\frac{3}{4}$ bu. each?

41. Make up five problems involving multiplication of fractions by whole numbers, mixed numbers, and fractions.

DIVISION OF FRACTIONS

Study Recitation

1. What is $\frac{1}{2}$ of 4 fifths; of 6 sevenths; of 8 ninths?

2. What is $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{4}{5} \div 2$? How is the result obtained?

3. What is $\frac{1}{3}$ of $\frac{3}{5}$, or $\frac{3}{5} \div 3$? Find the result in two ways.

First divide the numerator of $\frac{3}{5}$ by 3. Then multiply the denominator of $\frac{3}{5}$ by 3; and reduce the result, $\frac{3}{15}$, to its lowest terms. Compare the results.

To divide a fraction by a whole number, divide the numerator or multiply the denominator by the whole number.

Find the quotients by dividing numerators:

	a.	*b.*	*c.*	*d.*
4.	$\frac{6}{7} \div 2$	$\frac{8}{9} \div 4$	$\frac{12}{13} \div 6$	$\frac{10}{17} \div 5$
5.	$\frac{24}{25} \div 4$	$\frac{15}{16} \div 3$	$\frac{48}{49} \div 8$	$\frac{35}{37} \div 7$

Find the quotients and reduce to lowest terms:

6.	$\frac{8}{12} \div 2$	$\frac{6}{9} \div 2$	$\frac{10}{15} \div 5$	$\frac{9}{15} \div 3$
7.	$\frac{20}{25} \div 4$	$\frac{16}{18} \div 8$	$\frac{6}{8} \div 3$	$\frac{8}{10} \div 4$

Find the quotients by multiplying denominators:

8.	$\frac{3}{4} \div 2$	$\frac{5}{6} \div 4$	$\frac{3}{8} \div 5$	$\frac{7}{10} \div 3$
9.	$\frac{2}{3} \div 5$	$\frac{3}{4} \div 7$	$\frac{5}{7} \div 8$	$\frac{2}{9} \div 3$

To divide a whole number or a fraction by a fraction.

Study Recitation

1. How many times is $\frac{1}{2}$ contained in 1? $\frac{1}{3}$ in 1? $\frac{1}{4}$ in 1? $\frac{1}{5}$ in 1?

2. $1 \div \frac{1}{2} = ?$ $1 \div \frac{1}{3} = ?$ $1 \div \frac{1}{4} = ?$ $1 \div \frac{1}{5} = ?$

3. Since $1 \div \frac{1}{5} = 5$, $3 \div \frac{1}{5} = 3 \times 5$, or 15; $8 \div \frac{1}{5} = 8 \times 5$, or 40; $20 \div \frac{1}{5} = ?$

4. Since $3 \div \frac{1}{5} = 15$, $\frac{3}{4} \div \frac{1}{5} = \frac{1}{4}$ of $15 = ?$

5. Since $5 \div \frac{1}{8} = 40$, $\frac{5}{6} \div \frac{1}{8} = \frac{1}{6}$ of $40 = ?$

Find the quotients :

	a.	*b.*	*c.*	*d.*
6.	$2 \div \frac{1}{2}$	$16 \div \frac{1}{2}$	$\frac{2}{3} \div \frac{1}{2}$	$\frac{2}{5} \div \frac{1}{2}$
7.	$3 \div \frac{1}{3}$	$14 \div \frac{1}{4}$	$\frac{3}{4} \div \frac{1}{3}$	$\frac{3}{5} \div \frac{1}{3}$
8.	$5 \div \frac{1}{4}$	$15 \div \frac{1}{3}$	$\frac{5}{6} \div \frac{1}{4}$	$\frac{5}{7} \div \frac{1}{4}$
9.	$4 \div \frac{1}{5}$	$10 \div \frac{1}{5}$	$\frac{4}{5} \div \frac{1}{5}$	$\frac{4}{7} \div \frac{1}{5}$
10.	$8 \div \frac{1}{6}$	$16 \div \frac{1}{6}$	$\frac{8}{9} \div \frac{1}{9}$	$\frac{8}{15} \div \frac{1}{6}$
11.	$9 \div \frac{1}{10}$	$40 \div \frac{1}{10}$	$\frac{9}{10} \div \frac{1}{10}$	$\frac{9}{20} \div \frac{1}{10}$

When the fraction $\frac{1}{2}$ is changed to $\frac{2}{1}$, the fraction is said to be **inverted**. The inverted fraction shows how many times the given fraction is contained in 1. Thus, $\frac{1}{3}$ is contained in 1, $\frac{3}{1}$, or 3 times; $\frac{1}{7}$ is contained in 1, $\frac{7}{1}$, or 7 times; $\frac{2}{3}$ is contained in 1, $\frac{3}{2}$ times; $\frac{3}{4}$ is contained in 1, $\frac{4}{3}$ times; $\frac{4}{5}$ is contained in 1, $\frac{5}{4}$ times.

12. Divide $\frac{3}{8}$ by $\frac{4}{5}$.

$\frac{3}{8} \div \frac{4}{5} = \frac{3}{8} \times \frac{5}{4} = \frac{15}{32}$.

Since $1 \div \frac{4}{5} = \frac{5}{4}$, therefore $\frac{3}{8} \div \frac{4}{5} = \frac{3}{8}$ of $\frac{5}{4}$, or $\frac{15}{32}$.

The same result may be obtained by inverting the divisor $\frac{4}{5}$ to $\frac{5}{4}$, and then multiplying.

13. Divide $\frac{5}{6}$ by $\frac{5}{8}$.

$$\frac{5}{6} \div \frac{5}{8} = \frac{\cancel{5}}{\underset{3}{\cancel{6}}} \times \frac{\overset{4}{\cancel{8}}}{\cancel{5}} = \frac{4}{3} = 1\frac{1}{3}.$$

Invert the divisor and multiply, using cancellation.

Any number may be divided by a fraction by inverting the terms of the divisor and multiplying.

Since whole numbers and mixed numbers may be reduced to improper fractions, every case in division of fractions may be regarded as a case of dividing a fraction by a fraction, and may be changed to a case of multiplying a fraction by a fraction.

14. Divide $2\frac{1}{2}$ by $3\frac{3}{4}$.

Solution. $2\frac{1}{2} = \frac{5}{2}$, $3\frac{3}{4} = \frac{15}{4}$.

$$\frac{5}{2} \div \frac{15}{4} = \frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{15}}} = \frac{2}{3}.$$

15. Divide 196 by $\frac{4}{5}$.

Solution. $196 \div \frac{4}{5} =$

$$\overset{49}{\cancel{196}} \times \frac{5}{\cancel{4}} = 245.$$

Find the quotients:

	a.	*b.*	*c.*	*d.*
16.	$\frac{3}{8} \div 3$	$3 \div \frac{1}{5}$	$\frac{2}{3} \div \frac{2}{5}$	$2\frac{1}{2} \div 5$
17.	$\frac{4}{5} \div 4$	$4 \div \frac{2}{5}$	$\frac{3}{4} \div \frac{3}{8}$	$3\frac{1}{3} \div 10$
18.	$\frac{8}{9} \div 4$	$6 \div \frac{3}{8}$	$\frac{4}{5} \div \frac{4}{9}$	$4\frac{1}{2} \div 4\frac{1}{2}$
19.	$\frac{4}{9} \div 3$	$7 \div \frac{7}{8}$	$\frac{5}{6} \div \frac{5}{12}$	$4\frac{1}{2} \div 2\frac{1}{4}$
20.	$\frac{3}{4} \div 5$	$10 \div \frac{5}{6}$	$\frac{6}{7} \div \frac{6}{7}$	$3\frac{1}{3} \div 3\frac{1}{2}$

Written

Find the quotients:

	a.	*b.*	*c.*
21.	$20 \div \frac{5}{6}$	$\frac{3}{4} \div \frac{2}{3}$	$16\frac{2}{3} \div 14\frac{1}{2}$
22.	$24 \div \frac{4}{5}$	$\frac{3}{5} \div \frac{2}{3}$	$12\frac{1}{2} \div 8\frac{1}{3}$
23.	$28 \div \frac{4}{7}$	$\frac{2}{5} \div \frac{2}{3}$	$9\frac{3}{4} \div 6\frac{1}{2}$
24.	$30 \div \frac{6}{7}$	$\frac{2}{3} \div \frac{3}{8}$	$62\frac{1}{2} \div 16\frac{2}{3}$
25.	$18 \div \frac{3}{7}$	$\frac{3}{8} \div \frac{5}{8}$	$33\frac{1}{3} \div 4\frac{4}{5}$
26.	$10 \div 2\frac{1}{2}$	$\frac{5}{6} \div \frac{7}{8}$	$87\frac{1}{2} \div 37\frac{1}{2}$
27.	$25 \div 8\frac{1}{3}$	$\frac{7}{8} \div \frac{4}{5}$	$28\frac{4}{7} \div \frac{25}{49}$
28.	$\frac{3}{5} \div 6$	$\frac{3}{7} \div \frac{4}{7}$	$83\frac{1}{3} \div 3\frac{1}{3}$
29.	$\frac{7}{8} \div 9$	$\frac{4}{5} \div \frac{5}{9}$	$37\frac{1}{2} \div \frac{3}{4}$
30.	$\frac{8}{9} \div 15$	$\frac{7}{10} \div \frac{3}{10}$	$62\frac{1}{2} \div 33\frac{1}{3}$
31.	$\frac{7}{16} \div 14$	$\frac{2}{9} \div \frac{4}{5}$	$15\frac{7}{8} \div 3\frac{1}{4}$

	a.	*b.*	*c.*
32.	$2\frac{2}{5} \div 6$	$\frac{3}{20} \div \frac{2}{5}$	$\frac{5}{8} \div 12\frac{3}{4}$
33.	$8\frac{1}{3} \div 5$	$\frac{15}{16} \div \frac{15}{16}$	$\frac{7}{10} \div 8\frac{2}{5}$

34–66. Work each of the examples in 21 to 31 again, interchanging dividend and divisor.

67. $6393\frac{1}{6} \div 4$

$$\begin{array}{r} 4\overline{)6393\frac{1}{6}} \\ 1598\frac{7}{24} \end{array}$$

This is a convenient method of dividing a large mixed number by an integer. Dividing in the usual way gives 1598 as quotient and $1\frac{1}{6}$ remainder. $1\frac{1}{6} = \frac{7}{6}$, and this divided by 4 gives $\frac{7}{24}$.

68. $240\frac{3}{8} \div 3$ **70.** $564\frac{4}{5} \div 4$ **72.** $708\frac{1}{9} \div 15$

69. $785\frac{4}{5} \div 7$ **71.** $287\frac{4}{9} \div 11$ **73.** $1447\frac{2}{3} \div 30$

74. $\dfrac{\frac{2}{3}}{\frac{7}{8}}$ This form of fraction is called a **complex fraction**, and is read $\frac{2}{3}$ over $\frac{7}{8}$, or $\frac{2}{3} \div \frac{7}{8}$. Complex fractions are simply indicated divisions of fractions, the heavy line separating the dividend from the divisor.

75. $\dfrac{\frac{2}{3}}{3} = \frac{2}{3} \div 3$ **76.** $\dfrac{7}{\frac{3}{4}} = 7 \div \frac{3}{4} = ?$ **77.** $\dfrac{8}{\frac{4}{5}}$

PROBLEMS

Oral

1. How many jelly glasses holding $\frac{1}{2}$ pint each will be required for 3 gallons (24 pt.) of jelly?

2. In $2\frac{1}{4}$ acres, how many lots of $\frac{3}{8}$ of an acre each are there?

3. If it requires $5\frac{1}{3}$ yards of cloth for a skirt, how many skirts may be made from 32 yards?

4. If $\frac{3}{4}$ of a yard of ribbon is made into 6 badges of equal length, each badge is what part of a yard long? how many inches long?

5. Mrs. Brown filled 10 tumblers with $\frac{7}{8}$ of a gallon of jelly. What part of a gallon did she use in each?

6. A teacher divided $6\frac{2}{3}$ dozen buttons equally among 10 pupils. What part of a dozen did each receive? How many buttons?

7. A family burns $10\frac{1}{2}$ tons of coal in 4 months. What is the average number of tons burned per month?

8. If $\frac{3}{8}$ of a farm is divided equally among 3 children, what part does each get? If there are 120 acres in the farm, how many acres do the 3 children get? How many acres does each get?

9. If 2 yards of cloth will make a vest, how many vests will 10 yards make?

How is the problem solved? Answer: Divide the whole number of yards by the number of yards it takes to make 1 vest.

10. If $\frac{2}{3}$ of a yard of cloth will make a vest, how many vests will $1\frac{1}{3}$ yd. make? How is the problem solved?

11. If 5 men can build $15\frac{1}{2}$ rods of fence in a day, how many rods of fence can one man build in a day?

12. If 9 pounds of butter cost $\$1\frac{1}{2}$, how much do 4 pounds cost? What is the first question to be answered in this problem?

13. How long will it take a train that averages $33\frac{3}{4}$ miles an hour to run $101\frac{1}{4}$ miles?

14. At $37\frac{1}{2}$¢ a dozen how many dozen eggs can I buy for \$3? for \$1? for \$2?

15. How many pieces of land, each containing 3 acres, can be sold from a tract containing 15 acres? How is the problem solved?

16. How many village lots, each containing $\frac{3}{8}$ of an acre, can be sold from a tract containing 15 acres?

WRITTEN

17. If a family uses $\frac{4}{9}$ of a barrel of flour in a month, how long will $2\frac{2}{3}$ barrels last? Make a similar problem, using whole numbers. How is it solved?

18. If a bushel of corn costs $\$\frac{5}{8}$, how many bushels can be bought for $5?

19. How many sacks of salt weigh 612 pounds, if each weighs $12\frac{3}{4}$ pounds?

20. If a barrel holds $2\frac{3}{4}$ bushels of apples, how many barrels will hold 132 bushels?

21. If a man's steps are $2\frac{1}{2}$ feet each, how many steps will he take in walking 150 feet?

22. At $\$1\frac{3}{4}$ each how many pairs of gloves can be bought for $21?

23. If $\frac{7}{8}$ of a bushel of nuts is placed in each bag, how many bags will be required for 14 bushels?

SUGGESTION. If $\frac{7}{8}$ of a bushel of nuts is placed in each bag, it will require as many bags to hold 14 bushels as $\frac{7}{8}$ is contained times in 14.

24. If one man can build a fence in $9\frac{3}{5}$ days, in what time can 8 men build it? In what time can 3 men build it?

25. If $17\frac{1}{2}$ pounds of cream produce $4\frac{1}{2}$ pounds of butter fat, how many pounds of butter fat should 35 pounds of cream produce?

26. How many $3\frac{3}{4}$-inch strips of wood must be used to make a drawing board 17 inches wide, and how much waste will there be?

27. Miss Smith bought $1\frac{2}{3}$ yards of canvas 1 yard wide to make holders. Each holder was cut 12 inches by 6 inches. How many girls did she have in her class if she had just enough holders to go around?

28. I buy a remnant of $2\frac{1}{2}$ yards of lace for 15¢. How much is that a yard?

29. Mary is cutting bandages from an old tablecloth $1\frac{1}{2}$ yards by 2 yards. How many yards of bandages $1\frac{1}{2}$ inches wide can she make?

30. Make up two problems involving division of a fraction by a whole number or a fraction, and division of a whole number or a mixed number by a mixed number.

31. If a barrel holds $2\frac{3}{4}$ bushels, how many barrels will be required for shipping 330 bushels of apples?

32. If I exchange 9 bushels of beans at $1.60 a bushel for sugar at $5\frac{1}{3}$ cents a pound, how many pounds of sugar do I receive?

33. How many times the cost will be gained by buying apples at the rate of 3 for 1¢, and selling them at 3¢ apiece?

34. A can build a fence in 8 days, B can build it in 6 days, and C, in 4 days. In how many days can the three complete the work?

If A can build a fence in 8 days, what part can he do in 1 day? *Ans.* $\frac{1}{8}$.

If B can build a fence in 6 days, what part can he do in 1 day? *Ans.* $\frac{1}{6}$.

If C can build a fence in 4 days, what part can he do in 1 day? *Ans.* $\frac{1}{4}$.

If A can build $\frac{1}{8}$ of a fence in 1 day, B $\frac{1}{6}$ of it, and C $\frac{1}{4}$ of it, what part can they all do in 1 day? *Ans.* $\frac{13}{24}$ of it.

If A, B, and C can build $\frac{13}{24}$ of a fence in 1 day, how long will it take them to do $\frac{24}{24}$ of it? *Ans.* $\frac{24}{13}$, or $1\frac{11}{13}$ days.

The last question may be separated into two questions: If A, B, and C can build $\frac{13}{24}$ of a fence in 1 day, how long will it take to build $\frac{1}{24}$ of the fence? *Ans.* $\frac{1}{13}$ of a day.

If A, B, and C can build $\frac{1}{24}$ of a fence in $\frac{1}{13}$ of a day, how long will it take them to build $\frac{24}{24}$ of the fence? *Ans.* $\frac{24}{13}$, or $1\frac{11}{13}$ days.

35. How many strips of carpet $\frac{3}{4}$ of a yard wide will it take to cover a room $7\frac{1}{2}$ yards wide?

36. How many bows, each requiring $\frac{2}{5}$ of a yard, can be made from a piece of ribbon 10 yards long?

37. If 25 men earn $\$312\frac{1}{2}$ in $6\frac{1}{4}$ days, what is the wage of each per day?

38. At $\frac{7}{8}$ of a dollar a yard, how many yards of cloth can be bought for $8\frac{3}{4}$ dollars?

39. If a horse travels $1\frac{1}{5}$ miles in 12 minutes, in how many minutes will it travel 1 mile? In how many minutes will it travel $4\frac{3}{4}$ miles?

40. If a train of cars runs $1\frac{1}{5}$ miles in $1\frac{1}{2}$ minutes, in how many minutes will it run 4 miles?

41. If a train of cars runs $1\frac{1}{5}$ miles in $1\frac{1}{2}$ minutes, how far will it run in 5 minutes?

42. How many tickets were sold for an excursion, if $\$318\frac{3}{4}$ was received for all the tickets, and the price of each ticket was $\$3\frac{3}{4}$?

43. At $4\frac{1}{4}$¢ a pound, how many pounds of sugar can be bought for $1.70?

44. If 6 men can build a barn in $3\frac{2}{3}$ days, how long will it take 8 men to do it?

45. If an automobile goes 1 mile in $85\frac{5}{7}$ seconds, how many miles will it go in 1 hour, at the same rate?

46. If a 6-foot pole casts a shadow $8\frac{1}{2}$ feet long, what is the length of a pole which casts a shadow 34 feet long at the same time? Solve in two ways.

47. Make up five problems involving division of a fraction by a fraction, and division of a whole number or a mixed number by a mixed number or a fraction.

GENERAL PROBLEMS IN FRACTIONS

To find a fractional part of a number.

STUDY RECITATION

1. Find $\frac{3}{8}$ of 24.

SOLUTION. $\frac{1}{8}$ of $24 = 3$. $\frac{3}{8}$ of $24 = 3 \times 3$, or 9.
Or, $\frac{3}{8}$ of $24 = \frac{1}{8}$ of 3×24. $3 \times 24 = 72$. $\frac{1}{8}$ of $72 = 9$.
Use the second analysis when the denominator will not exactly divide the number.

Analyze the following:

	a.	*b.*	*c.*
2.	$\frac{3}{5}$ of 25.	$\frac{3}{4}$ of 11.	$\frac{6}{7}$ of 84.
3.	$\frac{7}{8}$ of 40.	$\frac{5}{9}$ of 12.	$\frac{5}{9}$ of 63.
4.	$\frac{2}{3}$ of 16.	$\frac{5}{6}$ of 16.	$\frac{4}{5}$ of 80.
5.	$\frac{5}{12}$ of 24.	$\frac{3}{7}$ of 63.	$\frac{3}{4}$ of 32.
6.	$\frac{7}{10}$ of 60.	$\frac{8}{9}$ of 54.	$\frac{5}{8}$ of 80.
7.	$\frac{4}{7}$ of 35.	$\frac{3}{10}$ of 20.	$\frac{3}{16}$ of 32.

Divide the number by the denominator of the fraction and multiply the quotient by the numerator.

Or, *Multiply the number by the numerator, and divide the product by the denominator.*

NOTE. Review also pp. 120 to 123.

To find what fractional part one number is of another.

STUDY RECITATION

1. 1 is what part of 2? of 3? of 4? of 5? of 7? of 15?

To express what fractional part 1 is of any number, write 1 as the numerator and the other number as the denominator of the fraction. $\frac{1}{8}$ means $1 \div 8$; $\frac{1}{6}$ means $1 \div 6$; $\frac{1}{4}$ means $1 \div 4$; $\frac{1}{10}$ means $1 \div 10$.

2. 2 is what part of 3? of 5? of 7? of 9? of 4? of 8? of 11? $\frac{2}{3}$ means $2 \div 3$; $\frac{2}{5}$ means $2 \div 5$.

3. How do you express what part 2 is of any number? Give examples.

4. How do you express what part 3 is of any number? 5? Give examples.

5. 4 is what part of 8? Or, what part of 8 is 4?

SOLUTION. 1 is $\frac{1}{8}$ of 8. 4 is $\frac{4}{8}$, or $\frac{1}{2}$, of 8.

6. 8 is what part of 4? Or, what part of 4 is 8?

Notice that these examples can be stated in two ways. In either case, make the number following the phrase, "what part of" the denominator and the other number the numerator. Thus, in "What part of 8 is 4, or 4 is what part of 8," make 8 the denominator and 4 the numerator. *Ans.* $\frac{4}{8}$ or $\frac{1}{2}$. In "8 is what part of 4," make 4 the denominator and 8 the numerator. *Ans.* $\frac{8}{4}$ or 2. In this case the answer is a whole number.

In expressing the fractional part one number is of another, give the fraction in its lowest terms.

7. 5 is what part of 10?

8. 6 is what part of 12?

9. 7 is what part of 21?

10. 6 is what part of 8?

11. 10 is what part of 5?

12. 12 is what part of 8?

13. What part of 14 is 8?

14. What part of 15 is 6?

15. What part of 15 is 10?

16. What part of 21 is 18?

17. What part of 6 is 12?

18. What part of 6 is 10?

Notice that the answers to examples 12 and 18 are improper fractions.

19. 2 is what part of $3\frac{1}{2}$?

SOLUTION. $\frac{2}{3\frac{1}{2}} = 2 \div \frac{7}{2} = 2 \times \frac{2}{7} = \frac{4}{7}$.

20. $2\frac{1}{2}$ is what part of $3\frac{1}{3}$?

SOLUTION. $\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \times \frac{3}{10} = \frac{3}{4}$.

21. 3 is what part of $8\frac{1}{3}$?

22. 4 is what part of $6\frac{1}{2}$?

23. $2\frac{1}{4}$ is what part of $4\frac{1}{2}$?

24. $2\frac{3}{4}$ is what part of $3\frac{1}{4}$?

PROBLEMS

Oral

1. John earns \$12 and Henry earns \$9. Henry's earnings are what part of John's? John's earnings are how many times Henry's?

Suggestion. \$9 is what part of \$12? \$12 is what part of \$9?

2. A farmer tests 32 kernels of corn to see how many will grow. 8 kernels fail to grow. What part of the whole number of kernels fail to grow?

3. In the month of November, James attended school 18 days. What part of the month did he attend school?

4. If there are 14 problems in to-day's lesson, and Mary solves 10 of them, what part of the lesson has she mastered?

5. A watch cost \$40 and a chain \$5? What part of the cost of both was the cost of each?

6. A boy hoes 12 rows of corn and has 4 rows yet to hoe. What part of the work has he done? What part has he still to do?

7. Rice that costs 8¢ a pound is sold for 10¢ a pound. The gain is what part of the cost?

8. A grocer pays 25¢ for a gallon of vinegar and sells it for 30¢. What part of the cost does he gain? How many hundredths of the cost does he gain?

9. A man can do a piece of work in 6 days. What part of the work can he do in 2 days? *Ans.* $\frac{2}{6} = \frac{1}{3}$. In 5 days? *Ans.* $\frac{5}{6}$. In $3\frac{1}{2}$ days? *Ans.* $\frac{3\frac{1}{2}}{6} = \frac{7}{2} \div 6 = \frac{7}{12}$.

10. A farmer builds a fence in 9 days. What part of it does he build in 1 day? in 3 days? in 8 days? in $\frac{2}{3}$ of a day?

Written

11. Cloth costing \$.25 a yard is sold for \$.45 a yard. What part of the cost is gained? How many hundredths of the cost are gained?

12. A book that costs 75¢ is sold for \$1. What fraction of the cost is the selling price? What part of the cost is gained? How many hundredths of the cost are gained?

13. A boy is in school 6 hours in a day and sleeps 8 hours. What part of the day is he in school? What part of the day does he sleep?

14. In an orchard there are 80 trees: 15 of them are apple trees, 30 of them are pear trees, and the rest are plum trees. What part of the whole number of trees are plum trees? Ask and answer two other questions on this problem.

15. If I buy a horse for \$80 and sell it for \$60, what part of the cost do I lose? How many hundredths of the cost do I lose?

To the Teacher. By being asked to give the number of hundredths, as in problems 8, 11, 12, and 15, pupils are being prepared for work in percentage. Ask similar questions when proper, in connection with other problems.

16. If $\frac{2}{3}$ of a chest of tea sells for what the whole costs, what fraction of the cost should the whole chest sell for?

Suggestion. $\frac{3}{3}$ is what fractional part of $\frac{2}{3}$?

17. If $\frac{2}{3}$ of a bushel of apples sells for what $\frac{4}{5}$ of a bushel costs, the whole should sell for what fraction of the cost?

18. If peaches that cost 2¢ each are sold at the rate of 3 for 10¢, the gain is what part of the cost?

19. If a piece of land costs \$540, and $\frac{2}{3}$ of it is sold for the cost of the whole, what part of the cost would be gained if the entire piece were sold at the same rate?

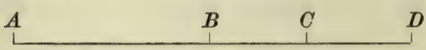
20. If $\frac{1}{4}$ of what a merchant receives for a pound of butter is gain, what part of the cost is gained? Fix the selling price of a pound of butter and prove your solution to be correct.

21. From a farm of 275 acres, there is sold to one man 40 acres, and to another 85 acres. What part of the farm remains?

22. A man buys 5 horses at \$90 each and 25 cows at \$65 each. He sells the horses at \$98 each, and the cows at \$75 each. How many dollars does he gain all together? His gain is what part of the cost?

23. If wood is worth $\$4\frac{3}{4}$ a cord, and coal is worth $\$7\frac{1}{2}$ a ton, 4 cords of wood are worth what fraction of the value of 4 tons of coal? Work in two ways.

24. A father is 25 years older than his son. His son's age is 10 years. What part of the father's age is the son's age?



25. The distance from A to B is 48 mi.; from B to C 24 mi.; from C to D 26 mi. The distance from A to B is what part of the distance from A to D? Make and answer three other questions on this problem.

To find a number when a fractional part of it is given.

STUDY RECITATION

1. If 3 eggs cost 12¢, how much does 1 egg cost?

2. If 3 fifths of my money is 12¢ how much is 1 fifth of it? 5 fifths of it?

3. 12 is $\frac{3}{5}$ of what number?

SOLUTION. If $12 = \frac{3}{5}$ of some number, $\frac{1}{5}$ of that number $= \frac{1}{3}$ of 12, or 4. $\frac{5}{5}$ of the number $= 5 \times 4$, or 20.

The same result may be obtained by dividing 12 by $\frac{3}{5}$. $12 \div \frac{3}{5} = 12 \times \frac{5}{3} = 20$.

4. $\frac{3}{4}$ of a number is 21. What is the number?

ANALYSIS BY QUESTION AND ANSWER: If 21 is $\frac{3}{4}$ of a number, what is $\frac{1}{4}$ of the number? *Ans.* $\frac{1}{3}$ of 21, or 7. If 7 is $\frac{1}{4}$ of a number, what is the number? *Ans.* 4×7, or 28.

Or, $21 \div \frac{3}{4} = 21 \times \frac{4}{3} = 28$.

TO THE TEACHER. If, in solving this example, pupils are inclined to say, "If 21 is $\frac{3}{4}$ of a number, $\frac{1}{4}$ is $\frac{1}{3}$ of 21," the teacher should give the example without naming the fractional unit; as: "If 21 is 3 of the equal parts of a number, what is 1 of these parts?" When this question is answered, then follow with: "If 7 is $\frac{1}{4}$ of a number, what is the number?" Then have the original example analyzed. Do not allow the pupils to say $\frac{1}{3} = 7$. $\frac{1}{3}$ does not equal 7, but $\frac{1}{3}$ of $21 = 7$.

Analyze the following examples:

5. 10 is $\frac{5}{8}$ of what number?

6. 6 is $\frac{3}{5}$ of what number?

7. 15 is $\frac{5}{7}$ of what number?

8. 21 is $\frac{7}{8}$ of what number?

9. If $\frac{7}{9}$ of a number is 14, what is the number?

To find a number when a fractional part of it is given, divide the given number by the fraction.

Solve the following, giving results orally:

10. 36 is $\frac{9}{10}$ of what number?	45?	18?	54?	27?	63?
11. 24 is $\frac{8}{9}$ of what number?	32?	56?	64?	24?	56?
12. 28 is $\frac{7}{8}$ of what number?	35?	56?	63?	49?	42?
13. 30 is $\frac{2}{3}$ of what number?	50?	44?	36?	50?	60?
14. 25 is $\frac{5}{6}$ of what number?	55?	75?	45?	70?	65?

TO THE TEACHER. Pupils may be required to prove these results by finding the fractional part of the number found in each case.

In example 11, having found the required number to be 27, find $\frac{8}{9}$ of it. In this work the pupil is finding a fractional part of a number.

The work may also be proved by finding what fraction of the whole number the given part is.

In example 11, having found 27, find what fraction 24 is of 27. In this case, it is required to find what fractional part one number is of another.

In the following solve the example given, and in each case, by using the answer, make two other examples and solve.

15. 16 is $\frac{4}{5}$ of what number? *Ans.* 20.
What is $\frac{4}{5}$ of 20? *Ans.* 16.
What part of 20 is 16? *Ans.* $\frac{4}{5}$.

16. 24 is $\frac{3}{7}$ of what number?

17. $\frac{5}{9}$ of 36 are how many?

18. 15 is what part of 24?

19. What part of 18 is 8?

20. 22 is $\frac{2}{3}$ of what number?

21. What part of 20 is 8?

22. Find $\frac{3}{8}$ of 24, 32, 48, 64, 56, 96, 72.

23. 24 is $\frac{2}{5}$ of —. $\frac{4}{5}$ of —. $\frac{6}{8}$ of —. $\frac{8}{9}$ of —.

24. 12 is what part of 16? 18? 20? 22? 32?

25. Find $\frac{2}{3}$ of $\frac{3}{4}$; of $\frac{6}{7}$; of $\frac{7}{8}$; of $\frac{8}{9}$; of $\frac{5}{12}$.

26. 12 is $\frac{3}{5}$ of —. $\frac{3}{4}$ of —. $\frac{3}{7}$ of —. $\frac{3}{8}$ of —.

27. $2\frac{1}{2}$ is $\frac{1}{2}$ of —. $\frac{1}{3}$ of —. $\frac{1}{4}$ of —. $\frac{1}{5}$ of —.

PROBLEMS

Oral

1. If $\frac{3}{4}$ dozen eggs costs 12 cents, what is the price of $\frac{1}{4}$ dozen? $\frac{1}{2}$ dozen? 1 dozen? $\frac{2}{3}$ dozen?

2. If $\frac{7}{8}$ of a yard of cloth costs 21 cents, what is the cost of 1 yard? $2\frac{1}{2}$ yards? $\frac{2}{3}$ yard?

3. If $\frac{3}{5}$ of the cost of a hat equals 90¢, what is the cost?

4. If $\frac{9}{7}$ of the price of a house equals \$6300, what is the price of the house?

5. $\frac{6}{5}$ of the cost of an article equals 24¢. What is the cost?

6. John has $2\frac{1}{2}$ doz. pencils, which are $\frac{5}{6}$ of the number I have. How many have I?

7. \$300 is $2\frac{1}{2}$ times the price of my watch. What is its price?

8. $3\frac{3}{4}$ acres are $\frac{3}{5}$ of Mr. Jay's land. How many acres has he?

9. A farmer sold 40 sheep, which were $\frac{5}{7}$ of his flock. How many sheep had he before the sale?

10. Mary's standing in arithmetic is 80, and Mary's standing is $\frac{8}{9}$ of her brother's. What is her brother's standing?

11. A room is 18 feet wide; its width is $\frac{3}{4}$ its length. How long is it?

12. When your age is $\frac{1}{3}$ greater than it is now, what fraction of your present age will it be? If your age then will be 16 years, what is it now?

Written

13. The difference between $\frac{3}{4}$ and $\frac{1}{3}$ of a number is 15. What is the number?

SUGGESTION. What is the difference between $\frac{3}{4}$ and $\frac{1}{3}$ of any number?

14. A boy spent $\frac{3}{7}$ of his money for a suit of clothes and had \$20 left. How much did his suit cost?

SUGGESTION. If he spent $\frac{3}{7}$ of his money, he had $\frac{7}{7} - \frac{3}{7}$, or $\frac{4}{7}$, of it left.

15. A fruit dealer sells in one day 75 oranges, which are $\frac{5}{6}$ of the number of apples he sells. How many apples does he sell?

16. If to A's age there is added $\frac{2}{3}$ and $\frac{2}{5}$ of his age, what fraction of his age will the sum be? If the sum is 62 years, what is his age?

17. In the fifth grade in a school there are 20 boys; $\frac{3}{8}$ of all the pupils in the grade are girls. How many pupils are there in the fifth grade? how many girls?

18. In five grades of the same school there are 216 boys, and $\frac{7}{15}$ of the pupils are girls. How many pupils are there in the five grades? how many girls?

19. $\frac{2}{9}$ of a ship is worth $20,000. Find the value of $\frac{2}{3}$ of it at the same rate.

20. A man sold $\frac{2}{3}$ of a flock of sheep to one man, and $\frac{1}{2}$ the remainder to another, and then had 44 sheep left. How many were there at first?

REVIEW

1. What is a fractional unit? What is a fraction?

2. What are the terms of a fraction? Define each term.

3. Define proper fraction; improper fraction; mixed number.

4. What is meant by the reduction of a fraction?

5. In any fraction whose denominator is 4, the value of the numerator is 4 times the fraction. In the fraction $\frac{3}{4}$, the value of 3 is $\frac{12}{4}$. $\frac{12}{4}$ is 4 times $\frac{3}{4}$. Show in the same way that in the fraction $\frac{2}{3}$ the value of the numerator is 3 times the fraction.

6. If a multiplier is used that is 4 times the correct multiplier, the product is 4 times the correct product. What must be done to the first product to get the correct product?

7. In multiplying by a fraction, if we multiply by its numerator, the product obtained is as many times the correct

product as its denominator is times 1; therefore this product must be divided by the denominator to secure the correct product.

8. Show the application of the statements in 7 in multiplying $\frac{5}{7}$ by $\frac{3}{4}$.

9. If a divisor is used that is 4 times the correct divisor, the quotient obtained is $\frac{1}{4}$ of the correct quotient. What must be done to the first quotient to get the correct quotient?

10. In dividing by a fraction, if we divide by its numerator, the quotient obtained is such a part of the correct quotient as 1 is a part of the denominator of the fraction. Therefore the first quotient must be multiplied by the denominator of the fraction to secure the correct quotient.

11. Show the application of the statements in 10 in dividing $\frac{5}{7}$ by $\frac{3}{4}$. Show that the operation is the same as inverting the divisor and proceeding as in multiplication.

Problems without Numbers

12. How is a fraction reduced to a fraction of a higher denomination?

13. How is a fraction reduced to its lowest terms?

14. How is a whole number reduced to a fraction?

15. Give the rule for the reduction of a mixed number to an improper fraction.

16. Give the rule for the reduction of an improper fraction to a mixed number.

17. How do you reduce fractions to equivalent fractions having the least common denominator?

18. Give the rule for the addition of proper fractions.

19. Give the rule for the addition of mixed numbers.

20. Give the rule for subtracting one fraction from another.

21. Show that in any simple fraction, the numerator is as many times the fraction as there are units in the denominator.

22. How may a fraction be multiplied by a whole number?

23. How may a fraction be divided by a whole number?

24. If you know the cost of a fractional part of a pound of sugar, how can you find the cost of any number of pounds of sugar?

25. If one yard of cloth costs a fractional part of a dollar, how can you find how many yards can be purchased for a given number of dollars?

26. How can you find a fractional part of a number?

27. How can you find what part one number is of another?

28. How can you find a number when a fractional part of it is given?

GENERAL REVIEW

Write from dictation and add, timing yourself. Test by adding in the opposite direction, and try to better your time record in the testing.

	a.	*b.*	*c.*	*d.*	*e.*
1.	2198	341239	653041	$2570.25	$2530 | 53
	2237	600523	983409	8561.54	140 | 05
	3365	751689	847638	7280.94	2030 | 07
	5422	832045	732746	2310.57	49 | 62
	6549	710009	528805	5000.09	531 | 42

NOTE. Bookkeepers sometimes use lines to separate dollars and cents, instead of decimal points, as shown in *e*.

Subtract, and test by adding. Time yourself.

2.	690572	983052	547369	$2563.25	$4075.82
	547219	894999	439438	1578.69	3987.94
3.	500000	405032	700050	$6050.00	$7000.53
	256789	356678	599999	5888.88	5678.99

Find products, and test by division. Time yourself.

	a.	*b.*	*c.*
4.	356 × 9345	213 × $2.15	987 × $22.25
5.	598 × 6208	394 × $3.14	654 × $33.75
6.	675 × 5754	487 × $5.16	823 × $16.25
7.	822 × 3219	652 × $7.98	742 × $15.14

Find quotients, and test by multiplication. Time yourself.

8.	94032 ÷ 144	$896.93 ÷ 257	$763.83 ÷ 207
9.	75555 ÷ 219	$1777.73 ÷ 389	$896.93 ÷ 349
10.	91476 ÷ 198	$755.55 ÷ 345	$4211.24 ÷ 748
11.	94032 ÷ 653	$1584.15 ÷ 537	$3407.04 ÷ 416

12. Find the prime factors of:
36, 42, 49, 56, 64, 72, 75, 81, 90, 100, 121, 144, 300.

Change to whole or to mixed numbers:

	a	*b.*	*c.*	*d.*	*e.*
13.	$\frac{16}{4}$	$\frac{44}{11}$	$\frac{50}{16}$	$\frac{49}{12}$	$\frac{29}{16}$
14.	$\frac{48}{25}$	$\frac{36}{18}$	$\frac{75}{25}$	$\frac{62}{16}$	$\frac{48}{20}$
15.	$\frac{90}{16}$	$\frac{48}{16}$	$\frac{50}{20}$	$\frac{54}{13}$	$\frac{63}{11}$

16. Change 5, 7, 6, 8, 9, 10 to 4ths; to 3ds; to 6ths; to 9ths; to 12ths.

Change to improper fractions:

	a.	*b.*	*c.*	*d.*	*e.*
17.	$5\frac{1}{3}$	$16\frac{2}{3}$	$62\frac{1}{2}$	$14\frac{2}{7}$	$41\frac{2}{3}$
18.	$6\frac{1}{4}$	$87\frac{1}{2}$	$83\frac{1}{3}$	$28\frac{4}{7}$	$58\frac{2}{3}$
19.	$8\frac{1}{3}$	$37\frac{1}{2}$	$66\frac{2}{3}$	$42\frac{6}{7}$	$75\frac{3}{4}$

20. Change $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{3}{4}$ each to 16ths; to 32ds; to 64ths.

21. Change $\frac{2}{5}$, $\frac{3}{10}$, $\frac{4}{15}$ each to 30ths.

22. Change $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{9}$ each to 36ths.

23. Change $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{6}$ each to 24ths.

24. Change $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ each to 12ths.

25. Change $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{8}$ each to 24ths.

26. Change $\frac{4}{5}$, $\frac{2}{3}$, $\frac{3}{5}$ each to 15ths.

27. Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ each to 12ths.

28. Change $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{6}$ each to 24ths.

29. Change $\frac{5}{6}$, $\frac{1}{2}$, $\frac{2}{3}$ each to 12ths.

Reduce to lowest terms:

	a.	*b.*	*c.*	*d.*	*e.*
30.	$\frac{16}{20}$	$\frac{12}{20}$	$\frac{14}{20}$	$\frac{11}{22}$	$\frac{18}{24}$
31.	$\frac{24}{36}$	$\frac{25}{30}$	$\frac{8}{32}$	$\frac{9}{27}$	$\frac{15}{27}$
32.	$\frac{14}{18}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$	$\frac{12}{16}$

	a.	*b.*	*c.*	*d.*	*e.*
33.	$\frac{7}{14}$	$\frac{14}{21}$	$\frac{15}{24}$	$\frac{25}{40}$	$\frac{15}{30}$
34.	$\frac{25}{50}$	$\frac{8}{40}$	$\frac{16}{32}$	$\frac{18}{27}$	$\frac{27}{30}$

35. Find the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{1}{6}$.

36. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{11}{12}$.

37. Find the sum of $\frac{3}{8}$, $\frac{7}{9}$, $\frac{9}{10}$, and $\frac{7}{12}$.

38. Find the sum of $2\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{9}$, and $\frac{8}{11}$.

39. Find the sum of $12\frac{3}{4}$, $14\frac{6}{7}$, and $10\frac{3}{10}$.

40. Find the sum of $36\frac{7}{8}$, $\frac{3}{5}$, and $12\frac{3}{4}$.

41. Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$.

42. Subtract $\frac{5}{12}$ from $\frac{5}{7}$.

43. Subtract $\frac{8}{9}$ from $\frac{11}{12}$.

44. Subtract $\frac{3}{25}$ from $\frac{17}{100}$.

45. Subtract $\frac{3}{4}$ from $12\frac{4}{7}$.

46. Subtract $\frac{5}{6}$ from $3\frac{5}{9}$.

47. Subtract $2\frac{1}{3}$ from $7\frac{1}{4}$.

48. Find the value of $18\frac{3}{4} + 37\frac{1}{2} - 16\frac{2}{3}$.

49. Add $30\frac{1}{4}$, $267\frac{1}{2}$, and $3230\frac{3}{8}$.

50. Subtract $2\frac{5}{8}$ from 7.

51. Subtract $250\frac{3}{5}$ from 300.

52. Subtract $90\frac{1}{3}$ from $108\frac{1}{2}$.

53. Subtract $10\frac{3}{5}$ from $14\frac{9}{40}$.

54. Find the sum of $30\frac{7}{15}$, $40\frac{3}{5}$, and $6\frac{8}{15}$.

55. Find the sum of $120\frac{1}{25}$ and $306\frac{1}{50}$.

56. Find the sum of $72\frac{8}{25}$ and $360\frac{9}{20}$.

57. Find the sum of $20\frac{7}{40}$, $34\frac{9}{20}$, and $6\frac{45}{50}$.

58. Find the sum of $304\frac{5}{6}$, $27\frac{7}{16}$, and $30\frac{5}{12}$.

59. Subtract $12\frac{3}{16}$ from 17.

60. $90\frac{5}{16} - 75\frac{3}{4} = ?$

61. $105\frac{1}{50} - 16\frac{1}{25} = ?$

62. $96\frac{1}{32} - 72\frac{3}{16} = ?$

63. $93\frac{2}{50} - 46\frac{1}{20} = ?$

Add or subtract as indicated:

64. $\frac{3}{8} + \frac{3}{4} + \frac{2}{3} = ?$

65. $\frac{5}{6} + \frac{3}{8} + \frac{1}{2} = ?$

66. $\frac{5}{21} + \frac{2}{3} + \frac{1}{7} = ?$

67. $\frac{5}{8} - \frac{1}{3}$

68. $\frac{7}{12} - \frac{1}{4}$

69. $\frac{4}{15} - \frac{1}{5}$

70. $\frac{7}{8} + \frac{5}{7}$

71. $6\frac{4}{5} + \frac{5}{6} + 1\frac{2}{3}$

72. $1\frac{7}{12} + 2\frac{3}{8} + 6\frac{1}{3}$

73. $15\frac{1}{3} - 6\frac{3}{4}$

74. $25\frac{1}{8} - 14\frac{3}{4}$

75. $3\frac{1}{2} + 5\frac{5}{8} - 1\frac{1}{3}$

76. $6\frac{5}{6} + 2\frac{1}{3} - 1\frac{1}{2}$

77. $5\frac{3}{4} + 2\frac{1}{8} - 1\frac{1}{16}$

78. $\frac{7}{24} + \frac{5}{12} + \frac{1}{2}$

79. $\frac{5}{9} + \frac{2}{3} + \frac{5}{6}$

80. $\frac{7}{16} + \frac{1}{8} + \frac{3}{4}$

81. $\frac{5}{16} - \frac{1}{8}$

82. $\frac{11}{24} - \frac{3}{8}$

83. $\frac{1}{2} - \frac{3}{8}$

84. $\frac{3}{4} + \frac{2}{3} - \frac{1}{2}$

85. $3\frac{5}{6} - 1\frac{1}{2} + \frac{2}{3}$

86. $2\frac{5}{7} + 3\frac{2}{3}$

87. $10\frac{1}{8} - 3\frac{2}{3}$

88. $16\frac{2}{3} - 12\frac{1}{2}$

89. $8\frac{5}{12} - 2\frac{1}{16} + 4\frac{1}{4}$

90. $105\frac{5}{6} - 3\frac{1}{12} + 8\frac{1}{4}$

91. $10\frac{3}{4} + 1\frac{1}{8} - 3\frac{5}{16}$

Give products:

	a.	*b.*	*c.*	*d.*
92.	$6 \times \frac{4}{5}$	$9 \times \frac{10}{11}$	$25 \times \frac{5}{7}$	$15 \times \frac{3}{5}$
93.	$7 \times \frac{3}{8}$	$10 \times \frac{4}{7}$	$16 \times \frac{3}{8}$	$\frac{3}{4}$ of $\frac{5}{6}$
94.	$12 \times \frac{5}{6}$	$15 \times \frac{5}{6}$	$18 \times \frac{4}{9}$	$\frac{3}{10}$ of $\frac{5}{9}$
95.	$\frac{5}{6}$ of 24	$\frac{5}{12}$ of 60	$\frac{2}{15}$ of 75	$\frac{5}{9}$ of 63
96.	$\frac{7}{8}$ of 48	$\frac{3}{20}$ of 100	$\frac{5}{11}$ of 121	$\frac{3}{7}$ of 77
97.	$\frac{9}{16}$ of 64	$\frac{2}{25}$ of 50	$\frac{5}{7}$ of 63	$\frac{9}{10}$ of 100
98.	$\frac{3}{4}$ of $\frac{4}{5}$	$\frac{8}{9}$ of $\frac{9}{10}$	$\frac{11}{12}$ of $\frac{5}{7}$	$\frac{5}{9}$ of $\frac{4}{7}$
99.	$\frac{7}{8}$ of $\frac{5}{9}$	$\frac{9}{10}$ of $\frac{5}{6}$	$\frac{2}{7}$ of $\frac{3}{8}$	$\frac{4}{7}$ of $\frac{7}{8}$
100.	$\frac{5}{6}$ of $\frac{10}{11}$	$\frac{10}{11}$ of $\frac{5}{7}$	$\frac{3}{8}$ of $\frac{5}{6}$	$\frac{7}{12}$ of $\frac{16}{25}$

	a.	*b.*	*c.*	*d.*
101.	$24 \times 1\frac{5}{8}$	$16 \times 2\frac{3}{5}$	$32 \times 1\frac{3}{4}$	$1\frac{5}{8} \times 39$
102.	$18 \times 2\frac{5}{6}$	$29 \times 5\frac{8}{9}$	$56 \times 4\frac{7}{8}$	$2\frac{2}{3} \times 47$
103.	$15 \times 3\frac{3}{5}$	$28 \times 1\frac{7}{8}$	$63 \times 6\frac{7}{9}$	$3\frac{3}{5} \times 56$
104.	$10 \times 4\frac{5}{8}$	$17 \times 3\frac{5}{6}$	$72 \times 5\frac{3}{8}$	$4\frac{5}{16} \times 32$
105.	$24\frac{1}{3} \times 2\frac{3}{4}$	$12\frac{1}{2} \times 62\frac{1}{2}$	$26\frac{1}{2} \times 7\frac{1}{8}$	$21\frac{1}{8} \times 13\frac{3}{4}$
106.	$17\frac{1}{2} \times 5\frac{5}{6}$	$87\frac{1}{2} \times 33\frac{1}{3}$	$19\frac{3}{5} \times 16\frac{1}{8}$	$17\frac{7}{9} \times 15\frac{4}{5}$
107.	$18\frac{2}{3} \times 3\frac{3}{8}$	$62\frac{1}{2} \times 16\frac{2}{3}$	$17\frac{3}{10} \times 18\frac{5}{12}$	$16\frac{3}{4} \times 14\frac{5}{16}$

	a.	*b.*	*c.*
108.	$\frac{2}{5} \times \frac{5}{6} \times \frac{2}{3}$	$5\frac{1}{2} \times 2\frac{2}{3} \times 3\frac{1}{6}$	$2\frac{5}{16} \times 3\frac{1}{2} \times 2\frac{5}{8}$
109.	$\frac{3}{7} \times \frac{1}{5} \times \frac{3}{4}$	$7\frac{1}{8} \times 3\frac{1}{4} \times 2\frac{1}{16}$	$1\frac{5}{9} \times 2\frac{1}{6} \times 3\frac{2}{3}$
110.	$\frac{8}{9} \times \frac{7}{8} \times \frac{6}{7}$	$3\frac{1}{5} \times 2\frac{1}{10} \times 4\frac{2}{5}$	$1\frac{7}{15} \times \frac{1}{3} \times \frac{4}{5}$

Find the quotients:

	a.	*b.*	*c.*	*d.*
111.	$\frac{9}{10} \div 3$	$\frac{3}{4} \div 5$	$\frac{9}{10} \div 8$	$\frac{11}{12} \div 5$
112.	$\frac{8}{9} \div 4$	$\frac{7}{8} \div 6$	$\frac{8}{9} \div 7$	$\frac{15}{16} \div 8$
113.	$\frac{6}{7} \div 3$	$\frac{15}{16} \div 4$	$\frac{6}{7} \div 8$	$\frac{7}{12} \div 6$
114.	$5 \div \frac{5}{6}$	$18 \div \frac{3}{4}$	$24 \div \frac{8}{9}$	$72 \div \frac{5}{7}$
115.	$7 \div \frac{7}{8}$	$19 \div \frac{4}{5}$	$36 \div \frac{9}{10}$	$24 \div \frac{6}{7}$
116.	$15 \div \frac{15}{16}$	$25 \div \frac{5}{7}$	$54 \div \frac{6}{7}$	$18 \div \frac{4}{5}$
117.	$\frac{3}{4} \div \frac{7}{8}$	$\frac{7}{8} \div \frac{4}{5}$	$\frac{9}{25} \div \frac{4}{10}$	$\frac{14}{15} \div \frac{7}{8}$
118.	$\frac{15}{16} \div \frac{3}{4}$	$\frac{11}{14} \div \frac{5}{6}$	$\frac{9}{16} \div \frac{8}{9}$	$\frac{9}{16} \div \frac{9}{10}$
119.	$\frac{5}{9} \div \frac{3}{5}$	$\frac{7}{16} \div \frac{3}{4}$	$\frac{8}{9} \div \frac{8}{9}$	$\frac{8}{11} \div \frac{8}{9}$
120.	$16\frac{2}{3} \div 3$	$17\frac{1}{4} \div 9$	$25 \div 4\frac{3}{5}$	$25 \div 2\frac{3}{4}$
121.	$15\frac{1}{2} \div 5$	$33\frac{1}{3} \div 6$	$62 \div 8\frac{2}{3}$	$16 \div 7\frac{1}{8}$
122.	$75\frac{1}{4} \div 4$	$62\frac{1}{2} \div 8$	$60 \div 5\frac{1}{7}$	$19 \div 5\frac{1}{4}$
123.	$8\frac{2}{3} \div 3\frac{1}{5}$	$2\frac{1}{3} \div 3\frac{3}{4}$	$8\frac{1}{7} \div 4\frac{1}{6}$	$26\frac{2}{5} \div 3\frac{3}{10}$
124.	$7\frac{1}{6} \div 5\frac{1}{4}$	$6\frac{1}{5} \div 5\frac{2}{5}$	$5\frac{1}{2} \div 3\frac{1}{4}$	$12\frac{3}{4} \div 8\frac{1}{2}$
125.	$8\frac{1}{9} \div 3\frac{4}{5}$	$7\frac{1}{8} \div 3\frac{2}{3}$	$6\frac{1}{4} \div 5\frac{1}{8}$	$16\frac{2}{3} \div 4\frac{4}{9}$

PROBLEMS

1. Balance the following account:

Receipts: \$217.65, \$525.17, \$654.01, \$256.75.

Expenses: \$106.25, \$506.19, \$321.75, \$143.44.

2. If I raise 240 heads of lettuce and sell them at 4 for 6¢, how much do I get from the sale?

3. If you buy eggs at 3 for 5¢ and sell them at 3 for 10¢, how much do you gain on 10 dozen eggs?

4. How many feet of wire will be needed to inclose a field with a 5-wire fence, if the field is 50 feet long and 40 feet wide?

5. If I need 50 inches of linen for the skirt of a cooking apron and 10 inches for the bib and the straps, how many such cooking aprons can I cut from 10 yards of linen?

6. How many bunches of radishes at 3 for 5¢ must Frank raise to pay for a tool chest costing \$12?

7. Ruth used for a dress 5 yards of crepe de chine @ \$1.32, 2 yards of messaline @ \$1.00, 2 spools of silk @ 10¢, 1 card of hooks and eyes @ 10¢, and collar stays @ 10¢. Find the cost of the dress.

8. If 3 dozen oranges can be bought for \$.75 what is the price of half a dozen?

9. The playhouse windows were 38 inches high. How much scrim did Edith buy to make 2 curtains for each of 3 windows, if she allowed $2\frac{3}{8}$ inches for shrinkage, $3\frac{1}{4}$ inches for hem at bottom, and $2\frac{1}{8}$ inches for hem at top?

10. The length of a picture frame is $18\frac{3}{4}$ inches; its width is $12\frac{1}{2}$ inches. What is the entire distance around it? Draw the outline of the frame on the board, exact size.

11. If you sell to R. H. Fiske $12\frac{3}{4}$ yards of velvet at \$2.50 a yard, $20\frac{1}{4}$ yards of ribbon at 10 cents a yard, $6\frac{1}{3}$ dozen

buttons at 15 cents a dozen, and $24\frac{1}{2}$ yards of lace at 56 cents a yard, how much will he owe you? Make out a receipted bill. If Mr. Fiske hands you a fifty-dollar bill, what amount should you return to him?

12. If $2\frac{1}{2}$ bushels of apples cost \$5 (\$3), how much will 1 bushel cost? How much will $3\frac{1}{2}$ bushels cost? Solve in two ways. First solve the problem with \$5, then with \$3.

13. At \$2 ($\frac{5}{8}$ of a dollar) per bushel, how many bushels of apples can be bought for \$12 (\$$12\frac{1}{2}$)?

14. The quotient is 4, the divisor is 3. What is the dividend? The quotient is $\frac{3}{4}$, the divisor is $\frac{5}{6}$. What is the dividend?

15. The divisor is $\frac{7}{8}$, the quotient is $\frac{2}{3}$. What is the dividend? Having a quotient and divisor given, how can you find the dividend?

16. The quotient is 7, the dividend is 28. What is the divisor? Make and solve a similar example with fractions.

17. If 18 yards of ribbon are cut into pieces $\frac{3}{4}$ of a yard long, how many pieces will it make?

18. How many times can I fill a pail holding $1\frac{1}{4}$ gallons from a tank containing 20 gallons?

19. How many strips of carpet $\frac{3}{4}$ of a yard wide will be needed for a room 27 feet wide? How many yards wide is the room?

20. From a piece of cloth containing $25\frac{1}{2}$ yards, there were sold $3\frac{1}{2}$ yards, $7\frac{1}{8}$ yards, and $12\frac{3}{4}$ yards. How many yards remained?

21. Wheat weighs 60 pounds to a bushel. A farmer sells first 3620 pounds and then 3580 pounds of wheat at 75 ¢ a bushel. How much should he receive?

22. Corn weighs 56 pounds to a bushel. If a farmer sells 6720 pounds of corn at 35 ¢ a bushel, and 4260 pounds of wheat at 75 ¢ a bushel, how much should he receive?

23. What is the cost of 1340 fence posts at $7 per C? (C means 100.)

24. In the working days of a week, a man works $9\frac{1}{2}$ hours, $8\frac{3}{4}$ hours, $10\frac{2}{3}$ hours, $8\frac{3}{4}$ hours, $9\frac{3}{4}$ hours, and $8\frac{1}{4}$ hours. How much are his wages for the week, at 30 ¢ an hour?

25. At $1.50 a day, how much does a man earn in 3 weeks, if he works $5\frac{3}{4}$ days the first week, $4\frac{3}{5}$ days the second week, and $5\frac{2}{3}$ days the third week?

26. A milkman sells $\frac{1}{2}$ of a pint of cream and $1\frac{1}{2}$ quarts of milk daily to each of three families. What is the value of the cream and the milk sold to the three families in 4 days, if cream is worth 20 ¢ a quart and milk 6 ¢ a quart?

27. From a lot of $6\frac{1}{2}$ dozen pairs of shoes, $3\frac{1}{3}$ dozen pairs were sold at $2.75 per pair, and the rest at $2.50 per pair. How many pairs were sold at $2.50 per pair? How much was received for the entire lot?

28. A woman buys a $6\frac{3}{8}$-yard remnant of silk worth $1.20 a yard for $6.25. How much does she save by buying the remnant?

29. What is the total cost of 2 turkeys at $25\frac{1}{2}$ ¢ a pound, one weighing $12\frac{1}{4}$ pounds and the other $11\frac{3}{4}$ pounds, and 3 chickens at $24\frac{1}{2}$ ¢ a pound, the first weighing 4 pounds, the second, $5\frac{1}{8}$ pounds, and the third $4\frac{7}{8}$ pounds?

30. If a number is diminished by $\frac{3}{8}$ of itself, 480 remains. What is the number?

SUGGESTION. When a number is diminished by $\frac{3}{8}$ of itself, $\frac{5}{8}$ of itself remains. 480 is $\frac{5}{8}$ of what number?

31. If a number is increased by $\frac{3}{8}$ of itself, the sum is 165. What is the number?

SUGGESTION. 165 is $\frac{11}{8}$ of what number?

32. If 4 farm-hands can do a piece of work in $9\frac{1}{2}$ days, how long will it take 1 farm-hand to do it? How long will it take 7 farm-hands to do it?

33. If 5 pounds of tea cost $\$3\frac{1}{4}$, how much will 24 pounds cost? Work in two ways.

34. If $2\frac{1}{2}$ bushels of corn cost 95¢, how much will $3\frac{1}{4}$ bushels cost?

35. A man lost $\frac{5}{16}$ of his money, and afterward gained $\frac{1}{5}$ of what he had lost. What part of the original amount had he then?

36. If a merchant buys goods for \$420 and sells them at a profit of $\frac{1}{5}$ of the cost, for how much does he sell them?

37. If a merchant buys goods for \$420 and sells them for \$504, what part of the cost does he gain?

38. If one farmer can build a fence in 5 days and another can build it in 4 days, how long will it take them working together?

39. A farmer can build a fence in $5\frac{1}{4}$ days. What part of the work can he do in $\frac{1}{4}$ of a day? in 1 day?

40. One farmer can plant a field of corn in $2\frac{1}{3}$ days, and his neighbor can do it in 3 days. What part of the work can they both do in 1 day? How many days will it take both to do the work?

41. If a farmer can build a fence in 5 days working $8\frac{1}{2}$ hours a day, in how many days can he do it working 9 hours a day?

42. A can dig a cellar in 12 days, and B in 8 days. What part can each finish in 1 day? What part can both finish in 1 day?

43. A does $\frac{1}{7}$ of a piece of work in 1 day. In how many days can he complete such a job?

44. B does $\frac{2}{5}$ of a piece of work in 1 day. In how many days can he complete such a job?

45. C does $\frac{7}{8}$ of a piece of work in 1 day. In how many days can he complete such a job?

46. D does $\frac{3}{16}$ of a piece of work in 1 day. In how many days can he complete such a job?

47. E does $\frac{7}{20}$ of a piece of work in 1 day. How long will it take him to do the whole work?

48. A merchant sold goods for \$350, or $\frac{1}{6}$ more than they cost. What was the cost? What part of the cost would he have gained if he had sold them for \$375?

SUGGESTION. \$350 = $\frac{7}{6}$ of the cost.

49. A man sold sheep at \$5 each, and gained $\frac{1}{4}$ of the cost. What was the cost? If he had lost $\frac{1}{4}$ of the cost by selling them at \$3, what would have been the cost?

50. A man sells two cows at \$60 each; on one he gains $\frac{1}{4}$ of what it cost him, and on the other he loses $\frac{1}{4}$ of its cost. Does he gain or lose, and how much?

51. A bin containing 500 cubic feet is $\frac{3}{5}$ full of wheat. If a bushel is equal to $1\frac{1}{4}$ cubic feet, how much are the contents worth at 70 cents a bushel?

52. A bin containing 306 bushels when full is $\frac{2}{3}$ full of wheat. What is the value of the wheat at 75¢ a bushel?

DECIMAL FRACTIONS

Since 10 dimes equal a dollar, 1 dime is $\frac{1}{10}$ of a dollar. Since 100 cents equal a dollar, 1 cent is $\frac{1}{100}$ of a dollar. Since 1000 mills equal a dollar, 1 mill is $\frac{1}{1000}$ of a dollar. These are called decimal divisions of the dollar.

The point between dollars and cents is called a decimal point; as, \$8.654. The first place to the right of the point represents *dimes* or *tenths* of a dollar; the second place, *cents* or *hundredths* of a dollar; the third place, *mills* or *thousandths* of a dollar.

Any unit may be divided into tenths, hundredths, thousandths, etc. Thus, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$. These are **decimal divisions** of the unit. *One tenth* may be written .1 instead of $\frac{1}{10}$; *one hundredth* may be written .01 instead of $\frac{1}{100}$; *one thousandth* may be written .001 instead of $\frac{1}{1000}$.

A **decimal fraction** or a **decimal** is one whose denominator is 10 or some power of 10. The term *decimal* is more commonly applied to a decimal fraction written with a decimal point.

The point separating the units and parts of units is called the **decimal point.** The orders to the right of the decimal point are called **decimal places.**

How many tenths are there in one unit? how many hundredths in 1 tenth? how many thousandths in 1 hundredth?

Ten units of any place equal 1 *unit of the next place to the left.*

A **mixed decimal** is one having an integral and a decimal part; as, 12.75, 5.045.

DECIMAL NOTATION AND NUMERATION

Study Recitation

In integral or whole numbers, the value of a figure in any order is $\frac{1}{10}$ the value of the same figure in the next higher order, and 10 times the value of the same figure in the next lower order. In the following numbers, show that the value of any figure is $\frac{1}{10}$ the value of the figure in the next higher order, and 10 times the value of the figure in the next lower order :

1111 2222 3333 4444 5555 9999

In whole numbers, the name of the lowest order is units, or ones. In the number 1111, $\frac{1}{10}$ of the value of the figure in the fourth order is 1 hundred, or the value of the figure in the third order ; $\frac{1}{10}$ of the value of the figure in the third order is 1 ten, or the value of the figure in the second order ; $\frac{1}{10}$ of the value of the figure in the second order is 1 unit, or the value of the figure in the first order.

$\frac{1}{10}$ of the value of the figure in the first order is $\frac{1}{10}$. As a decimal it is written without the denominator by placing a decimal point at the right of the units' order and then writing 1 at its right ; thus, 1111.1.

The name of the *first* order at the right of the decimal point is *tenths*. The value of .1 is $\frac{1}{10}$.

$\frac{1}{10}$ of the value of 1 in tenths' order is $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$, which is written at the right of the 1 in tenths' order ; thus, .11.

The name of the *second* order at the right of the decimal point is *hundredths*.

$\frac{1}{10}$ of the value of 1 in hundredths' order is $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$, written at the right of the 1 in hundredths' order ; thus, .111.

The name of the *third* order at the right of the decimal point is *thousandths*.

In the same way it may be shown that $\frac{1}{10}$ of $\frac{1}{1000}$ is $\frac{1}{10000}$, that it is written at the right of the thousandths' order, and that the *fourth* order at the right of the decimal point is *ten-thousandths.*

The name of the *fifth* order is *hundred-thousandths.*

The name of the *sixth* order is *millionths.*

The name of the *seventh* order is *ten-millionths*, and so on.

To write decimals.

By using the decimal point, decimal fractions may be written in the same manner as whole numbers are written, the denominator being indicated by the number of places at the right of the decimal point.

	Tenths, Hundredths, Thousandths, Ten-thousandths, Hundred-thousandths, Millionths	Learn:
$\frac{1}{10}$ =	.1	1 place or order is required to write tenths. Note that there is *one* zero in the denominator of $\frac{1}{10}$.
$\frac{1}{100}$ =	.0 1	2 places or orders are required to write hundredths. Note that there are *two* zeros in the denominator of $\frac{1}{100}$.
$\frac{1}{1000}$ =	.0 0 1 . . .	3 places or orders are required to write thousandths. How many zeros are there in the denominator of $\frac{1}{1000}$?
$\frac{1}{10000}$ =	.0 0 0 1 . .	4 places or orders are required to write ten-thousandths.
$\frac{1}{100000}$ =	.0 0 0 0 1 .	5 places or orders are required to write hundred-thousandths.
$\frac{1}{1000000}$ =	.0 0 0 0 0 1	6 places or orders are required to write millionths.
$5\frac{123456}{1000000}$ =	5.123456	This is the way to write 5 and 123,456 millionths.

The number of places required to write a decimal is the same as the number of zeros in its denominator.

Written

NOTE. It is rarely necessary to use decimal orders beyond millionths.

1. Write 56 thousandths. Write the numerator 56. There are three 0's in the denominator 1000. Therefore there are three decimal places. Place a cipher before 56. Thus, .056.

2. Write 4 ten-thousandths. Write 4. As the denominator has four 0's, there are four decimal places. Write three 0's before 4 to make 4 places; thus, .0004.

Write the numerator, and point off from the right as many decimal places as there are zeros in the denominator.

3. Write three thousand two hundred fifteen and twenty-seven ten-thousandths. Write the whole number and then write the decimal.

4. How many figures are required to write thousandths? tenths? millionths? ten-thousandths? hundredths? hundred-thousandths?

5. Write the names of the decimal orders from the decimal point to the sixth order to the right.

6. A decimal has 4 places. Write its denominator. Write the name of its denomination.

7. Write the denomination of a decimal of 3 places; of 5 places; 2 places; 6 places; 4 places; 1 place.

8. Write the denominations of the following decimals: 475; .0361; .00045; .05; .6; .275046.

Write in decimal notation:

9. 25 hundredths.

10. 9 thousandths.

11. 19 thousandths.

12. 199 thousandths.

13. Three hundred six ten-thousandths.

14. Seventy-three and eight hundred-thousandths.

15. Nine hundred six and forty-three millionths.

16. Seven ten-thousandths.
17. 830 and 46 hundredths.
18. 47 thousandths.
19. 82 hundredths.
20. 834 hundred-thousandths.
21. 400 and 6 hundredths.
22. 1 and 15 millionths.
23. $\frac{7}{10}$, $\frac{32}{1000}$, $\frac{70}{10000}$, $\frac{5}{100}$, $\frac{15}{100}$, $\frac{15}{1000}$, $\frac{115}{1000}$, $\frac{115}{100}$.
24. Nine hundred eight thousand, and two thousandths.
25. Nine thousand, and nine ten-thousandths.
26. 864 million, and 84 thousandths.
27. 300 million 64, and 64 hundredths.
28. $\frac{83}{1000000}$, $\frac{7}{1000}$, $\frac{20}{10000}$.
29. $\frac{7}{1000}$, $\frac{83}{100}$, $\frac{65}{1000}$.
30. $\frac{875}{1000000}$, $96\frac{3}{10}$, $\frac{1}{100000}$.
31. Seven hundred four thousand one, and one thousandth.
32. Ten thousand, and four ten-thousandths.
33. Forty-five million, and forty-five thousandths.
34. 798 million 98, and 98 hundredths.
35. 873 ten-thousandths.
36. 56 and 65 millionths.
37. 412 and 78 thousandths.
38. 8706 and 168 ten-thousandths.
39. 99 and 3 thousandths.
40. 2 and 12 ten-thousandths.
41. 135 millionths.
42. One thousand and one thousandth.
43. 861 and 861 thousandths.

To read decimals.

1. Read .215. The name of the right-hand order is thousandths. As a whole number you read: Two hundred fifteen. Add the name of the right-hand order, and read: Two hundred fifteen thousandths.

2. Read .21567. The name of the right-hand order is hundred-thousandths. As a whole number you read: Twenty-one thousand five hundred sixty-seven. Add the name of the right-hand order and read: Twenty-one thousand five hundred sixty-seven hundred-thousandths.

3. Read .0032. The name of the right-hand order is ten-thousandths. The decimal is read: Thirty-two ten-thousandths.

Read the decimal as though it were a whole number, and give it the name of the right-hand order.

In reading a whole number and a decimal combined, as, 156.27, read the whole number first; then the word "and," and then the decimal. Thus, one hundred fifty-six and twenty-seven hundredths.

Read; then write from dictation:

	a.	*b.*	*c.*	*d.*
4.	.5	.27	26.9	.03
5.	.24	.9	.064	.3
6.	.06	.376	.0007	.83
7.	.05	.27	.8566	.2706
8.	.0005	.328	.4728	.3405
9.	.00075	.0486	.275	.125
10.	.023	.2056	.0006	.9807
11.	.07805	.2005	.008	.004
12.	.27506	.2063	.0009	.7805
13.	.325	$2.8\frac{1}{2}$	.9006	.0064
14.	.025	19.002	.7	.0008

	a.	*b.*	*c.*	*d.*
15.	67.74	4.3646	.8607	.08
16.	153.0004	86.0009	986.27	.001
17.	4.4067	14.3609	$124.004\frac{1}{3}$	1.00032
18.	875.64	800.08	9807.007	5647.0276
19.	907.875	25.025	2000.0002	40.045
20.	807.0019	$208.8\frac{1}{2}$	$560.37\frac{1}{2}$	5.00055
21.	918.00016	$563.005\frac{1}{2}$	$21708.006\frac{1}{4}$	27.003256
22.	76.2705	$4301.07\frac{1}{4}$	35912.257	9006.0004

Decimals that have the same number of decimal places are *similar* decimals.

To change the denomination of a decimal.

$.5 = \frac{5}{10} = \frac{1}{2}$
$.50 = \frac{50}{100} = \frac{1}{2}$
$.500 = \frac{500}{1000} = \frac{1}{2}$
$.5000 = \frac{5000}{10000} = \frac{1}{2}$

What is the effect of annexing zeros to a decimal? of omitting zeros?

1. Change to similar decimals: .3, .05, .045, and .3560.

.3 = .300
.05 = .050
.045 = .045
.3560 = .356

The smallest denomination by which all these numbers can be expressed is thousandths, since .3560 = .356. Reduce the first two decimals to thousandths by annexing zeros and the last by omitting zero.

2. Change to similar decimals: .5, .65, .530; .4, .55, .6300.

Change:

3. .3 to thousandths.

4. .27 to ten-thousandths.

5. .105 to ten-thousandths.

6. .020 to hundredths.

7. .55 to thousandths.

8. .3400 to hundredths.

9. How are decimals changed to the same denomination?

10. Change to hundredths; to thousandths:

$\frac{1}{2}$, $\frac{1}{5}$, $\frac{4}{5}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{10}$, $\frac{5}{10}$, $\frac{6}{10}$.

REDUCTION OF DECIMALS TO COMMON FRACTIONS

Study Recitation

1. Reduce .75 to a common fraction in its lowest terms.

$.75 = \frac{75}{100} = \frac{3}{4}$ Express the decimal as a decimal fraction, $\frac{75}{100}$. Reduce this fraction to its lowest terms, $\frac{3}{4}$.

2. Reduce .025 to a common fraction in its lowest terms.

Solution. $.025 = \frac{25}{1000} = \frac{1}{40}$.

3. Reduce $.05\frac{1}{2}$ to a fraction in its lowest terms.

$$\text{Solution.}\quad .05\tfrac{1}{2} = \frac{5\frac{1}{2}}{100} = \frac{\frac{11}{2}}{100} = \frac{11}{2} \times \frac{1}{100} = \frac{11}{200}.$$

Or, $.05\frac{1}{2} = .055 = \frac{55}{1000} = \frac{11}{200}$.

Write the denominator of the decimal underneath the decimal. Remove the decimal point and reduce the fraction to its lowest terms.

Written

Reduce the following decimals to common fractions in their lowest terms:

	a.	*b.*	*c.*	*d.*	*e.*
4.	.5	.375	.3125	$.6\frac{1}{4}$	$.31\frac{1}{4}$
5.	.4	.625	.5625	$.33\frac{1}{3}$	$.062\frac{1}{2}$
6.	.25	.875	.0375	$.16\frac{2}{3}$	$.08\frac{1}{3}$
7.	.75	.555	.4375	$.14\frac{2}{7}$	$.43\frac{3}{4}$
8.	.30	.125	.0625	$.11\frac{1}{9}$	$.58\frac{1}{3}$
9.	.20	.150	.8125	$.12\frac{1}{2}$	$.05\frac{5}{9}$
10.	.10	.225	.0875	$.66\frac{2}{3}$	$.00\frac{1}{2}$
11.	.35	.700	.6875	$.37\frac{1}{2}$	$.00\frac{5}{6}$
12.	.40	.025	.0125	$.28\frac{4}{7}$	$.00\frac{3}{4}$
13.	.45	.075	.1975	$.06\frac{2}{3}$	$.062\frac{1}{2}$
14.	.50	.235	.1425	$.06\frac{1}{4}$	$.56\frac{1}{4}$

ADDITION AND SUBTRACTION OF DECIMALS

Study Recitation

1. Add: .27, 6.05, 3.756, .326.

$$\begin{array}{r} .27 \\ 6.05 \\ 3.756 \\ .326 \\ \hline 10.402 \end{array}$$

Write the decimals so that units of the same order stand in the same column. The decimal points will then be in a column.

Add as in whole numbers, and place the decimal point under the points above.

2. From 2.76 take .081.

$$\begin{array}{r} 2.760 \\ .081 \\ \hline 2.679 \end{array}$$

Write the numbers with the decimal points in a column. Annex one zero to the minuend to change it to thousandths. Subtract as in whole numbers, and place the decimal point under the points above.

Written

Write from dictation and add:

3. $.2 + .02 + .002 + .0002$

4. $.5 + .25 + .013 + .0009$

5. $.7 + .03 + .024 + .0305$

6. $.08 + .6 + .015 + .0567$

7. $3.08 + 5.5 + 3.027 + 6.0123 + 56.0312 + 72.005$

8. $8.8 + 8.88 + 8.888 + 8.8888 + 2.0045 + 64.0102$

9. $1.08 + 14.6 + 3.009 + 16.005 + 13.6304 + 15.7583$

Write from dictation and subtract:

	a.	*b.*	*c.*
10.	$.6 - .3$	$.05 - .02$	$.07 - .0025$
11.	$.7 - .5$	$.06 - .003$	$.5 - .007$
12.	$14.3 - .6$	$17.05 - .015$	$16.04 - .105$
13.	$52.08 - .7$	$16.04 - .0005$	$17.05 - .045$
14.	$43.008 - .06$	$57.0002 - .05$	$25.057 - .019$

Write from dictation and add:

15.	*a.*	*b.*	*c.*	*d.*
	.275	.4	25.3	64.7
	.056	.0045	1.75	9.08
	.105	.00065	.2507	23.10
	.30852	.278	15.003	16.00

16.	8.15	3.42	7.703	19.119
	3.9	9.204	63.424	5.09
	90.81	6.17	8.58	.635
	4.006	89.008	6.011	.006

17.	8.004	3.050	59.400	.908
	6.057	98.010	45.800	18.103
	521.119	.709	361.612	6.084
	4.028	29.014	1246.000	7.402

18, 19. Add the first lines of examples, 15 *abcd*, and 16 *abcd* (8 addends); of 16 *abcd* and 17 *abcd*.

20, 21. Add the second lines of 15 *abcd* and 16 *abcd*; of 16 *abcd* and 17 *abcd*.

22, 23. Add the third lines of 15 *abcd* and 16 *abcd*; of 16 *abcd* and 17 *abcd*.

24, 25. Add the fourth lines of 15 *abcd* and 16 *abcd*; of 16 *abcd* and 17 *abcd*.

Write from dictation and subtract. Prove your results by adding subtrahends and remainders:

26.	2.0005	25.34	.9437	6.001
	1.2316	21.275	.72891	5.0432

27.	35.04	1.027	5.	7.0095
	32.163	.9863	3.356	5.1098

28.	8.095	5.204	48.009	63.
	6.213	3.18	32.456	45.198
29.	28.	51.001	800.	900.
	18.049	49.079	469.384	802.345
30.	2.100	854.106	76.592	1000.
	1.789	659.378	43.486	620.002
31.	568.39	56.294	938.35	1200.
	394.364	37.458	519.343	853.075

32–55. Subtract the remainders in examples 26 to 31 from the subtrahends.

56. Find the sum of twenty-five thousandths, three hundred four ten-thousandths, five hundredths, eighteen ten-thousandths.

57. Find the sum of 25 millionths, 305 ten-thousandths, 28 thousandths, 8 tenths.

Read, write units of the same order in the same columns, and perform the required operations:

58. $3.26 + 83.08 + .9 + .026 + .07 + 3.0072 + 47.8305$.

59. Subtract 86.0764 from 193.08696.

60. Find the sum of .36, 3.7, 86.03, .476, 8.304, and .0005.

61. Find the difference between 6.83 and 4.7638.

62. Add 7.5, .83, .936, 90, 8.7506, 12.065, and 16.007.

63. From 78.00067 take 24.999999.

64. From three and eight tenths take sixty-four thousandths.

PROBLEMS

Written

1. A farmer has 5 fields containing, respectively, 12.625 acres, 30.75 acres, 4.4 acres, 190.375 acres, and 12.125 acres. How many acres are there in the five fields?

2. From 24.875 tons of coal I sell 7.9 tons. How many tons remain?

3. A has 46.05 tons of hay; B has 2.375 tons less than A. How many tons of hay have A and B?

4. Ida bought a ready-made waist for \$4.95 and paid \$.50 for alterations. I had a similar waist made to order, paying \$1.25 for making, \$1.80 for crepe, and \$1.50 for trimming, buttons, etc. How much cheaper was my waist?

5. If the temperature on July 3 was 87.9 degrees and on July 4, 94.75 degrees, how much did it increase from July 3 to July 4?

6. A train runs 35.125 miles in one hour and 30.625 miles the next hour. How much faster does it run the first hour than the second hour?

7. If I start out to walk 10.125 miles, how much farther must I walk after walking 3.875 miles?

8. The sum of two numbers is 204.56 and one of the numbers is 56.0057. What is the other number?

9. If I cut 4.875 yards from a piece of cloth containing 9.625 yards, how much is left?

Cooking Problems

The Stout Institute at Menomonie, Wisconsin, gave an exhibition of cookery for which the students made the following articles of food at the prices indicated. Find the cost of each article:

NOTE. c., cup; tsp., teaspoonful; tbsp., tablespoonful; lb., pound.

10. *Graham Bread*

1 c. graham flour	$.0075
$\frac{3}{4}$ c. white sauce	.016
$\frac{1}{2}$ tsp. soda	.0001
1 c. milk (sour)	.015

11. *Asparagus on Toast*

3 slices bread	$.012
$\frac{1}{2}$ can asparagus	.125
1 c. milk	.017
$\frac{1}{2}$ tsp. salt	.0005
2 tbsp. flour	.004
2 tbsp. butter	.0118
$\frac{1}{2}$ bunch parsley	.025

12. *Mince Pie*

2 c. flour	$.015
$\frac{1}{2}$ tsp. salt	.0005
2 tbsp. lard	.0078
2 tbsp. butter	.0118
2 c. mince meat	.20

13. *Parker House Rolls*

$\frac{1}{2}$ c. milk	$.0085
$\frac{1}{4}$ cake compressed yeast	.005
1 tbsp. lard	.004
1 tbsp. sugar	.0017
2 c. flour	.015
$\frac{1}{2}$ tsp. salt	.0005

14. *Stuffed Peppers*

6 green peppers	$.10
1 c. corn	.05
1 c. thick white sauce	.025
1 c. bread crumbs	.006
$\frac{1}{4}$ tsp. salt	.00025

15. *Roast Beef and Franconian Pototoes*

3 lb. beef	$.54
6 potatoes	.03
2 tsp. salt	.002
$\frac{1}{2}$ c. flour	.0038

16. *Meat Croquettes*

1 c. meat	$.15
$\frac{1}{2}$ c. milk	.0085
2 tbsp. butter	.0118
2 tbsp. flour	.004
1 egg	.0266
$\frac{1}{2}$ c. bread crumbs	.003

17. *Potatoes in Half Shell*

2 tbsp. butter	$.0118
3 tbsp. milk	.006
1 egg	.025
$\frac{1}{2}$ c. bread crumbs	.003
6 potatoes	.03

18. *Twin Mountain Muffins*

$\frac{1}{4}$ c. butter	$.04$\frac{1}{2}$
$\frac{1}{4}$ c. sugar	.00$\frac{5}{8}$
1 egg	.02$\frac{5}{12}$
$\frac{1}{4}$ c. milk	.00$\frac{3}{8}$
2 c. flour	.01$\frac{1}{2}$
4 tsp. baking powder	.01$\frac{1}{3}$

19. *Spinach Mold*

1 can spinach	$.15
1 tbsp. butter	.0118
lemon juice	.05
$\frac{1}{2}$ tsp. salt	.0005
3 eggs (garnish)	.075

20. *Salmon Loaf*

1 can salmon	$.20
2 eggs	.046$\frac{2}{3}$
$\frac{1}{2}$ c. milk	.007$\frac{3}{4}$
1 c. stale bread crumbs	.006

21. *Roast Chicken*

4 lb. chicken	$.70

DRESSING

2 c. bread and cracker crumbs	$.012
$\frac{1}{2}$ c. melted butter	.10
1 tbsp. sage and pepper	.005
1 tsp. salt	.001
1 c. scalded milk	.0175

22. *Fondant Candy*

2 c. cane sugar	$.05
2 tsp. cream of tartar	.0166
2 oz. almonds	.0863
2 oz. walnuts	.075

23. *Invalid Tray for Dietetic Patient*

1 c. salmon	$.10
2 eggs	.05
1 grapefruit	.10
1 slice bread	.0033
1 c. cream	.075
3 stalks celery	.03

24–36. How much more did the chicken cost than each of the other articles?

37–48. How much more did the roast beef cost than each of the articles except the chicken?

Find the difference in the cost of:

49. Spinach and asparagus.

50. Salmon and meat croquettes.

51. Mince pie and Parker house rolls.

52. Candy and muffins.

53. Meat croquettes and potatoes.

54. Asparagus and peppers.

55. Muffins and Graham bread.

56. Mince pie and muffins.

57. Salmon and potatoes.

58. Muffins and potatoes.

MULTIPLICATION OF DECIMALS

To multiply a decimal by a whole number.

STUDY RECITATION

1. Multiply $1.25 by 4; 1.25 by 4.

2. Multiply $1.375 by 5; 1.375 by 5.

Multiply as in whole numbers, placing the decimal point under the point in the multiplicand.

Find the products:

	a.	b.	c.
3.	4×1.2	7×2.15	9×3.054
4.	3×2.65	8×1.44	8×2.1356
5.	6×5.43	12×1.375	5×3.0004
6.	5×4.32	7×2.625	12×1.0505
7.	9×7.28	6×3.875	8×2.0059
8.	$7\frac{1}{5} \times 3.25$	$3\frac{2}{3} \times 4.125$	$12\frac{1}{4} \times 1.2344$

To multiply a decimal by 10, 100, 1000, etc.

1. Multiply 3.75 by 10; by 100; by 1000.

$3.75 \times 10 = 37.5$
$3.75 \times 100 = 375$
$3.75 \times 1000 = 3750$

In multiplying by 10, the decimal point is moved 1 place to the right; by 100, 2 places to the right; by 1000, 3 places to the right.

Move the decimal point as many places to the right as there are zeros in the multiplier, annexing zeros, if necessary.

Multiply the following numbers by 10; by 100; by 1000.

	a.	b.	c.	d.
2.	7.5	.25	.157	25.06
3.	.0003	20.	.046	2.03
4.	98.7	11.6	1.003	150.3

	a	b.	c.	d.
5.	1.003	25.6	40.009	10.05
6.	.501	4.001	50.067	1.005
7.	.004	13.75	3.02	.1005

To multiply a decimal by a decimal.

STUDY RECITATION

1. $\frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$
   $.7 \times .3 = .21$

2. $18 \times \frac{2}{10} = \frac{36}{10} = 3\frac{6}{10}$
   $18 \times .2 = 3.6$

3. $\frac{3}{10} \times \frac{7}{100} = \frac{21}{1000}$
   $.3 \times .07 = .021$

4. $\frac{3}{100} \times \frac{4}{100} = \frac{12}{10000}$
   $.03 \times .04 = .0012$

Compare the sum of the decimal places in the multiplicand and the multiplier with those in the product.

Point off in the product as many decimal places as there are in the multiplicand and multiplier together.

5. Multiply 2.456 by .24.

$$\begin{array}{r} 2.456 \\ .24 \\ \hline 9824 \\ 4912 \\ \hline .58944 \end{array}$$

Multiply as in integers. The sum of the decimal places in the multiplier and the multiplicand is 5; hence place the decimal point 5 places from the right.

6. Multiply .00176 by .015.

$$\begin{array}{r} .00176 \\ .015 \\ \hline 880 \\ 176 \\ \hline .00002640 \end{array}$$

Multiply as before. The sum of the decimal places in the multiplier and multiplicand is 8; there are but four figures in the product; hence, 4 decimal zeros must be prefixed to the product obtained by the multiplication. The last zero to the right may be omitted.

Without multiplying, give the number of places to be pointed off in the following products. Then find the products.

WRITTEN

	a.	b.	c.	d.
7.	$.3 \times .4$	$.5 \times 4$	$1.5 \times .04$	$1.5 \times .00004$
8.	$.12 \times 2$	$.12 \times .2$	14.2×2	$1.42 \times .04$
9.	$.04 \times .3$	$.06 \times .04$	$.151 \times .03$	$1510 \times .0003$
10.	$1.2 \times .2$	$.2 \times 6$	$.7 \times 10$	$.07 \times 10$
11.	$.07 \times 11$	$.11 \times .07$	7.8×100	$.006 \times 10$
12.	$.012 \times .3$	$.012 \times .003$	$.008 \times 100$	$.87 \times 1000$

Multiply:

	a.	b.	c.	d.	e.	f.	g.
13.	.275 .15	1.05 .08	27.05 .04	.256 1.8	3.75 .004	125 .004	.256 .001
14.	9.87 .13	.654 1.5	8.05 116	74.2 .005	3.01 15.2	6.98 3.49	.757 56.7

	a.	b.	c.
15.	$.18 \times .37$	450×4.65	$.006 \times .005$
16.	$.29 \times .19$	3.34×7.89	$.9201 \times .806$
17.	$.25 \times .25$	9.23×614	86.52×9.37
18.	$.82 \times .63$	$21.149 \times .321$	$.056 \times .054$
19.	$.61 \times .59$	18.018×8.09	$.0045 \times .098$
20.	$.245 \times .826$	$5.637 \times .175$	6.5×1.78
21.	$.039 \times .154$	65.4×8.55	89.5×1.34
22.	$.225 \times .348$	$7.022 \times .051$	12.34×2.961
23.	$.418 \times .809$	9.832×7.72	50.65×3.04
24.	$.324 \times .009$	4.94×2.398	15.07×5.002
25.	$.872 \times 1.02$	4.877×5.03	$90.083 \times .0128$
26.	$.609 \times 148$	$2.526 \times .0408$	57.006×4.9
27.	$.14 \times 8.09$	$7.412 \times .0096$	4.32×1.024

	a.	b.	c.
28.	.58 × 90.35	6.72 × 8.0407	58.69 × 2.5
29.	.76 × .005	3.908 × 4.006	10.009 × 6.09
30.	.121 × .016	5.07 × .076	62.4 × 8.04
31.	.218 × .014	7.66 × .018	96.135 × 9.52
32.	.96 × .009	8.015 × .025	5.408 × 1.98
33.	.315 × 6.04	4.324 × .007	789.362 × 44.9
34.	.536 × .079	3.271 × .095	43.053 × 14.205
35.	402.6 × .0091	5.0002 × .009	2022 × 11.09
36.	7.265 × .018	2.0503 × .64	34.005 × 6.043
37.	30.007 × 7.25	1.031 × .026	4.106 × 19.89
38.	1907.06 × 32.06	9.0746 × .014	45.003 × 26.543
39.	90.075 × .756	$87.58 × 1.756	21.402 × 45.109
40.	2700 × 2.003	$126.05 × .0839	40.049 × 9.009
41.	6.706 × 230	7.763 × 10	86.46 × 6.02
42.	97.063 × .00075	.3576 × 100	8.042 × 1.406
43.	.101 × .019	.067 × 1000	8.239 × 3.703
44.	3.6 × 79.75	.00763 × 100	.0995 × 5000
45.	5.712 × .093	.0906 × 1000	63.021 × 1.0354
46.	6.863 × .008	7.83 × 10000	72.005 × 21.007

PROBLEMS

Written

1. What is the value of 8.5 tons of hay at $7.50 per ton?

2. At $.75 a day, how much will a boy earn in 4.5 days?

3. A cubic foot of water weighs 62.5 pounds. How much do 9.25 cubic feet weigh?

4. If a ton of iron ore yields .65 of a ton of pure iron, how much iron will 376.48 tons yield?

5. Find the cost of 24.375 yards of cloth at \$1.25 a yard.

6. James earned \$38.65 and John earned \$2.75 more than .40 as much as James. How much did John earn?

7. How far will a locomotive go in 6 hours at the rate of 24.75 miles an hour, if 2 stops of 7.5 minutes each are made?

8. A farmer exchanged 84.75 bushels of potatoes at \$.45 a bushel for 450 pounds of sugar at \$.06¼ a pound, and the balance in cash. How much cash did he receive?

9. J. T. Jones bought 24.75 yards of silk ribbon at \$.125 a yard, 12.25 yards of alpaca at \$1.20 a yard, and 10.5 yards of silesia at \$.16 a yard. Write out, foot, and receipt the bill.

10. A store valued at \$12,000 is insured for .625 of its value. What is Mr. Smith's share of the insurance, if he owns .375 of the store?

11. An engine pumped 37.75 barrels of 31.5 gallons each from a reservoir containing 31,500 gallons of water. How many barrels remained?

12. Milk is 1.03 times as heavy as water. If a gallon of water weighs 8.5 lb., how much will 5 gallons of milk weigh?

Value of Fertilizers

Protein contains the nitrogen, and the carbohydrates consist mostly of starches and sugars. The carbon is secured by the plants from the air, but the other substances are furnished by the soil. For every 1000 pounds of corn raised, 79 pounds of nitrogenous compounds, 7 pounds of phosphoric acid, and 4 pounds of potash are withdrawn from the soil. When these substances are exhausted from the land, fertilizers which restore them to the soil become a necessity.

The leading artificial fertilizers are wood ashes, rock phosphate, ground bones, ammonium sulphate, and potassium sulphate. Barnyard manure contains all the substances mentioned in sufficient quantities for restoring fertility to the soil.

Amount of Leading Fertilizer Components in 100 Pounds.

	Protein Pounds	Carbohydrates Pounds	Phosphoric Acid Pounds	Potash Pounds
Corn	7.9	66.7	.7	.4
Wheat	10.2	69.2	.8	.5
Rye	9.9	67.6	.8	.5
Barley	8.7	63.6	.7	.5
Oats	9.2	47.3	.8	.6
Silage corn	.9	11.3	.5	.9
Fodder corn, cured	2.5	34.6	.5	.9
Potatoes	.9	16.3	.1	.5
Timothy hay	2.8	43.4	.3	.6
Red clover	6.8	35.8	.1	.5

13. How many pounds of the leading substances are withdrawn from 10 acres of corn averaging 100 bushels of 70 pounds each per acre?

14. How many pounds of these substances are absorbed by 2 acres of potatoes averaging 300 bushels of 60 pounds each per acre?

15. If the value of the barnyard manure of a horse is $27.25 per year, of a cow, $18.50, and of a hog, $12.25, what is the annual value of the manure of 4 horses, 22 cows, and 40 hogs?

16. From 1 acre of firm soil 159.46 lb. of nitrates, 4.59 lb. of phosphates, and 333.17 lb. of sulphates were washed by rains, while from an acre of similar soil tilled to a depth of 3 in., only 67 lb. of nitrates, 2.69 lb. of phosphates, and 31.94 lb. of sulphates were washed at the same time. How much of these components are saved by keeping a 30-acre tract of corn well tilled?

17. What is the value of the components saved in problem 16, at an average price of $.00625 a pound?

18. A ton of barnyard manure increases the yield of shelled corn 70 lb. per acre. What is the gain on 20 acres if 10 tons are applied per acre, corn selling at \$.55 a bushel? (56 lb. = 1 bu.)

19. If 1000 lb. of grain remove 5.2 lb. of potash from the soil, how much will the grain of 40 bu. of wheat remove? (60 lb. = 1 bu.)

20. If the straw weighs 1.5 times as much as the grain, and 1000 lb. of straw remove 6.3 lb. of potash, how much will the straw of 40 bu. of wheat remove?

21. If 1000 lb. of grain remove 7.9 lb. of phosphoric acid and 1000 lb. of straw remove 2.2 lb. of phosphoric acid, how much phosphoric acid will be removed by 40 bu. of wheat and the straw?

22. The amount of nitrogen in wheat is 20.8 lb. for 1000 lb. of grain and 4.8 lb. for 1000 lb. of straw. How much nitrogen will be removed by 40 bu. of grain and its straw, if the straw weighs $1\frac{1}{2}$ times as much as the grain?

23. 1000 lb. of tobacco take from the soil 42 lb. of nitrogen, 5 lb. of phosphoric acid, and 57 lb. of potash. The nitrogen is worth 15¢ a pound, and each of the other minerals 5¢ a pound. Find the total value of the minerals removed by 1000 lb. of tobacco.

24. 1000 lb. of corn take from the soil 18 lb. of nitrogen, 17 lb. of phosphoric acid, and 4 lb. of potash. Find the total value of the substances removed by 1000 lb. of corn at the same prices as in problem 23.

25. If 1 ton of barnyard manure contains 10 lb. of nitrogen, how much manure would it be necessary to apply to replace the nitrogen used by the 1000 lb. of corn? (See problem 24.)

DIVISION OF DECIMALS

Study Recitation

1. Divide \$.84 by 4.

$$4\overline{)\$.84}$$
$$\$.21$$

2. Divide .84 by 4.

$$4\overline{).84}$$
$$.21$$

3. Divide \$1.32 by 4.

$$4\overline{)\$1.32}$$
$$\$.33$$

4. Divide 1.32 by 4.

$$4\overline{)1.32}$$
$$.33$$

The product of the divisor and the quotient always gives the dividend. The dividend, therefore, has as many decimal places as there are decimal places in both the divisor and the quotient. If the divisor is a whole number, the quotient has as many decimal places as the dividend.

Illustrations:

$4\overline{)2.4}$	$4\overline{)2.36}$	$4\overline{).5268}$	$12\overline{)27.984}$
.6	.59	.1317	2.332

In the above examples, the decimal point of the quotient is placed below the decimal point in the dividend before beginning the division. The first quotient figure is placed below the right-hand figure in the first partial dividend, and the other quotient figures below succeeding figures of the dividend.

$4\overline{).24}$	$4\overline{).0036}$	$12\overline{).1104}$	$12\overline{).0036}$
.06	.0009	.0092	.0003

In the above examples, the decimal point of the quotient is placed below the decimal point in the dividend before beginning the division. A 0 is placed below each figure of the dividend from the left until the last figure of the first partial dividend is reached; below this figure, the first significant figure of the quotient is placed, and the other figures in the quotient are placed in order below the remaining figures of the dividend.

Divide, placing the decimal point in the quotient before beginning the division, and prove the correctness of the work by multiplying the quotient by the divisor.

	a.	b.	c.	d.
5.	$8\overline{)3.84}$	$6\overline{).276}$	$21\overline{)4.431}$	$16\overline{)32.32}$
6.	$8\overline{)38.4}$	$9\overline{).1152}$	$32\overline{).1184}$	$15\overline{)30.45}$
7.	$9\overline{)2.457}$	$11\overline{).1001}$	$42\overline{).2688}$	$14\overline{)2.814}$
8.	$7\overline{)326.9}$	$4\overline{).0028}$	$18\overline{).0126}$	$13\overline{).0026}$
9.	$12\overline{)26.04}$	$5\overline{).0275}$	$16\overline{).0032}$	$12\overline{)3.648}$
10.	$11\overline{)22.11}$	$8\overline{).0224}$	$14\overline{).0028}$	$11\overline{).3344}$

11. Divide .18 by 4.

$$\begin{array}{r} 4\overline{).180} \\ .045 \end{array}$$

Since there is a remainder after dividing .18 by 4, annex a zero, changing .18 to .180 (without altering its value) and continue the division.

Find the quotients and the remainders:

	a.	b.	c.	d.
12.	$.16 \div 5$	$1.2 \div 5$	$5.4 \div 5$	$3.2 \div 5$
13.	$.17 \div 2$	$4.6 \div 4$	$4.4 \div 8$	$.28 \div 8$

Written

	a.	b.	c.
14.	39.06 by 18.	9.80 by 49.	137.92 by 32.
15.	43.68 by 39.	312.4 by 44.	3.285 by 15.
16.	4.956 by 14.	12.978 by 21.	16.566 by 33.
17.	9.18 by 18.	1.25 by 25.	73.5 by 21.
18.	6.096 by 12.	.54 by 18.	24.864 by 32.
19.	.747 by 15.	.02 by 5.	51.17 by 43.
20.	89.25 by 17.	9.4 by 80.	4.08 by 24.

	a.	*b.*	*c.*
21.	.888 by 24.	3.64 by 140.	41.5 by 25.
22.	46.4 by 16.	.01 by 50.	3.535 by 35.
23.	6.03 by 15.	.03 by 50.	2.337 by 41.
24.	93.6 by 18.	.08 by 200.	47.6 by 34.

To divide a number by 10 or any power of 10.

Multiply by moving the decimal point:

1. .27 by 10.

2. .045 by 100.

3. 2.006 by 1000.

4. .15 by 1000.

A number is multiplied by 10 or any power of 10 by moving the decimal point as many places to the right as there are zeros in the multiplier, zeros being annexed if necessary.

5. Divide each of the above products by the multiplier used in producing it, but without going through the process of division.

A number is divided by 10, 100, 1000, *etc., by moving the decimal point toward the left as many places as there are zeros in the divisor.*

Divide by 10; by 100; by 1000:

	a.	*b.*	*c.*	*d.*	*e.*
6.	2.78	50.4	5.675	25	5275
7.	10.07	2.9	27.80	275	470.5
8.	253	275	27.15	126	126.5
9.	25.3	27.5	271.5	22.6	12.65
10.	2.53	2.75	2.715	1.26	1.265
11.	4.56	27.56	198.7	4.625	.4625

To divide a decimal by a decimal.

STUDY RECITATION

In the multiplication of decimals, every product has as many decimal places as the number of decimal places in both its factors.

If the number of decimal places in a product or a dividend and in one factor or a divisor are known, how can you find the number of decimal places in the other factor?

In division of decimals the number of places in the quotient is the difference between the number in the dividend and the number in the divisor.

1. Divide (*a*) $\frac{6}{100}$ by $\frac{3}{100}$; (*b*) $\frac{6}{10}$ by $\frac{3}{10}$; (*c*) 6 by 3.

a. $\frac{6}{100}$ divided by $\frac{3}{100} = \frac{\overset{2}{\cancel{6}}}{\cancel{100}} \times \frac{\cancel{100}}{\cancel{3}} = 2.$

b. $\frac{6}{10}$ divided by $\frac{3}{10} = \frac{\overset{2}{\cancel{6}}}{\cancel{10}} \times \frac{\cancel{10}}{\cancel{3}} = 2.$

c. 6 divided by $3 = 2$.

In (*b*) the dividend and the divisor are ten times those in (*a*); in (*c*) they are ten times those in (*b*) and 100 times those in (*a*). The quotients are all the same.

Multiplying both dividend and divisor by the same number does not alter the quotient.

When the divisor is a whole number, the quotient has the same number of decimal places as the dividend. The divisor may always be made a whole number by moving the decimal point to the right of its lowest order. When this is done, the decimal point in the dividend must be moved the *same number of places to the right.* Place the decimal point in the quotient below the decimal point in the dividend in short

division, and above it in long division, before the division is begun.

If this plan is followed, there is little likelihood of error in pointing off the decimal in the quotient.

2. Change .4).28 to 4)2.8; .04).28 to 4)28. Then divide.

3. Change .006)1.8 to 6)1800; .06).018 to 6)1.8. Then divide.

Explain what you have done in each case.

First change the divisors to whole numbers and make corresponding changes in the dividend. Then divide:

	a.	*b.*	*c.*	*d.*	*e.*
4.	$4.8 \div .8$	$1.44 \div .12$	$.056 \div .07$	$.0081 \div .09$	$.0096 \div .12$
5.	$6.3 \div .9$	$12.2 \div .11$	$.0063 \div .9$	$.0072 \div .08$	$.048 \div .06$
6.	$3.6 \div 1.2$	$13.2 \div .12$	$.0072 \div .009$	$.0096 \div .8$	$4.8 \div .08$
7.	$6.3 \div .7$	$.084 \div 1.2$	$.072 \div 1.2$	$.064 \div .8$	$5.4 \div .09$

ILLUSTRATION:

8. Divide .01536 by .012.

$$
\begin{array}{r}
\cancel{.}012.\overline{)\cancel{.}015.36} \\
1.28
\end{array}
$$

First Step. Make the divisor a whole number by moving the decimal point to the right of the 2. Move the decimal point in the dividend also 3 places to the right, placing it to the right of the 5. In order to avoid confusion as to the place of the points, cancel the original decimal points, as shown in the illustration. The divisor is now 12 (the zero at its left not affecting its value), and the dividend is 15.36. Place the decimal point for the quotient under the one in the dividend, before beginning the division, as shown.

Second Step. The first partial dividend is 15; 12 in 15 gives 1 for the first quotient figure, which is placed under 5, the last figure in the first partial dividend. The remaining figures in the quotient are placed under the successive figures in the dividend.

The first step is the moving of the decimal points in divisor and dividend, canceling the original points, and placing the decimal point in the quotient.

The second step is the division, each quotient figure being written under the last figure of the partial dividend used to produce it.

In the actual work, the numbers need not be written more than once.

9. Divide 33.07 by 2.12.

$$\begin{array}{r} 15.6 \\ 2_{\wedge}12.\overline{)33_{\wedge}07.2} \\ \underline{212} \\ 1187 \\ \underline{1060} \\ 1272 \\ \underline{1272} \end{array}$$

Without rewriting the example, the decimal points are moved two places to the right in divisor and dividend, the first points being canceled. The decimal point for the quotient is placed above the new point in the dividend, and the division is performed. Each figure in the quotient is placed above the last figure of the partial dividend used to produce it.

10. Divide .00136 by .4.

$$\begin{array}{r} {}_{\wedge}4.\underline{)_{\wedge}0.0136} \\ .0034 \end{array}$$

After moving the decimal points, it is required to find $\frac{1}{4}$ of .0136. $\frac{1}{4}$ of 0 tenths is 0, $\frac{1}{4}$ of 1 hundredth is 0, with 1 hundredth remaining. 1 hundredth = 10 thousandths; 10 thousandths and 3 thousandths = 13 thousandths; $\frac{1}{4}$ of 13 thousandths is 3 thousandths, with 1 thousandth remaining. 1 thousandth = 10 ten-thousandths. 10 ten-thousandths + 6 ten-thousandths = 16 ten-thousandths; $\frac{1}{4}$ of 16 ten-thousandths is 4 ten-thousandths.

11. Divide 25 by .008.

$$\begin{array}{r} {}_{\wedge}008.\underline{)25_{\wedge}000.} \\ 3\,125 \end{array}$$

It is required to divide the whole number, 25, by .008. A decimal point is not usually written at the right of a whole number, but it may be regarded as having its place at the right of units. When there are not as many decimal places in the dividend as the number of decimal places in the divisor, the decimal point must be moved to the right, the vacant places being filled with zeros.

Written

Find the quotients:

	a.	b.	c.
12.	2.36 ÷ 2	.0025 ÷ 50	$196.02 ÷ 2.42
13.	.678 ÷ 6	.07828 ÷ .04	$.306 ÷ .006
14.	.66 ÷ .015	5.2 ÷ .005	12.75 ÷ 2.5
15.	1.69 ÷ .0013	.84 ÷ 1.2	3.91 ÷ 1.7
16.	17.5)40.25	.0115)80.5	.125)8.75
17.	4.2)4704	.84)9.408	8.44).633
18.	4.425).2832	.032)25.6	8.75)3.85
19.	.07521 ÷ 3	.3276 ÷ 1.2	$.287 ÷ 4.1
20.	.00176 ÷ 4	.0814 ÷ .22	3 ÷ 8
21.	7.62 ÷ .6	321.3 ÷ .21	4.004 ÷ 7
22.	.0728 ÷ .8	3 ÷ .125	7.02 ÷ 9
23.	.0236 ÷ .4	7 ÷ 8	5 ÷ 8
24.	7.788 ÷ .12	23.256 ÷ 290.7	1.008 ÷ 12
25.	27.071 ÷ .11	3.6 ÷ 90	2.04 ÷ 17
26.	.045 ÷ 900	.77794 ÷ 8.02	4.76 ÷ 28
27.	$8 ÷ $.0625	$9.24 ÷ .08	.1 ÷ 16

28–75. Interchange dividends and divisors in examples 12 to 27, and find the quotients to four decimal places.

To the Teacher. Require pupils to write the numbers for division from dictation.

PROBLEMS

Written

1. A boat goes at the rate of 17 (7.25) miles per hour. In how many hours will it go 85 (65.25) miles?

2. At $.37 per bushel, how many bushels of potatoes can be bought for $4.81?

3. Into how many building lots, each containing .125 of an acre, can 9 acres be divided?

4. If the average rainfall is 2.72 inches per month, in how many months does the rainfall amount to 32.64 inches?

5. If a boy earns $12.25 a week, in how many weeks will he earn $122.50?

6. How many yards of carpeting at $1.75 a yard can be bought for $87.50?

7. If a train travels .6 of a mile a minute, how many minutes will it take to travel 20.4 miles?

8. A grade Holstein produced during a year 13097.2 pounds of milk containing 449.66 pounds of butter fat. What was the average test of the milk?

9. The Minnesota Experiment Station purchased two cows at the same price and kept them for a period of six years, giving them the same care and feed. The following data show their records during this period:

Year	Roxy		Sella	
	Milk	Butter	Milk	Butter
1904	3939.5	202.9	4488.9	226.4
1905	4772.1	232.8	3490.6	178.0
1907	7831.6	392.7	4553.3	232.6
1908	4784.6	216.5	4459.6	197.0
1909	7539.9	366.5	3602.3	153.3
1910	6104.6	297.4	3634.4	157.0

Find the average number of pounds of milk and butter produced by each cow during the period. If butter was worth 25 cents per pound, what was the difference in value of the product of the two cows during the period?

REDUCTION OF COMMON FRACTIONS TO DECIMALS

Study Recitation

This is a special case of the reduction of fractions to higher denominations. The denominator of a decimal is always 10 or some power of 10.

To reduce a common fraction to a decimal is to reduce the fraction to a fraction having 10 or some power of 10 for a denominator, writing the result as a decimal.

1. Reduce $\frac{3}{4}$ to hundredths.

1st Solution. $1 = \frac{100}{100}$. $\frac{1}{4} = \frac{25}{100}$. $\frac{3}{4} = \frac{75}{100} = .75$.

2d Solution. $3 = \frac{300}{100}$. $\frac{1}{4}$ of $\frac{300}{100} = \frac{75}{100} = .75$.

3d Solution. Since $\frac{3}{4}$ indicates the division of 3 by 4, example 1 may be stated: Divide 3 by 4, expressing the quotient as a decimal.

$$\begin{array}{r} 4\overline{)3.00} \\ \hline .75 \end{array}$$

3 is not divisible by 4.

$3 = 30$ tenths. $\frac{1}{4}$ of 30 tenths $= 7$ tenths with 2 tenths remaining.

2 tenths $= 20$ hundredths. $\frac{1}{4}$ of 20 hundredths $= 5$ hundredths. $\frac{3}{4} = .75$.

As it is not always easy to tell to what decimal denomination a fraction is to be reduced before performing the operation, the process of division given last is the one ordinarily used.

To reduce a common fraction to a decimal, place a decimal point at the right of the numerator, annex zeros, and divide by the denominator, pointing off as in division of decimals.

Usually not more than three places are required in the decimal. If the division is not then exact, the remainder may be written as in division of whole numbers, or the + sign may be written at the right of the third figure in the quotient, to denote that the decimal is not exact, but somewhat larger. Sometimes a decimal figure equal to, or greater than, 5 is considered as a whole unit of the next order. Thus, .5366 may be considered as .537, while .5364 is considered as .536.

2. Reduce $\frac{3}{7}$ to a decimal.

$$7\overline{)3.000} \atop .428\tfrac{4}{7} \qquad \textit{Or,} \quad 7\overline{)3.000} \atop .428^{+} \text{ or } .43^{-}$$

In these examples do not carry the decimal beyond the second place. Give exact results by writing the remainders, if any, with the decimal. Thus, $\frac{1}{3} = 1 \div 3 = 3\overline{)1.00}$
$.33\frac{1}{3}$

WRITTEN

Reduce to decimals of not more than three places:

	a.	*b.*	*c.*
3.	$\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$	$\frac{5}{6}$; $\frac{1}{7}$; $\frac{2}{7}$	$\frac{1}{9}$; $\frac{2}{9}$; $\frac{4}{9}$
4.	$\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$	$\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$	$\frac{5}{9}$; $\frac{7}{9}$; $\frac{1}{12}$
5.	$\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{6}$	$\frac{7}{8}$; $\frac{3}{10}$; $\frac{5}{10}$	$\frac{1}{20}$; $\frac{1}{40}$; $\frac{4}{25}$
6.	$\frac{4}{7}$; $\frac{4}{11}$; $\frac{4}{15}$	$\frac{7}{9}$; $\frac{5}{12}$; $\frac{6}{7}$	$\frac{27}{35}$; $\frac{19}{101}$; $\frac{77}{81}$
7.	$\frac{3}{16}$; $\frac{3}{17}$; $\frac{3}{22}$	$\frac{9}{17}$; $\frac{25}{45}$; $\frac{11}{103}$	$\frac{9}{5}$; $\frac{26}{14}$; $\frac{93}{35}$

ALIQUOT PARTS OF $1, 1, AND 100

$\$.50 = \$\frac{1}{2}$; $\$.25 = \$\frac{1}{4}$; $\$.33\frac{1}{3} = \$\frac{1}{3}$; $\$.20 = \$\frac{1}{5}$.

These are called aliquot parts of $1.

An **aliquot part** of a number is a part that divides it exactly.

$.50 = \frac{1}{2}$; $.25 = \frac{1}{4}$; $.33\frac{1}{3} = \frac{1}{3}$; $.20 = \frac{1}{5}$.

These are aliquot parts of 1.

$50 = \frac{1}{2}$ of 100; $25 = \frac{1}{4}$ of 100; $33\frac{1}{3} = \frac{1}{3}$ of 100; $20 = \frac{1}{5}$ of 100.

These are aliquot parts of 100.

NOTE. Multiples of these parts, as, $\$.75 = \$\frac{3}{4}$, $\$.60 = \$\frac{3}{5}$, or $.75 = \frac{3}{4}$, $.60 = \frac{3}{5}$, are also usually called aliquot parts.

1. How many times is $.05 contained in $1?

2. What part of a dollar is $\$.12\frac{1}{2}$? $\$.16\frac{2}{3}$? $\$.33\frac{1}{3}$? $\$.08\frac{1}{3}$? $\$.06\frac{1}{4}$? $\$.62\frac{1}{2}$? $\$.37\frac{1}{2}$? $\$.83\frac{1}{3}$? $\$.87\frac{1}{2}$? $\$.18\frac{3}{4}$?

Learn the following table:

$\$.05 = \frac{1}{20}$ of a dollar	$.05 = \frac{1}{20}$
$\$.06\frac{1}{4} = \frac{1}{16}$ of a dollar	$.06\frac{1}{4} = \frac{1}{16}$
$\$.08\frac{1}{3} = \frac{1}{12}$ of a dollar	$.08\frac{1}{3} = \frac{1}{12}$
$\$.10 = \frac{1}{10}$ of a dollar	$.10 = \frac{1}{10}$
$\$.12\frac{1}{2} = \frac{1}{8}$ of a dollar	$.12\frac{1}{2} = \frac{1}{8}$
$\$.16\frac{2}{3} = \frac{1}{6}$ of a dollar	$.16\frac{2}{3} = \frac{1}{6}$
$\$.20 = \frac{1}{5}$ of a dollar	$.20 = \frac{1}{5}$
$\$.25 = \frac{1}{4}$ of a dollar	$.25 = \frac{1}{4}$
$\$.30 = \frac{3}{10}$ of a dollar	$.30 = \frac{3}{10}$
$\$.33\frac{1}{3} = \frac{1}{3}$ of a dollar	$.33\frac{1}{3} = \frac{1}{3}$
$\$.37\frac{1}{2} = \frac{3}{8}$ of a dollar	$.37\frac{1}{2} = \frac{3}{8}$
$\$.40 = \frac{2}{5}$ of a dollar	$.40 = \frac{2}{5}$
$\$.50 = \frac{1}{2}$ of a dollar	$.50 = \frac{1}{2}$
$\$.60 = \frac{3}{5}$ of a dollar	$.60 = \frac{3}{5}$
$\$.62\frac{1}{2} = \frac{5}{8}$ of a dollar	$.62\frac{1}{2} = \frac{5}{8}$
$\$.75 = \frac{3}{4}$ of a dollar	$.75 = \frac{3}{4}$
$\$.80 = \frac{4}{5}$ of a dollar	$.80 = \frac{4}{5}$
$\$.87\frac{1}{2} = \frac{7}{8}$ of a dollar	$.87\frac{1}{2} = \frac{7}{8}$

3. Make and learn a similar table of the aliquot parts of 100, of 10; of 1000. Thus, $25 = \frac{1}{4}$ of 100; $12\frac{1}{2} = \frac{1}{8}$ of 100; $2.5 = \frac{1}{4}$ of 10; $250 = \frac{1}{4}$ of 1000, etc.

4. Find the cost of 8 yd. of cloth at $\$.37\frac{1}{2}$ a yard.

$$\cancel{8} \times \$\frac{3}{\cancel{8}} = \$3$$

Express $\$.37\frac{1}{2}$ as $\$\frac{3}{8}$; $8 \times \$\frac{3}{8} = \$3$.

5. At $\$.37\frac{1}{2}$ a yard, how many yards of cloth can be bought for \$3?

$$\$3 \div \$\frac{3}{8} = \cancel{3} \times \frac{8}{\cancel{3}} = 8$$

Express $\$.37\frac{1}{2}$ as $\$\frac{3}{8}$. $\$\frac{3}{8}$ is contained in \$3, 8 times. Therefore, 8 yards can be bought.

PROBLEMS

Oral

1. At 25¢ a pound, how much will 16 pounds of butter cost?

2. At $87\frac{1}{2}$¢ a bushel, how many bushels of wheat can be bought for \$7?

3. How much will 240 pounds of raisins cost at $12\frac{1}{2}$¢ a pound? at $16\frac{2}{3}$¢ a pound? at 25¢ a pound?

4. What is the cost of 4 dozen eggs at $16\frac{2}{3}$¢ a dozen? at $33\frac{1}{3}$¢ a dozen?

5. A dealer buys 120 readers at $37\frac{1}{2}$¢ each, and sells them at 50¢ each. How much does he gain?

6. At $37\frac{1}{2}$¢ an hour, how much will a man earn in 2 days of 8 hours each?

7. A laborer receives \$2.25 a day. What are his wages for 24 days?

SUGGESTION. \$2.25 = \$2.00 + \$.25   24 × \$2.00 + 24 × \$$\frac{1}{4}$ = ?

8. What part of \$1 added to \$1 equals \$$1.12\frac{1}{2}$? \$$1.37\frac{1}{2}$? \$1.75? \$$1.87\frac{1}{2}$?

WRITTEN

Find the cost of 120 articles at the following prices each:

	a.	*b.*	*c.*	*d.*	*e.*
9.	\$$.33\frac{1}{3}$	\$$.37\frac{1}{2}$	\$.20	\$.40	\$$.66\frac{2}{3}$
10.	\$.25	\$.30	\$.80	\$$.62\frac{1}{2}$	\$$.12\frac{1}{2}$
11.	\$.50	\$.75	\$.60	\$$.16\frac{2}{3}$	\$$.83\frac{1}{3}$

12. How many articles can be bought for \$120 at each of the prices mentioned in examples 9 to 11?

Find the total costs:

	NUMBER OF ARTICLES	PRICE OF EACH		NUMBER OF ARTICLES	PRICE OF EACH
13.	50	\$.40	**20.**	200	\$$.87\frac{1}{2}$
14.	72	\$$.37\frac{1}{2}$	**21.**	144	\$$.87\frac{1}{2}$
15.	42	\$$.16\frac{2}{3}$	**22.**	240	\$$.12\frac{1}{2}$
16.	75	\$.20	**23.**	300	\$$.16\frac{2}{3}$
17.	18	\$$.33\frac{1}{3}$	**24.**	600	\$.60
18.	80	\$$.12\frac{1}{2}$	**25.**	660	\$$.83\frac{1}{3}$
19.	180	\$$.83\frac{1}{3}$	**26.**	800	\$$.62\frac{1}{2}$

Find how many articles can be purchased for the given amounts at the given prices:

	Price	Amount		Price	Amount
27.	$\$.37\frac{1}{2}$	$21.00	**34.**	$\$.06\frac{1}{4}$	$32
28.	$\$.87\frac{1}{2}$	$49.00	**35.**	$\$.12\frac{1}{2}$	$40
29.	$\$.62\frac{1}{2}$	$15.00	**36.**	$\$.08\frac{1}{3}$	$36
30.	$\$.33\frac{1}{3}$	$3.00	**37.**	$.75	$15
31.	$\$.83\frac{1}{3}$	$60.00	**38.**	$\$.62\frac{1}{2}$	$50
32.	$.40	$8.00	**39.**	$\$.37\frac{1}{2}$	$75
33.	$\$.66\frac{2}{3}$	$12.00	**40.**	$\$.66\frac{2}{3}$	$20

COST OF ARTICLES BY THE C, M, OR TON

To find the cost of articles when the number of units and price per hundred are given.

1. What is the cost of 7256 pounds of pork at $15 per hundred pounds?

Suggestion. What is the unit? What is the given number of units? *Ans.* 7256.

What is the quotient of $7256 \div 100$? *Ans.* 72.56.

To divide by 100, *move the decimal point two places to the left.*

What is the product of the cost per hundred by the number of hundreds? $72.56 \times \$15 = \$1088.40$.

2. What is the cost of 1200 pounds of flour at $2.25 per hundred pounds?

What is the first step required? What is the result?

What is the second step required? What is the result?

Give the rule. Make and solve three examples to which the rule applies.

To find the cost of articles when the number of units and price per thousand are given.

To divide by 1000, *move the decimal point three places to the left.*

1. Find the cost of 25,575 bricks at $8.75 per M.

What is the first step required by the rule? What is the result? What is the second step required by the rule? What is the result? Give the rule. Make and solve three examples to which the rule applies.

To find the cost of articles sold by the ton or 2000 pounds.

Since there are 2 times 1000 pounds in a ton, the cost of 1000 pounds is $\frac{1}{2}$ the cost of a ton.

I. *Divide the cost of* 1 *ton by* 2 *to find the cost of* 1000 *pounds, and apply the rule for finding cost of articles sold by the thousand.*

In a given number of pounds there are $\frac{1}{2}$ as many tons as there are times 1000 pounds, therefore

II. *Divide the given number of pounds by* 1000, *and this result by* 2, *to find the number of tons. Find the product of the number of tons by the price per ton.*

1. What is the cost of 4425 pounds of coal at $8 per ton?

SOLUTION BY THE FIRST RULE. 2)$8 = $4, cost of 1000 pounds. 1000)4425 = 4.425, number of thousand. 4.425 × $4 = $17.70.

SOLUTION BY THE SECOND RULE. 1000)4425 = 4.425. 2)4.425 = 2.2125, number of tons. 2.2125 × $8 = $17.70.

2. What is the cost of 850 fence posts at $7.50 per C?

SOLUTION. 100)850 × $7.50 = 8.5 × $7.50 = $63.75. *Ans.*
850 posts = 8.5 C; 8.5 times $7.50, the cost per C, gives $63.75.

3. Find the cost of 16375 bricks at $6.50 per M.

SOLUTION. 1000)16375 bricks = 16.375 M bricks.
16.375 × $6.50 = $106.44. *Ans.*

4. Find the cost of 5780 fence posts at $27 per C.

5. What is the cost of 3500 feet of lumber at $16.50 per M?

6. At $7.50 a ton, what is the cost of 4530 pounds of hard coal?

7. At $4.25 per ton, what is the cost of 6400 pounds of soft coal?

8. Bought of R. J. Green the following:

6000 lb. nut coal at $7.25 per ton.
4600 lb. soft coal at $4.50 per ton.
8750 lb. chestnut at $6.50 per ton.
4500 lb. screenings at $2.50 per ton.

Write out a receipted bill.

9. A contractor furnishes the following materials for the construction of a house:

32,600 bricks at $7.75 per M.
9800 feet flooring at $18.50 per M.
12,500 feet lumber at $16 per M.
7900 shingles at $9.80 per M.
5400 laths at $.50 per C.

Find the cost of the materials and make out bill.

10. If the cost of any number of cords of wood and the number of cords are known, how can you find the cost of 1 cord? Illustrate.

The number of cords may be expressed as a whole number, 4 cords; a mixed number, $3\frac{2}{3}$ cords; a fractional number, $\frac{3}{4}$ cord; a decimal, .8 cord; or a mixed decimal, 2.2 cords.

11. If 4 cords of wood cost $11, what is the cost of 1 cord?

12. Make four examples by substituting for 4 in the last example each of the following numbers: $3\frac{2}{3}$, $\frac{3}{4}$, .8, 2.2. Solve them, showing that they are all solved like the 11th.

13. Make and solve similar examples in the same way, using other numbers and other expressions than "cords of wood."

Of the five different forms of numbers expressed in the statement preceding example 11, the first is a whole number; the other four may be expressed as common fractions. Thus, $3\frac{2}{3} = \frac{11}{3}$, $\frac{3}{4}$, $.8 = \frac{4}{5}$, $2.2 = \frac{11}{5}$.

Analyze each of the examples called for under example 12, treating each number as a common fraction, and using the following form of analysis:

If $3\frac{2}{3}$ cords of wood cost \$11, how much will 1 cord cost?

SOLUTION. $3\frac{2}{3} = \frac{11}{3}$. If $\frac{11}{3}$ cords of wood cost \$11, $\frac{1}{3}$ of a cord costs $\frac{1}{11}$ of \$11, or \$1. If $\frac{1}{3}$ of a cord of wood costs \$1, $\frac{3}{3}$ of a cord costs 3 times \$1, or \$3.

14. 6.5 yards of cloth cost \$3.25. What is the cost per yard?

15. How can you find the cost of 3.5 cords of wood from the cost of .4 of a cord? Suppose the cost of .4 of a cord to be \$3.20, find the cost of 3.5 cords.

16. A coal dealer sold to a lumberman 3 loads of coal weighing 4400 pounds, 5775 pounds, and 3800 pounds, respectively. He took in exchange 3875 feet of lumber at \$16 per M, and 350 fence posts at \$7.50 per C. If coal is \$6.50 a ton, which is in debt to the other, and how much?

17. A sold B, in exchange for potatoes at \$.45 a bushel, 80 fence posts at \$6 per C, and 450 feet of lumber at \$15 per M. How many bushels did B receive?

18. The product of four factors is 16.95. Three of the factors are 1.2, .25, and .113. What is the fourth factor?

19. Frame and solve two practical problems, each involving at least two operations with decimals.

REVIEW

1. Define decimal; mixed decimal.

2. What is the use of the decimal point in writing decimals?

3. The value of a unit in any order is —— the value of the same unit in the next lower order. The value of a unit in any order is —— the value of the same unit in the next higher order. Fill the blanks and illustrate.

4. Show why the first order at the right of the decimal point is tenths; why the second order is hundredths.

5. Write a decimal of three places; write it as a decimal fraction with the denominator expressed. What is the relation of the number of places in a decimal to the number of zeros in its denominator?

6. How may any decimal be changed to a common fraction? Illustrate.

7. Annexing a zero to a decimal has what effect upon its denominator? upon its denomination? upon its value? Illustrate.

8. Cutting off a zero from the right of a decimal has what effect upon its denominator? upon its denomination? upon its value? Illustrate.

9. Moving the decimal point to the right has what effect upon the decimal? Illustrate.

10. Moving the decimal point to the left has what effect upon the decimal? Illustrate.

11. Moving the decimal point the same number of places to the right in both divisor and dividend has what effect upon each? upon the quotient? Illustrate.

12. Write five decimals of different denominations. Reduce them to the same denomination.

13. Why is $\frac{7}{10}$ a decimal fraction? Why is $\frac{5}{8}$ not a decimal fraction?

14. How may $\frac{5}{8}$ be changed to a decimal? What is done in making the change?

15. In 31.23, what is the value of the 3 at the right of the decimal point as compared with the 3 at the left of the decimal point?

Problems without Numbers

16. How can you change a decimal to a common fraction?

17. How can you change a common fraction to a decimal?

18. Give the rule for writing decimals; for reading decimals.

19. Give the rule for adding decimals; for subtracting decimals.

20. Give the rule for multiplying decimals; for dividing decimals.

21. How can you find the cost of articles sold by the hundred? by the thousand?

22. How can you find the cost of articles sold by the ton?

23. How can you find the cost of a number of yards of lace when the cost of one yard is an aliquot part of a dollar?

24. How can you find the number of yards of lace that can be purchased for a given amount when the cost of 1 yard is an aliquot part of a dollar?

GENERAL REVIEW

Time yourself with these exercises. Then work them again and see whether you can beat your record.

Write from dictation and add. Test by adding in reverse order:

	a.	*b.*	*c.*	*d.*	*e.*
1.	15698	86754	97685	\$230.52	\$143.28
	54272	70921	74654	\$691.04	\$876.52
	38314	61832	83732	\$570.00	\$500.09
	57629	52743	62810	\$430.95	\$605.98
	14244	43654	51902	\$823.74	\$721.34

Subtract and test:

2.	43215	29164	48000	40050	60309
	28679	16478	34707	21236	59345
3.	\$871.11	\$700.00	\$500.00	\$1596.50	\$1428.32
	\$654.32	\$354.89	\$498.95	\$ 759.28	\$ 649.25

Multiply and test:

4.	5437	8025	6039	7542	8138
	1297	1602	2008	3050	7095
5.	\$162.55	\$137.89	\$200.50	\$305.75	\$890.60
	1209	1056	3057	2187	6134

Divide and test:

6.	89667 by 123.	124609 ÷ 353.	\$1578.28 by 187.
7.	260967 by 301.	163620 ÷ 405.	\$7238.40 by 104.
8.	143812 by 229.	642720 ÷ 309.	\$1971.84 by 948.
9.	449460 by 198.	104624 ÷ 503.	\$8989.20 by 227.
10.	988036 by 994.	152934 ÷ 718.	\$3544.92 by 687.

Change to lowest terms:

11.	$\frac{18}{27}$	$\frac{30}{45}$	$\frac{25}{100}$	$\frac{48}{90}$	$\frac{18}{48}$	$\frac{45}{72}$	$\frac{27}{36}$	$\frac{16}{24}$
12.	$\frac{25}{60}$	$\frac{24}{32}$	$\frac{12}{64}$	$\frac{20}{25}$	$\frac{56}{64}$	$\frac{49}{56}$	$\frac{32}{48}$	$\frac{26}{28}$

Change to improper fractions:

13. $5\frac{3}{4}$ $16\frac{2}{3}$ $37\frac{1}{2}$ $62\frac{1}{2}$ $10\frac{7}{8}$ $15\frac{2}{3}$ $12\frac{3}{8}$ $15\frac{3}{5}$

Change to similar fractions, having the l. c. d. and add or subtract as indicated:

	a.	*b.*	*c.*	*d.*
14.	$\frac{2}{3}+\frac{1}{2}$	$\frac{1}{3}+\frac{3}{4}+\frac{3}{8}$	$\frac{7}{8}-\frac{3}{16}$	$\frac{9}{16}-\frac{3}{8}$
	$\frac{3}{4}+\frac{7}{8}$	$\frac{2}{5}+\frac{3}{4}+\frac{3}{16}$	$\frac{3}{4}-\frac{5}{12}$	$\frac{7}{15}-\frac{2}{5}$
	$\frac{3}{5}+\frac{2}{15}$	$\frac{3}{8}+\frac{3}{8}+\frac{4}{5}$	$\frac{5}{6}-\frac{3}{8}$	$\frac{4}{5}-\frac{3}{4}$

First add; then subtract:

15.	$35\frac{1}{8}$	$74\frac{2}{3}$	$83\frac{1}{4}$	$42\frac{2}{3}$	$79\frac{7}{10}$
	$16\frac{3}{4}$	$35\frac{5}{6}$	$56\frac{5}{8}$	$16\frac{5}{6}$	$35\frac{4}{5}$
16.	$98\frac{5}{6}$	$56\frac{1}{2}$	$72\frac{3}{4}$	$60\frac{1}{2}$	$50\frac{1}{2}$
	$34\frac{2}{3}$	$42\frac{1}{3}$	$67\frac{7}{8}$	$58\frac{4}{5}$	$39\frac{3}{10}$

Multiply, canceling when possible: Divide:

17.	$\frac{3}{4}\times\frac{5}{6}$.	$4\frac{3}{5}\times 1\frac{2}{3}$.	$\frac{7}{8}$ by 5.	$3\frac{7}{8}$ by $2\frac{1}{2}$.
18.	$16\times\frac{5}{8}$.	$6\frac{3}{4}\times 12\frac{3}{8}$.	5 by $\frac{7}{8}$.	$19\frac{3}{4}$ by $3\frac{7}{8}$.
19.	$\frac{2}{3}$ of $\frac{7}{10}$.	$7\frac{1}{2}\times 15\frac{3}{8}$.	$\frac{2}{3}$ by $\frac{4}{5}$.	$25\frac{7}{8}$ by $4\frac{1}{4}$.

Write from dictation and add; then subtract:

20.	356.005	421.	87.057	521.4
	195.3674	16.574	1.3462	413.205
21.	524.0082	209.04	957.	875.05
	315.4367	93.675	642.035	276.945

22. Add the four lines in examples 20 and 21, as though they were one example.

Find products or quotients, as indicated:

	a.	*b.*	*c.*
23.	$750 \times .037$	$.0572 \times 31.257$	$7.182 \div .057$
24.	$.456 \times .789$	1.256×4.009	$10.3707 \div 1.503$
25.	$.0234 \times 1.235$	$7.14 \times .0562$	$38.4804 \div 3.054$
26.	$1.054 \times .0274$	$154 \div .25$	$8113.03 \div 12.109$
27.	25.125×5.08	$26.18 \div 1.54$	$9072.144 \div 3.012$

PROBLEMS

Written

Farm Problems

1. The annual wheat crop of the United States is about 500,000,000 bushels. If this could be increased .02 by seed selection, how much would the addition be worth at 80¢ per bushel?

2. How many agricultural experiment stations would this increase support at $500,000 a year for each station?

3. If grass seed costs 10¢ a pound and is .20 weed seed, what is the real cost of clean grass seed? How much cheaper per pound would it be to buy a clean seed at 12¢ a pound?

4. If 4 days extra at $2 a day were spent each year in selecting, caring for, and testing seed corn, and the yield of a 40-acre field were thereby increased 10 bushels per acre, how much would be gained, when corn sells at $.71 a bushel.

5. If there are 40 birds per square mile, how many birds are there in the state of Indiana, which has an area of 36,350 square miles?

6. Allowing each bird 50 insects a day for the 120 days of summer, how many insects would be destroyed during the year by 1,454,000 birds?

7. Three golden-winged woodpeckers were found by

King to have in their stomachs, respectively, 255 ants, 220 ants, and 200 ants. Suppose this to represent 1 day's ration, how many ants would these 3 birds eat in 120 days?

8. During an outbreak of Rocky Mountain locusts, a marsh wren was seen to carry 30 locusts to her young in 1 hour. At this rate, how many locusts would she carry in 7 hours?

9. If there are 20 broods of marsh wrens to the square mile, how many locusts would be destroyed in 7 hours in Wisconsin, which has an area of 56,040 square miles, at the rate of 30 locusts per hour for each brood?

10. If each of the 235,368,000 locusts were to consume its own weight of standing crops each day, how much would be saved by the birds, allowing $10 per ton for the crops? Each locust weighs 15 grains, and there are 7000 grains in a pound.

11. If by extra cultivation a man can raise 100 bushels of potatoes more an acre, how many days' work would thus be paid for, allowing $3 per day for man and team, if the potatoes bring $.25 per bushel?

12. The Kansas Experiment Station fed 10 calves on skim milk for 126 days, and they gained an average of 2.02 pounds per day. Another lot of 10 calves, fed for an equal length of time on buttermilk, gained 1.79 pounds per day. What was the difference in total gain between the two lots?

Cooking Problems

Written

13. If the cost of milk is 3.9 ¢ per pound, and the cost of eggs is 13.3 ¢ per pound, and the fuel value is 290 and 590 heat units per pound, respectively, what would be the comparative fuel value in 10 ¢ worth of each?

Solution

$10¢ \div 3.9 = 2.6$, or number of pounds of milk to be obtained for $10¢$.
$10¢ \div 13.3 = .75$, or number of pounds of eggs to be obtained for $10¢$.
$290 \times 2.6 = 754$, fuel value of $10¢$ worth of milk.
$590 \times .75 = 443$, fuel value of $10¢$ worth of eggs.
$754 \div 443 = 1.7$ times as much fuel value in $10¢$ worth of milk as in $10¢$ worth of eggs.

14. Mary wishes to make gingerbread. How much of each ingredient should she use for $\frac{1}{2}$ the following recipe? for $\frac{1}{4}$ of it?

$\frac{1}{2}$ cup butter	1 level teaspoonful cinnamon
$\frac{1}{2}$ cup sugar	1 level teaspoonful ginger
$\frac{1}{2}$ cup New Orleans molasses	1 level teaspoonful cloves
$2\frac{1}{2}$ cups flour	1 cup boiling water
$2\frac{1}{2}$ level teaspoonfuls soda	2 eggs

$\frac{1}{2}$ teaspoonful salt

15. At current prices in your locality, how much would it cost to make the gingerbread from the above recipe?

16. A loaf of bread weighs 1 pound. If the sugar, salt, butter, etc., weigh 4 ounces, and the rest is the weight of the flour, how many cups of flour are used in the loaf?

16 ounces = 1 pound
4 cups of flour = 1 pound

17. If butter is \$.30 per pound; sugar, \$1 per 20 pounds; flour, \$1.50 per 50 pounds; eggs, \$.25 per dozen; 2 teaspoonfuls baking powder, \$.00$\frac{2}{3}$; and 1 teaspoonful spices, \$.00$\frac{1}{3}$, determine the cost of the cookies in the following recipe:

1 cup butter	4 eggs
2 cups sugar	2 teaspoonfuls baking powder
3 cups flour	1 teaspoonful spices

2 cups butter = 1 pound
2 cups sugar = 1 pound
4 cups flour = 1 pound

18. How much cream of tartar, soda, and cornstarch must be substituted for 5 teaspoonfuls of baking powder, if the composition of baking powder is $\frac{1}{4}$ soda, $\frac{1}{4}$ cornstarch, and $\frac{1}{2}$ cream of tartar?

19. Reduce the quarts, pints, and pounds in the following recipe for reception chocolate to cups. Compute one fourth of the recipe.

2 quarts milk	1 pint cream
1 pound cocoa	2 eggs
3 tablespoonfuls white sugar	3 teaspoonfuls vanilla extract

2 cups = 1 pint
2 pints = 1 quart
2 cups cocoa = 1 pound

20. Find the cost of making 6 times the quantity of apple jam called for in the following recipe, if apples are \$.12 a quart ($1\frac{1}{2}$ pounds to a quart), sugar is \$.05 a pound, lemons are 3 for \$.05, and cinnamon enough for the entire quantity is \$.01.

4 pounds sour cooking apples	Grated rind and juice of 2 lemons
3 pounds sugar	
$\frac{1}{4}$ pint cold water	$\frac{1}{4}$ teaspoonful ground cinnamon

21. If 10 eggs weigh 1 pound and .15 of the egg is protein, how many ounces of protein are there in 10 eggs? If a man eats 3 eggs daily, how many ounces of protein will he eat? (16 ounces = 1 pound.)

22. The total amount of protein that is needed every day by an adult is 3.6 ounces. What part of this would be furnished by 2 eggs, weighing together $\frac{1}{5}$ of a pound?

23. In 1 pound of oatmeal there is .156 of a pound of protein and in 1 pound of rice .078 of a pound of protein. How many more ounces of protein are there in 10 pounds of oatmeal than in 10 pounds of rice?

DENOMINATE NUMBERS

A **denominate number** is a concrete number in which the unit of measure is established by law or custom.

A **simple denominate number** is one that is composed of units of only one denomination; as, 4 ounces, 6 hours, 8 feet, 20 pounds.

A **compound denominate number** is one that is composed of units of two or more denominations, each of which may be changed to the other; as, 4 bushels 3 pecks; 2 feet 3 inches; 3 gallons 1 quart.

A **quantity** is anything that can be measured; as, money, grain, wood.

A **measure** is a standard or fixed unit of some quantity; as, $1; 1 lb.; 1 in.

Measuring a quantity is the process of finding how many times the fixed unit of that quantity is contained in it.

Dollars and cents are measures of value; inches and feet, of length; ounces and pounds, of weight.

Usually not more than two denominations are used in stating the measure of any quantity; as, 2 pounds 3 ounces; 6 feet 2 inches.

In business transactions, one unit only is used, and this is the one in which the price of the quantity is quoted. To illustrate: The butcher sells meat at a certain price per pound. He does not usually say, 3 pounds 12 ounces, but $3\frac{3}{4}$ pounds. The merchant says $8\frac{3}{4}$ yards of ribbon instead of 8 yards 27 inches.

DRY MEASURE

Dry measure is used in measuring different kinds of vegetables, fruits, and other dry commodities. Some of these commodities, like grain, potatoes, fruits, and coal, are sometimes sold by weight.

Table

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

Note. A bushel contains 2150.4 cubic inches (cu. in.), or $1\frac{1}{4}$ cubic feet (cu. ft.), approximately. One quart, or $\frac{1}{32}$ of a bushel = 67.2 cubic inches. (See p. 239.)

PROBLEMS

Oral

How many

1. Pecks equal 6 bu.? 24 qt.? 8 bu.?

2. Pints equal 2 bu.? 72 qt.? 4 pk.?

3. Pints equal 2 pk.? 7 qt.? 1 pt.?

4. Bushels equal 32 pk.? 37 pk.? 19 pk.?

5. When potatoes are selling at 20¢ a peck, what is the price per bushel?

6. When apples are selling at 15¢ a half peck, what is the price per bushel?

7. If chestnuts are worth 15¢ a quart, what is the price per peck?

8. How many quarts are there in a bushel? What part of a bushel are 7 qt.? 8 qt.? 9 qt.?

9. How many pints are there in a bushel? What part of a bushel are 3 pt.? 1 pt.?

10. What is the cost of 2 bu. 3 pk. of oats at 40¢ a bushel?

11. How many quarts are there in 32 pt.? 24 pt.? 16 pt.? 9 pt.?

12. How many quarts are there in $2\frac{1}{2}$ pk.? $3\frac{1}{4}$ pk.? $1\frac{3}{4}$ pk.?

13. How many pecks are there in 64 qt.? 56 qt.? 24 qt.?

14. How many bushels are there in 16 pk.? 28 pk.?

15. A crate holds 36 qt. of berries. It holds 1 bu. and what part of a bushel?

Written

16. If apples cost $1.20 per bushel and are sold at 40¢ per peck, what is the gain on one bushel?

17. If peanuts are bought at $1.20 a bushel and are sold at 5¢ a pint, what is the gain on one bushel?

18. James planted 2 bu. 1 pk. of early potatoes which cost him 75¢ a bushel. He harvested and sold $15\frac{1}{2}$ bu. at 20¢ a half peck. How much did he realize for his labor?

19. If a grocer buys blueberries at $3 per bushel and sells them at 15¢ a quart, how much does he gain on each bushel?

20. A farmer had $15\frac{1}{2}$ bu. of seed corn. He sold to one man $2\frac{1}{4}$ bu., to another $1\frac{3}{4}$ bu. How much had he left?

21. A bushel contains 2150.4 cu. in. How many cubic inches are there in 1 pk.? in 1 qt.?

22. A box contains $2\frac{1}{4}$ cu. ft. It contains how many cubic feet more than 1 bu.?

23. Counting $1\frac{1}{4}$ cu. ft. as approximately 1 bushel, how many cubic feet must a box contain to hold 10 bu.?

24. How many cubic feet must a box contain to hold 25 bu.?

25. What is the difference in cubic inches in the capacity of the two boxes mentioned in problems 23 and 24?

LIQUID MEASURE

Liquid measure is used in measuring liquids.

TABLE

4 gills (gi.)	=	1 pint (pt.)
2 pints	=	1 quart (qt.)
4 quarts, 8 pints, 32 gills	=	1 gallon (gal.)

NOTES. 1. 1 gallon contains 231 cubic inches.

1 quart, or $\frac{1}{4}$ of a gallon, contains 57.75 cubic inches. As a quart, dry measure, contains 67.2 cu. in. it will be seen that it is larger by 9.45 cu. in. than a quart, liquid measure. A dry quart contains about 1.16 liquid qt.

2. In measuring the capacities of cisterns, reservoirs, etc., $31\frac{1}{2}$ gal. equal one barrel (bbl.) and 63 gal., one hogshead (hhd.); but in commerce the barrel and the hogshead vary in capacity.

PROBLEMS

ORAL

1. How many gallons equal 2 bbl.? 2 bbl. 10 gal.? 2 hhd.?

2. If milk is sold at 6¢ a quart, what is the value of 2 gal.? 2 gal. 3 qt.?

3. If cream is sold at $1.20 per gallon, what is the cost of a pint?

4. What part of a gallon are 3 qt.? 2 qt.?

5. How many pints are there in a gallon? What part of a gallon are 4 pt.? 7 pt.?

6. How much will 4 gal. of vinegar cost at 5¢ a quart?

7. How much will 4 gal. 2 qt. of oil cost at 8¢ a quart?

8. How much will 5 gal. 3 qt. of oil cost at 10¢ a quart?

9. What is the cost of $4\frac{1}{2}$ gal. of sirup at 20¢ a quart?

10. A milkman sells 3 qt. 1 pt. of milk to one customer and 3 pt. to another customer. How much does he sell to both?

11. How many quarts are there in one barrel?

12. If the liquid contained in a barrel is put into pint bottles, how many bottles will be required?

13. If bottles are used of such a size that five will hold 1 gal., how many bottles will be required to contain the liquid in the barrel?

14. How many cubic inches equal a gallon?

15. How many cubic inches equal a quart, liquid measure?

WRITTEN

16. How many more cubic inches are there in a quart, dry measure, than in a quart, liquid measure?

17. If a 5-gallon can of cream costs $4, at what price per pint must it be sold to gain $2?

18. How many cubic inches are there in a quart measure used for measuring liquids? How many cubic inches are there in a bushel measure?

19. Since 1 dry qt. contains 1.16 liquid qt., how many liquid quarts does a bushel contain? How many more liquid quarts than dry quarts are there in a bushel? If a grocer buys berries by the bushel and sells them by the liquid quart at 10¢ per quart, what is the gain on 1 bu.? on 25 bu.? on 60 bu.?

20. A caterer pays $1.25 per gallon for ice cream. If he serves six dishes per quart at 10¢ per dish, what is his profit on one gallon?

21. A milkman sells quart tickets at 6¢ each, or at 17 for \$1. If a family use $\frac{1}{2}$ gal. of milk each day, how much will they save in 34 days by buying tickets at the rate of 17 for a dollar?

22. A tank contains $7\frac{1}{2}$ cu. ft. How many gallons of water will it hold?

23. A tank has a capacity of 6 cu. ft. for each foot in depth. If it is 5 ft. deep, how many gallons does it contain? Omit the fraction in the answer.

24. How many bushels will the same tank contain? Omit the fraction in the answer.

AVOIRDUPOIS WEIGHT

Avoirdupois weight is used in weighing most articles bought and sold by weight.

TABLE

7000 grains	= 1 pound (lb.)
16 ounces	= 1 pound
100 pounds	= 1 hundredweight (cwt.)
20 cwt. / 2000 pounds	= 1 ton (T.)

NOTES. 1. Some scales show divisions of the ounce, as $\frac{1}{2}$ oz. or $\frac{1}{4}$ oz.

2. At the United States Custom House, and in weighing coal and iron at mines, the long ton of 2240 lb. is used.

The tables of Troy weight and Apothecaries' weight are so seldom used in ordinary business transactions, that they are not given here. They will be found in the Supplement.

TABLE OF MISCELLANEOUS WEIGHTS

200 pounds of pork	= 1 bbl.
196 pounds of flour	= 1 bbl.
100 pounds of nails	= 1 keg
56 pounds of butter	= 1 firkin

Most states have adopted the following :

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL

Oats,	32 lb.	Rye,	56 lb.
Wheat,	60 lb.	Barley,	48 lb.
Corn (shelled),	56 lb.	Clover seed,	60 lb.
Corn on the cob,	70 lb.	Timothy seed,	45 lb.
Potatoes,	60 lb.	Coal,	80 lb.

PROBLEMS

ORAL

1. A slice of steak weighs $2\frac{1}{2}$ lb. How much will it cost at 30¢ per pound? What is the weight in pounds and ounces?

2. What is the cost of $4\frac{1}{4}$ lb. of sugar at 6¢ per pound? What is the weight of the sugar in pounds and ounces?

3. What is the cost of $5\frac{3}{4}$ lb. of honey at 12¢ per pound? What is the weight in pounds and ounces?

4. How much will 12 oz. of tea cost at 48¢ per pound? What fraction of a pound does the tea weigh?

Express the following in pounds or a fraction of a pound:

	a.	*b.*	*c.*
5.	3 lb. 8 oz.	6 oz.	14 oz.
6.	4 lb. 4 oz.	2 lb. 12 oz.	10 lb. 10 oz.
7.	12 lb. 2 oz.	9 lb. 10 oz.	8 lb. 3 oz.
8.	10 oz.	4 oz.	6 lb. 9 oz.

9. Make and solve problems requiring the cost of commodities of the weights given in examples 5 to 8, at various prices per pound.

The following weights are what fraction of a ton? what decimal?

	a.	*b.*	*c.*	*d.*
10.	100 lb.	500 lb.	750 lb.	1200 lb.
11.	200 lb.	1000 lb.	800 lb.	250 lb.
12.	400 lb.	1500 lb.	900 lb.	1800 lb.

13. How many pounds are there in the following fractions of a ton? $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{3}{10}$, $\frac{1}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

14. A load of coal weighs 4200 lb. Give its weight in tons.

15. How much heavier are 20 bu. of corn in the ear than 20 bu. of shelled corn?

16. What is the cost of 2 bbl. of pork at $12\frac{1}{2}$ ¢ per pound?

17. What is the cost of $\frac{1}{2}$ bbl. of pork at 14 ¢ per pound?

18. What is the cost of 50 lb. of pork at $ 25 per barrel?

19. What is the cost of 1 firkin of butter at 30 ¢ per pound?

20. A hardware merchant buys 4 kegs of nails at $ 2.50 per keg and sells them at 4 ¢ per pound. How much does he gain?

21. What is the weight of 1 pk. of wheat? of 3 pk.?

22. What is the weight of 4 qt. of oats? of 3 pk. of corn?

23. What part of a pound are 12 oz.? 6 oz.? 14 oz.? 8 oz.?

24. What is the value of 12 oz. of canary seed at 40 ¢ a pound?

25. Find the cost of $\frac{7}{8}$ lb. of pepper at 4 ¢ an ounce.

26. A teamster had a load of stone weighing $2\frac{3}{4}$ tons. How many pounds were there in the load?

Written

27. A farmer sells a load of oats weighing 1296 lb. at 52¢ per bushel. How much does he receive for the load? (See p. 208.)

What fraction of 1 ton are the following weights? what decimal? Give each fraction in its lowest terms. Give no more than three places in the decimal.

28. 250 lb. **29.** 450 lb. **30.** 700 lb. **31.** 650 lb.

Find the value of the following amounts of coal at $7.50 per ton:

32. 2250 lb. **33.** 4450 lb. **34.** 6700 lb. **35.** 1650 lb.

Find the value of the following:

36. 3250 lb. of pork, at $24 per barrel.

37. 1617 lb. of flour, at $6 per barrel.

38. 8 kegs of nails, at $4\frac{1}{2}$¢ per pound.

39. 3040 lb. of wheat, at 90¢ per bushel.

40. 1239 lb. of shelled corn, at 72¢ per bushel.

41. 3045 lb. of potatoes, at 48¢ per bushel.

42. 1372 lb. of rye, at 60¢ per bushel.

43. 1160 lb. of barley, at 72¢ per bushel.

44. 1180 lb. of clover seed, at $5.75 per bushel.

45. 3065 lb. of timothy seed, at $4.50 per bushel.

46. A coal dealer bought 3 carloads of coal, at $2.25 per long ton. The weight of the coal in the three cars was: 64,800 lb., 72,450 lb., and 81,500 lb. He sold the coal at $4.15 per short ton. If it cost him 50¢ per short ton to deliver the coal to his customers, how much did he gain?

TIME MEASURE

Time measure is used in measuring time.

Table

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
365 days (or 52 weeks 1 day)	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

The solar year, or the time required for the earth to revolve around the sun, is 365 days, 5 hours, 48 minutes, and 46 seconds nearly. This is nearly $\frac{1}{4}$ of a day more than 365 days. This fractional part of the day is omitted each year until the fourth year, when the time lost amounts to nearly a day. In every year, the date number of which is divisible by 4, one day is added to the month of February, and that year is called **leap year**, except in the *centennial* years, in which the date numbers must be divisible by 400 to be leap years. This is because, in adding one day every 4 years, the error amounts to something over three days in 4 centuries, and is corrected by making the centennial year a leap year once in 400 years only.

Oral

1. How many minutes are there in 2 hours? in $\frac{1}{2}$ hour? in $1\frac{1}{2}$ hours? in 10 hours?

2. How many minutes are there from 9 A.M. to 11.30 A.M.? from 10 A.M. to 2.30 P.M.?

3. How many months are there in $3\frac{1}{2}$ years? in $4\frac{1}{3}$ years?

4. A man lived 15 years more than half a century. At what age did he die?

5. How many days are there in the spring months? in the winter months of 1908? of 1909?

6. How many days are there in the fall months?

7. Beginning with January, name the months having 31 days. How many of these months are there?

8. Beginning with April, name the months having 30 days. How many of these months are there? How many days has the remaining month?

9. One pupil may name the months in irregular order and require other members of the class to give the number of the months in the year. Example: Named — September. Number — 9th month.

10. What part of a day are 9 hours? 6 hours? 10 hours? 12 hours?

The calendar day begins at midnight and ends 24 hours later at midnight. The first twelve hours of the day constitute the forenoon; the twelve hours from noon to midnight constitute the afternoon. The abbreviation A.M. is used to indicate time between midnight and noon; P.M., to indicate the time from noon to midnight; M., to indicate noon.

11. What part of a day is the time from 6 A.M. to 10 A.M.? from 6 A.M. to 10 P.M.?

12. How many hours are there in the month of June? in the month of July? Answer the last question without multiplying the number of hours in a day by the number of days in July.

13. At 20¢ per hour, how much does a man earn laboring 8 hr. every working day in April if April 1 falls on a Saturday? If he works overtime, he receives $1\frac{1}{2}$ times the regular rate. If during the month he works $17\frac{1}{2}$ hr. overtime, how much does he earn that month?

14. How many hours of the day have passed at 5 A.M.? at 7.30 A.M.? at 10.45 A.M.? at 12 M.? at 1 P.M.? at 5 P.M.? at 8.45 P.M.? at 11.30 P.M.?

Time tables of railways in the United States give the time trains are due to arrive at stations as forenoon and afternoon time. The time

tables of the principal Canadian roads do not make this distinction. They give the time in terms of the number of hours and minutes past midnight. 1 P.M. is shown on these tables as 13.00; 4 P.M., as 16.00; 5.30 P.M., as 17.30; 11.45 P.M., as 23.45.

15. A train on the Canadian Pacific Railway is scheduled to arrive at

Calgary	at 5.55	A.M.	
Laggan	10.40		
Glacier	16.32	P.M.	
Revelstoke	19.30		
Scotch Hill	23.26		

Fill in, under A.M. and P.M., the times as they would appear in the United States.

What is the running time between each station and the next?

MISCELLANEOUS TABLE

12 units, or things	= 1 dozen (doz.)
12 dozen	= 1 gross
12 gross	= 1 great gross
20 units, or things	= 1 score
24 sheets of paper	= 1 quire
20 quires	= 1 ream

PROBLEMS

Oral

1. If oranges sell at the rate of 3 for 10 ¢, what is the selling price per dozen?

2. If apples sell at the rate of 2 for 5 ¢, how much must be paid for $2\frac{1}{2}$ doz.?

3. A dealer buys lead pencils at $5.75 per gross and sells them at 10 ¢ each. What is his profit on 1 gross?

4. A farmer sells 102 eggs for $2.55. How many cents does he receive per dozen? 102 = 8.5 doz.

5. A school board furnishes paper for written examinations. The paper costs $1.75 per ream. If $1\frac{1}{2}$ quires are used in an examination, what is the cost for the paper?

6. A man is 48 years old. In how many years will he be three score and ten years old?

7. If the cost of a given number of units of any kind is known, how can you find the cost of 1 doz.? of $\frac{1}{2}$ doz.? Give an example to illustrate.

8. If the cost of 1 doz. units of any kind is known, how can you find the cost of any given number of units of the same kind? Give two ways. Which do you prefer, and why? Give an example to illustrate.

9. If the cost of a gross of units of any kind is known, how can you find the cost of $2\frac{1}{2}$ doz. units of the same kind? Give an example to illustrate.

MEASURE OF LENGTH

Long measure, or linear measure, is used in measuring lengths or distances.

TABLE

12 inches (in. or ″)	= 1 foot (ft. or ′)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ ft.	= 1 rod (rd.)
320 rods 1760 yards 5280 feet	= 1 statute mile (mi.)

NOTE. The following measures are sometimes used:

4 in.	= 1 hand, used in measuring the height of horses.
6 ft.	= 1 fathom, used in measuring depths of water.
40 rd.	= 1 furlong.
6086 ft. (1.15+ statute mi.)	= 1 knot, or geographic mile, used in measuring distances at sea.
3 geographic mi.	= 1 league.

Surveyors' long measure is a table not much used. It will be found in the Supplement.

Questions on Units used in Measuring Lengths

1. What is the unit used by the merchant in measuring cloth? What is the unit used by the carpenter in stating the size of a room? the height of a house? What unit is used by the geographer in stating the height of a mountain? the distance between two cities? What unit is used by the farmer in stating the length and the width of a farm?

2. What are the dimensions of your arithmetic? the length, the width, and the height of the schoolroom? the width of the door? the height of the teacher's desk? the length of the school building? the length and the width of the school yard? the distance to your home? the distance between the two rear wheels of a wagon?

To the Teacher. Each pupil should be provided with a ruler, and should be required to measure and give the length and the width of the top of the desk; the height of the front edge of the desk from the floor; the height of the seat from the floor; the height and the width of the windows and the doors; the width and the length of the blackboards; the heights of different pupils. A yardstick should be provided for use in measuring yards.

These exercises should be given to make pupils familiar with different units of measurement and skillful in estimating short distances. Each pupil should also fix in his mind two points in the locality which are one mile apart. This will serve him as a standard in estimating greater distances.

PROBLEMS

Oral

1. What part of a yard is 27 in.? 9 in.? 4 in.? 8 in.? 2 ft. 3 in.? 2 ft. 6 in.? 1 ft. 6 in.? 1 ft. 8 in.?

2. How many feet are there in a mile? in $\frac{1}{8}$ of a mile? in $\frac{1}{4}$ of a mile? How many rods are there in 66 ft.?

3. What would be the cost of picture molding around a room 14 ft. by 16 ft., at 10 ¢ per foot?

4. Jane had $\frac{8}{9}$ yd. of ribbon. After using $\frac{3}{4}$ yd., how many inches of ribbon had she left?

5. A milliner cut 27 in. from a piece of lace containing $8\frac{5}{6}$ yd. How many yards were left in the piece?

6. How many ribbons 9 in. in length can be cut from a bolt $9\frac{3}{4}$ yd. in length?

7. How many feet are there in 2 rd.? in 3 rd.? in 4 rd.?

8. How many rods are there in 1 mi.? in 3 mi.? in 2 mi.?

9. How many feet are there in 60 in.? in 98 in.?

10. What is the total width, in feet and inches, of 4 windows, each 32 in. wide?

11. In walking, the length of a man's step is 30 in. How many steps would he take in walking 10 ft.? 100 ft.?

12. How many strips of wall paper 18 in. wide will be required to cover the walls of a room 18 ft. by 15 ft.? How many rolls will be required if the room is 8 ft. high?

NOTE. Wall paper is usually sold by the roll, each roll containing 8 yd. of paper $\frac{1}{2}$ yd. wide. Paper is laid on the wall in strips running up and down. As dealers do not sell less than a roll, a fractional part of a roll is regarded as an entire roll. How many yards are there in the entire distance around the room? How many strips will it take to paper the room? What is the length of each strip? How many strips of this length will one roll of paper make? How many rolls of paper will be required?

Find the distance in yards around the room, and reckon two strips for each yard. Divide this number of strips by the number of strips that can be made from a roll. The quotient is the number of rolls required.

13. How many rolls of paper 18 in. wide will it take for the walls of a room 21 ft. long, 18 ft. wide, and 12 ft. high?

14. How many rolls of paper 18 in. wide will it take for the walls of a room 15 ft. long, 12 ft. wide, and 8 ft. high?

WRITTEN

When a line is drawn 1 in. long to represent a line 4 in. long, it is drawn on a **scale** of 1 to 4. When a line is drawn 1 in. long to represent a line 1 ft. long, the scale is 1 in. to 1 ft., or 1 to 12.

NOTE. Sometimes the scale is given with the larger number first; as, 4 to 1, or 500 mi. to 1 in. "Scale 500 mi. to 1 in." means that a distance of 500 mi. is represented by 1 in. "Scale 1 in. to 500 mi." means that 1 in. represents 500 mi. In either case the smaller unit represents the line drawn.

15. Represent the height of a window pane 32 in. high by a line drawn on a scale of 1 to 8; of 1 to 16.

16. Represent the length and the width of a table $3\frac{1}{2}$ ft. long and $2\frac{1}{2}$ ft. wide on a scale of 1 in. to 1 ft.

17. What length is represented by a line 2 in. long drawn on a scale of 1 to 2? of 1 to 3? of 1 to 16? of 1 in. to 1 ft.? of 1 in. to 1 yd.? of 1 in. to 1 rd.?

18. A field of corn is 40 rd. long and 24 rd. wide. If the hills of corn are $3\frac{2}{3}$ ft. apart, and the first hill in each row is planted on the outer edge of the field, how many hills are there in each row running the length of the field? in each row running across the field? How many hills of corn are there in the field? Illustrate by drawing.

SUGGESTION. Add 1 hill after dividing. Why?

19. Line AH is $3\frac{1}{2}$ in. long and is divided into $\frac{1}{2}$-inch spaces. The line from A to C represents a distance of 20 mi. What distance does each of the following portions of the line represent? A to D; A to E; A to F; A to G; C to H; E to H.

20. Using a scale of $\frac{1}{2}$ in. to 2 mi., what distance would be represented by a line $2\frac{3}{4}$ in. long? $3\frac{1}{8}$ in. long? 6.5 in. long? 4.3 in. long? 2.8 in.? $7\frac{1}{2}$ in.?

21. If the line AC in problem 19 represents a distance of 35 rd., what distance is represented by AB? AD? AE? AF? AG? AH? CH? EH? DG?

22. Measure the length and the width of your schoolroom and draw a figure to represent it, on a scale of 1 in. to 8 ft.

23. Measure the length and the width of your school building, and draw a figure to represent it, on any scale you please.

24. A warship, on its trial trip, makes 22 knots per hour. How many miles does it travel per hour?

25. A tract of land has a frontage on a city street of 594 ft., and a depth of 132 ft. If the tract is to be divided into lots, each having a frontage of 4 rd., how many lots will there be? What is the depth of each lot in rods? Draw on a scale of 1 inch to 1 rod a plot of the tract, showing the boundary lines of each lot.

26. Give the dimensions of a room in feet, and find the number of yards around the room. For what purpose might you wish to know the distance in yards around the room?

27. Draw a diagram of a room 27 ft. long and 18 ft. wide, on a scale of 1 in. to 4 ft.

28. If carpet is $\frac{3}{4}$ of a yard wide, how many strips will it take to carpet a room 18 ft. by 27 ft., if the strips are laid lengthwise of the room? if they are laid crosswise?

29. How many yards of carpet 1 yd. wide will it take to carpet lengthwise a room 21 ft. long and 15 ft. wide?

30. Write the table of long measure, using the proper abbreviations.

31. Prove that a mile equals 5280 ft.; 63360 in.

32. Having the distance between two places in miles, how can you find, by one operation, the distance in feet?

SURFACE OR SQUARE MEASURE

Square measure is used in measuring surfaces.

A **surface** is that which has length and breadth only.

An **angle** is the difference in direction of two lines.

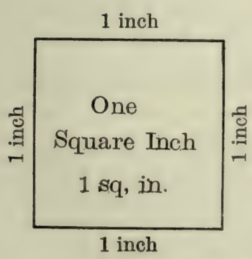
If one straight line meets another straight line, so as to make two equal angles, each angle is a **right angle**. A right angle is formed by two lines meeting in a square corner.

A **square** is a plane surface having four equal sides and four right angles.

A **rectangle** is a plane surface having four sides and four right angles. The opposite sides are equal, but the length may be greater than the width.

Table

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
30¼	square yards (272¼ sq. ft.)	= 1 square rod (sq. rd.)
160	square rods	= 1 acre (A.)
640	acres	= 1 square mile (sq. mi.)
36	square miles or sections	= 1 township



A square inch is a plane surface one inch square.

A square foot is a plane surface one foot square.

To the Teacher. Such a square should be drawn upon the blackboard, and divided into square inches, the number of square inches in the square foot should be ascertained, and the fact should be developed that the number of square inches in a square foot is found by multiplying the number of square inches in the row of squares on one side by the number of such rows.

1. Draw a square foot, and divide it into strips 1 in. wide. How many strips does it contain? How many square inches are there in each strip? How many square inches are there in a square foot?

2. Draw a rectangle 4 in. wide and 6 in. long. Divide it into inch squares. How many squares are there in the rectangle? how many square inches?

3. Draw a rectangle 8 in. wide and 12 in. long. How many strips 12 in. long and 1 in. wide does it contain? How many square inches are there in one strip? How many square inches are there in the rectangle? How may this number be obtained?

4. Draw another rectangle of the same size as the preceding one. How many strips 8 in. wide and 1 in. long does it contain? How many square inches are there in each strip? How many square inches are there in the rectangle? What is the unit of measurement? How can you find the area of rectangles?

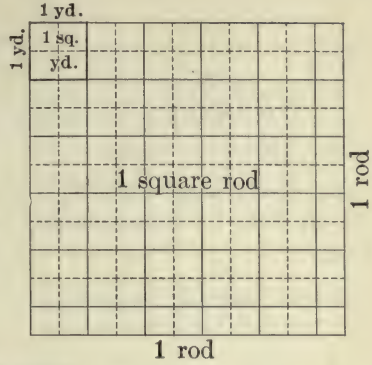
A **square yard** is a plane surface one yard, or three feet, square.

To the Teacher. Such a square should be drawn upon the blackboard, and the number of square feet in its surface should be determined in the same manner as the number of square inches in the square foot was determined.

A **square rod** is a plane surface one rod, or $16\frac{1}{2}$ feet, square, or $5\frac{1}{2}$ yards square.

To the Teacher. Make clear to pupils that the number of square feet or square yards in a square rod is found in the same way as the number of square inches in a square foot or of square feet in a square yard.

The unit of measurement in measuring surfaces is always a square. This square may be a square inch, a square foot, a square yard, etc.



What are the dimensions in yards of the square rod?

What are the dimensions in $\frac{1}{2}$ yards of the smallest of the squares? What are the dimensions in $\frac{1}{2}$ yards of the square rod?

The square $\frac{1}{2}$ yd. on each side is what part of the square yard?

How many of the $\frac{1}{2}$ yd. squares are there in the square rod?

As there are 4 of the $\frac{1}{2}$ yd. squares in each square yard, how many square yards equal the whole number of $\frac{1}{2}$ yd. squares?

The dimensions of a surface are given in linear units. If the dimensions are in inches, the unit of measurement is the square inch, and the **area** or surface is given in square inches. If the dimensions are in feet, the area is square feet, etc.

The **perimeter** is the distance around a surface.

From the explanation already given of the mode of finding the number of square inches in a surface 1 ft. or 12 in. square, the number of square feet in a surface 1 yd. or 3 ft. square, it is seen that the number of square units in such a surface equals the product of the two numbers which give the dimensions in like linear units; but what is really done is to multiply the number of square units on one side of the surface by the number of such units on one end of the surface. In actual work, the number of linear units in length is multiplied by the number of like linear units in width, and the product is the number of square units whose sides are the linear units used in giving the dimensions.

The ***area*** *of a rectangle is a number of square units equal to the product of its two dimensions when expressed in like units.*

PROBLEMS

Oral

1. What is the area of the floor of a room 10 ft. wide and 12 ft. long?

2. What is the area of a blackboard that is 4 ft. wide and 16 ft. long? What is the unit of measurement in each case?

Note. Estimates for painting are by the square of 100 sq. ft.; that is, a square surface 10 ft. on each side.

3. What is the cost of painting a solid board fence 10 ft. high and 150 ft. long at 50¢ per square?

4. The surface of a slate contains 96 sq. in.; the slate is 8 in. wide. How long is it?

5. What must be the width of a floor 6 yd. long in order to contain 30 sq. yd.? Give the width first in yards and then in feet.

6. A room is 9 ft. high and 12 ft. wide. How many square yards are there in the two ends?

7. A room is 15 ft. long, 12 ft. wide, and 9 ft. high. How many square yards are there in the two sides? How many square yards are there in the ceiling? How many square yards are there in the walls and the ceiling? How many square yards of carpet will it take to cover the floor?

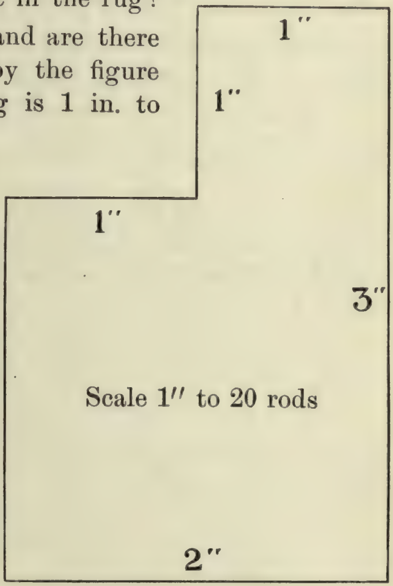
8. How many square rods are there in half an acre? in $\frac{3}{4}$ of an acre? in $\frac{5}{8}$ of an acre?

9. How many acres are there in a square mile? in $\frac{1}{2}$ of a square mile? in $\frac{1}{8}$ of a square mile? in $\frac{3}{4}$ of a square mile? in $\frac{5}{8}$ of a square mile?

WRITTEN

10. Draw a rectangle on a scale of $\frac{1}{2}$ in. to 1 ft. to represent a floor 16 ft. by 14 ft. Inside this rectangle draw another on the same scale to represent a rug, so laid as to leave an uncovered space 2 ft. wide around the room. How many square yards are there in the rug?

11. How many acres of land are there in the tract represented by the figure if the scale of the drawing is 1 in. to 20 rods?



12. How many rods of fence are required to inclose this land?

13. After the tract has been fenced, how many more rods of fence are needed to separate the tract into 2 rectangular lots?

14. Draw a dotted line to represent this additional fence.

15. It requires 30 sq. yd. of carpet to cover a floor. The floor is 18 ft. long. What is its width in feet?

SOLUTION. 18 ft. = 6 yd. $30 \div 6 = 5$, the number of yards in width; $3 \times 5 = 15$, the number of feet in width. Or, 30 sq. yd = 270 sq. ft. $270 \div 18 = 15$, the number of feet in width.

16. Figure 1 on p. 224 is a square $1\frac{1}{2}$ in. on each side. Find its area in the same way as you would find the area of a square 3 in. on each side. What are the dimensions and area of the square *A*? of the square *D*? of the rectangle *C*? of the rectangle *B*? What is the sum of their areas?

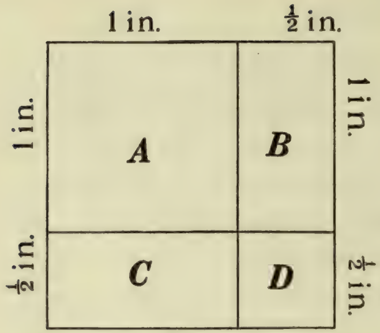


Fig. 1

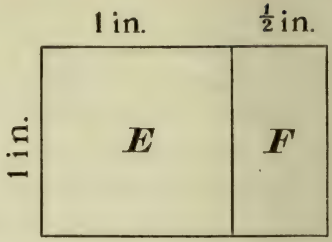


Fig. 2

17. Figure 2 is a rectangle $1\frac{1}{2}$ in. long and 1 in. wide. Find its area in the same way as you first found the area of Figure 1. What are the dimensions of the square *E*? of the rectangle *F*? What is the sum of their areas?

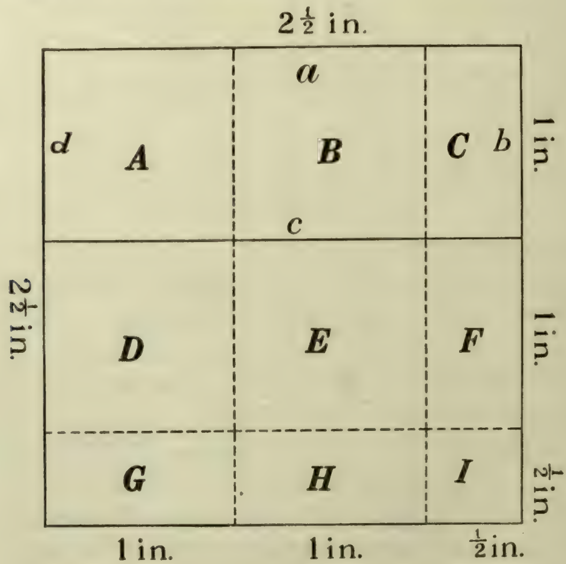


Fig. 3

18. What is the difference between $1\frac{1}{2}$ square inches and $1\frac{1}{2}$ inches square?

19. Figure 3 (p. 224) is a square $2\frac{1}{2}$ in. on each side. Find its area as you would find the area of a 3 in. square.

20. In Figure 3, the rectangle bounded by the lines *a*, *b*, *c*, *d*, is $2\frac{1}{2}$ in. long and 1 in. wide. What is its area?

21. Prove the correctness of your results by adding the areas of the squares and rectangles in Figure 3; by adding the areas of the squares *A* and *B* and the rectangle *C*.

22. What is the difference between $2\frac{1}{2}$ sq. in. and $2\frac{1}{2}$ in. square?

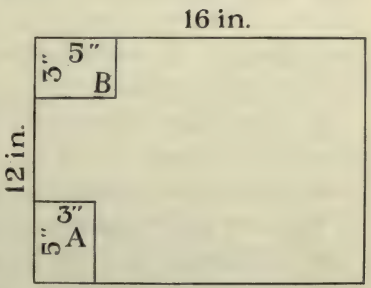
PAPER AND CARDBOARD WORK

In a box factory, pulp board in sheets of different sizes is used. The bottoms, tops, and sides of the boxes are cut from these boards so as to waste as little material as possible.

1. How many pieces 2 in. by 2 in. can be cut from a sheet 12 in. by 16 in.? how many pieces 2 in. by 4 in.?

In problem 1 there is no waste, because the dimensions, 12 in. and 16 in., of the sheet are multiples of the dimensions, 2 in. and 4 in., of the pieces.

WRITTEN



2. The pieces are 3 in. by 5 in. How many pieces can be cut from a sheet 12 in. by 16 in., if the pieces are cut so that the length of each piece runs in the same direction as the piece *B*? as in the piece *A*? How many square inches of the board are wasted in each case?

3. Draw two rectangles, each 12 in. by 16 in. In one, draw lines to show the number of pieces, B, that can be cut from the sheet, the boundary lines of each piece, and the portion of the sheet wasted. In the other, draw lines to show the number of pieces, A, their boundary lines, and the portion of the sheet wasted.

4. How many pieces 4 in. by 6 in. can be cut from a sheet of pulp board 22 in. by 28 in.?

5. Draw a rectangle on a scale of $\frac{1}{8}$ in. to 1 in. to represent the sheet, and draw one piece 4 in. by 6 in. showing the portion of the sheet cut out to make this piece. The lengths of the remaining pieces must run in the same direction as in the piece marked out. How many square inches of the sheet are wasted?

6. What is the largest number of pieces $5\frac{1}{4}$ in. by $7\frac{1}{2}$ in. that can be cut from a sheet of pulp board 22 in. by 28 in.?

7. How many book covers $7\frac{1}{2}$ in. $\times$ 5 in. can be cut from a sheet of binder's board 22 in. by 28 in.?

8. How many samples 2 in. by 3 in. can be cut from a sheet of cardboard 36 in. long and 30 in. wide?

9. A sheet of mounting board, $22'' \times 28''$, is to be used for mounting pictures. A piece $5'' \times 7''$ is needed for each picture. How many mounts of this size can be cut from the large sheet? How many square inches of the surface of the sheet will be wasted?

10. How many mounts $4'' \times 5\frac{1}{2}''$ can be cut from a sheet of mounting board $22'' \times 28''$?

11. If it takes a piece of boxboard $8'' \times 9''$ to make a pencil box, how many boxes can be made from a sheet $24'' \times 36''$?

12. A certain kind of bond paper measures $17'' \times 22''$. How many sheets of letter head size ($8\frac{1}{2}'' \times 11''$) can be cut from the large sheet?

13. A ream of paper usually contains 500 sheets. If we buy one ream of bond paper, $17'' \times 22''$, and cut it to letter-head size ($8\frac{1}{2}'' \times 11''$), how many reams have we?

14. If a customer wants 4000 sheets of letter-head paper ($8\frac{1}{2}'' \times 11''$), how many reams of the large size ($17'' \times 22''$) will be needed to supply him?

15. How many sheets of book paper $6'' \times 9''$ can be cut from a ream of 500 sheets, size $20'' \times 25''$?

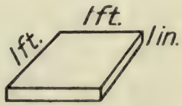
16. A ream of cover paper, weighing 60 pounds, costs 5 cents per pound. What is the cost per sheet?

17. A piece of cardboard $7\frac{1}{2}''$ long and $4\frac{1}{2}''$ wide is to be cut so as to make the largest possible number of squares $1\frac{1}{2}''$ on each side. How many such squares laid side by side will be equal in length to the length of the piece of cardboard? to the width of the piece of cardboard?

18. Draw a figure, $7\frac{1}{2}''$ long and $4\frac{1}{2}''$ wide, and then draw the remaining lines to show the outlines of the squares indicated in problem 17. Draw the figure on a scale of 1 in. to 2 in.

LUMBER MEASURE

The **board foot**, or a piece of board 1 ft. long, 1 ft. wide, and 1 in. or less in thickness, is the unit of lumber measure. Besides this unit, the lineal foot is often used, as in the case of moldings, etc. Lumber is also measured by the number of pieces or M pieces, as in the case of laths and shingles.



1 Board Foot

If a board is 1 in. or less in thickness, the number of *board feet* it contains is the number of *square feet* on one side of the board. If the board is $1\frac{1}{2}$ in. thick, the number of board feet is $1\frac{1}{2}$ times the number of square feet on one side of the board. If the board is 2 inches thick, the number of

board feet is 2 times the number of square feet on one side of the board, and so on.

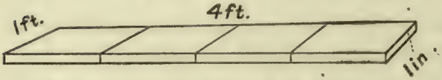
In making out a bill of lumber, the amount of each kind is entered as follows:

4 pieces $2'' \times 4'' - 12'$
8 pieces $2'' \times 6'' - 14'$
2 pieces $4'' \times 8'' - 16'$

The number of pieces is written first, followed by the thickness and width in inches and the length in feet. ($''$) is used instead of (in.) to denote "inches" and ($'$) instead of (ft.) to denote "feet."

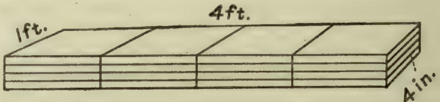
The *thickness* is always given first, then the *width*, and then the *length*.

In checking a bill of lumber a clerk would read the first item above "four pieces, two by four, twelve feet."



A piece of 1-inch board 1 ft. wide and 4 ft. long contains 4 board feet. If it is 6 in. or $\frac{6}{12}$ ft. wide, it contains $\frac{6}{12} \times 4$, or 2 board feet. If it is 3 in. or $\frac{3}{12}$ ft. wide, it contains $\frac{3}{12} \times 4$, or 1 board foot.

ORAL



1. A piece of 4-inch board is 1 ft. wide and 4 ft. long. How many board feet does it contain? If it is 6 in. wide, how many board feet does it contain? 4 in. wide? 3 in. wide? 8 in. wide? 9 in. wide?

The number of board feet in a board, $1''$ or less in thickness, and

2. $1'$ wide = the number of feet in the length of the board.

3. $6''$ wide = $\frac{1}{2}$ the number of feet in the length of the board.

Complete the table and give the number of board feet in a board, using each of the different widths given, if the board is

8 ft. long. 10 ft. long.
12 ft. long. 14 ft. long.
16 ft. long. 18 ft. long.

4. 3″ wide =
5. 2″ wide =
6. 4″ wide =
7. 8″ wide =
8. 9″ wide =
9. 10″ wide =
10. 14″ wide =
11. 16″ wide =
12. 18″ wide =

In lumber more than 1″ thick, the number of board feet in any piece may be found by *multiplying the length in feet* by *the width and thickness in inches, and dividing the product by* 12. Explain why this gives the correct result.

How many board feet are there in each of the following pieces of lumber:

13. 1 pc. $1\frac{3}{4}'' \times 6''$–$12'$ (Read $1\frac{3}{4}$ inches by 6 inches by 12 feet.)

SOLUTION. $\left(12 \times \frac{7}{4} \times 6\right) \div 12 = 10\frac{1}{2}$. *Ans.* $10\frac{1}{2}$ board feet.

14. 1 pc. $1'' \times 6''$–$12'$

15. 2 pcs. $1\frac{1}{2}'' \times 12''$–$5'$

16. 3 pcs. $1'' \times 5''$–$12'$

17. 4 pcs. $3'' \times 6''$–$16'$

18. 6 pcs. $2'' \times 3''$–$16'$

19. 3 pcs. $1'' \times 8''$–$20'$

20. 8 pcs. $1'' \times 9''$–$12'$

21. 10 pcs. $2'' \times 4''$–$12'$

22. 6 pcs. $2'' \times 8''$–$16'$

23. 4 pcs. $5'' \times 10''$–$12'$

WRITTEN

In lumber that is listed by the piece, an item in the bill of material would appear as follows:

25 pcs. $2'' \times 4'' - 16'$ #1 Pine @ $25.00 per M.

To find the cost of such an item, use the following formula:

$$\frac{\text{Number of pieces} \times \text{thickness in inches} \times \text{width in inches} \times \text{length in feet} \times \text{price per foot}}{12} = \text{Cost}$$

What is the cost of each of the following items:

24. 40 pcs. $2'' \times 6''$—$14'$ #1 Hemlock @ \$25.00 per M?

25. 90 pcs. $2'' \times 4''$—$16'$ #1 Hemlock @ \$27.50 per M?

26. 37 pcs. $2'' \times 10''$—$16'$ #1 Hemlock @ \$33.00 per M?

27. 28 pcs. $1'' \times 4''$—$10'$ #2 Pine @ \$24.00 per M?

28. 110 pcs. $1'' \times 6''$—$14'$ #2 Pine Drop Siding @ \$35.00 per M?

29. What amount of lumber is contained in 24 table legs $4'' \times 4'' \times 30''$?

In order to finish to $4'' \times 4''$, the size of the rough lumber must be $5'' \times 5''$, and at least $2''$ must be allowed on the length of each leg. The lumber can be procured $10'$, $12'$, $14'$, or $16'$ in length. Which length would cut to the best advantage? What would be the difference between the amount contained in the 24 legs and the amount that would appear on the order?

30. At \$55 per M, what is the cost of the amount of lumber that is dressed off on the 24 legs?

31. What is the cost of 20 boards $\frac{3}{4}$ in. thick, 9 in. wide, and 16 ft. long, at \$26 per M?

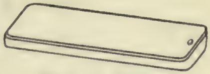
32. What is the cost of 60 planks 2 in. thick, 10 in. wide, and 14 ft. long, at \$24 per M?

33. What is the cost of sixty $1\frac{1}{2}$ in. flooring planks, 8 in. wide, 16 ft. long, at \$33 per M?

34. A 4-ft. sidewalk is to be built for a 50-ft. lot. 2-in. planks 6 in. wide are used for the surface, and two stringers $4'' \times 4''$ as supports. Find the cost of the material at \$24 per M.

WOODWORKING

NOTE. In working these examples, remember that lumber less than 1″ thick is counted the same as lumber 1″ thick.

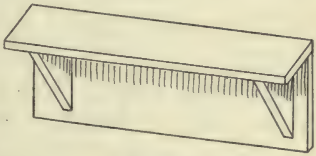


BREAD BOARD

1. **Bread board.** If a bread board is $\frac{3}{4}'' \times 6'' \times 10''$, how many board feet are contained in 1 doz. bread boards?

BOOK RACK

2. **Book rack.** This book rack is made of one piece, $1'' \times 5'' \times 14''$, and 2 pieces, $1'' \times 4'' \times 5''$. Allowing for waste it can be made from one piece $1'' \times 6'' \times 27''$. How many board feet will it take to make two dozen book racks?



SHELF

3. **Shelf.** The top of the shelf is $\frac{3}{4}'' \times 5'' \times 14''$; the back is $\frac{3}{4}'' \times 6'' \times 14''$; and the two brackets, in the shape of right triangles with one side 4″ and the other side 5″ long can be cut from a rectangle $4'' \times 5''$, making no allowance for waste in cutting. Allowing $\frac{1}{4}$ of the amount in the finished article for waste, how many board feet are required to make one dozen of the bracket shelves?

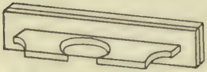
FLOWERPOT STAND

4. **Flowerpot stand.** The flowerpot stand requires 4 slats and 3 supports. The slats are $\frac{3}{4}''$ thick $\times$ $\frac{1}{2}''$ wide $\times$ 17″ long. Allowing $\frac{1}{4}''$ for waste in ripping and smoothing each piece, how wide must the piece be from which 4 slats may be cut?

5. Allowing for waste in length, the 4 slats can be made from a piece $1'' \times 3'' \times 18''$. How many board feet are needed to make the four slats?

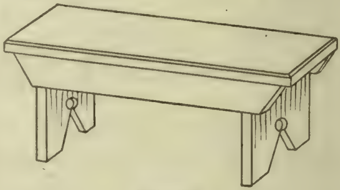
6. The supports for the flowerpot stand are $\frac{3}{4}'' \times \frac{3}{4}'' \times 5''$; but, allowing for waste, 12 supports (enough for 4 stands) can be cut from a board $1'' \times 6'' \times 1'$. If the 4 stands require in addition $1\frac{1}{2}$ board feet for slats, how many board feet do they require all together?

7. **Broom holder.** The broom holder is made from two pieces of stock each $\frac{7}{8}'' \times 4'' \times 12''$. How many broom holders can be made from 100 board feet of lumber?



BROOM HOLDER

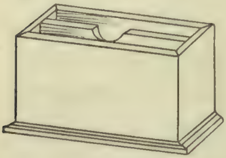
8. **Bench.** The bench has one top $\frac{7}{8}'' \times 10'' \times 29''$, two ends $\frac{7}{8}'' \times 8'' \times 8''$ (not allowing for $\wedge$ cuts), and two rails $\frac{7}{8}'' \times 3'' \times 27''$. How many board feet are there in 12 finished benches?



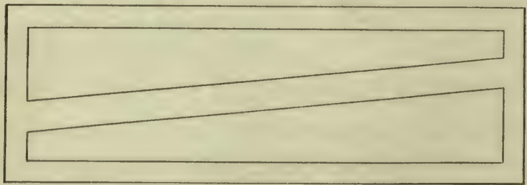
BENCH

9. **Bench hook.** The bench hook requires $\frac{3}{4}$ bd. ft. of white pine at $60 per M. How much does the stock in 6 bench hooks cost?

10. **Letter box.** If John can make one letter box from 1 board foot red gumwood which costs him $55 per M, and if Fred requires $1\frac{1}{2}$ board feet of oak which costs him $75 per M, how much more will one dozen boxes cost Fred than John?

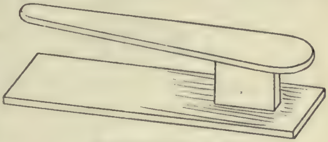


LETTER BOX

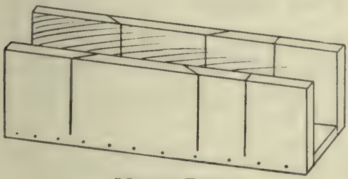


SLEEVE BOARD PATTERN (See Ex. 11.)

11. Sleeve board. By using a pattern for the tops, as shown on p. 232, you can cut 2 sleeve board tops from a board $1'' \times 10'' \times 2'$. If each bottom is cut from a board $1'' \times 6'' \times 2'$ and each block from a board $2'' \times 6'' \times 6''$, how much will it cost to make 100 sleeve boards of clear basswood, at $60 per M?



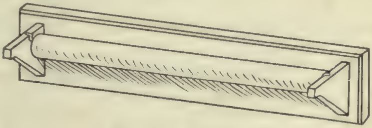
SLEEVE BOARD



MITER BOX

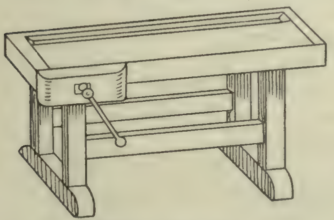
12. Miter box. This box contains $1\frac{1}{2}$ board feet of white pine at $75 per M. If 2 bd. ft. are used up in making each box, how much does the waste cost in 24 boxes?

13. Towel roller. The towel roller requires one piece $\frac{7}{8}'' \times 4'' \times 20''$, two triangular brackets that can be cut from one rectangular piece $\frac{7}{8}'' \times 4'' \times 4''$, and one roller that requires $\frac{1}{3}$ board foot. If the towel roller is made from $2\frac{1}{2}''$ clear oak at $75 per M, what is the cost of the material?

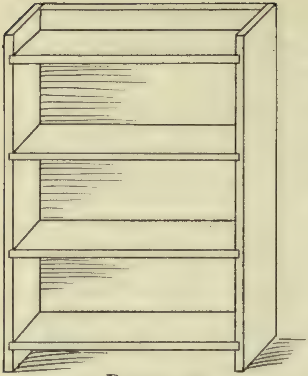


TOWEL ROLLER

14. Work bench. The top of the work bench requires 10 board feet of clear maple @ $65 per M, and is 5 ft. long. How much will it increase the cost of the top to increase the length 2 ft.? The bench sells for $12. If $\frac{3}{4}$ of this price is for labor, profit, etc., and $\frac{1}{4}$ is the cost of stock, how many feet were required, using clear maple at $65 per M?



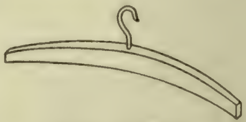
WORK BENCH



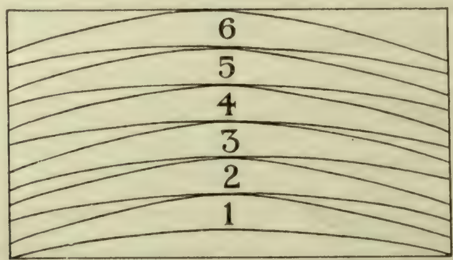
BOOKCASE

15. Bookcase. If the 2 ends of the bookcase require together $8\frac{1}{2}$ board feet, the back $12\frac{1}{2}$ board feet, and each of the 4 shelves $2\frac{1}{2}$ board feet, how much more will the bookcase cost if made of oak at \$85 per M than if made of basswood at \$60?

16. Coat hanger. If the coat hanger is made from a rectangular piece of stock, it requires a piece $\frac{3}{4}'' \times 4'' \times 16''$. If a pattern is made, it can be laid out upon a piece of stock $\frac{3}{4}'' \times 12'' \times 16''$, six times, as shown in the sketch. If the material used is worth \$50 per M, how much will the stock cost for two dozen coat hangers made by the first method? by the second method?



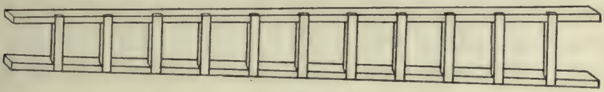
COAT HANGER



PATTERN FOR COAT HANGER

17. Ladder. The ladder is 12′ long, 18″ wide at the bottom, and 12″ wide at the top. The rungs are spaced one in every foot. How many are there? What is the average

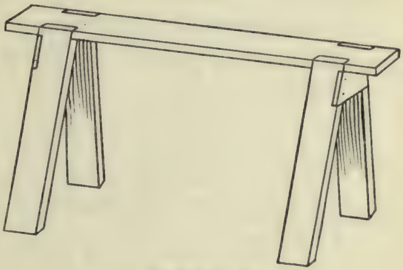
length of each one? The uprights are made from 2 pcs., $2'' \times 4''$—$12'$ #1 Pine @ \$45 per M, and the rungs are



LADDER

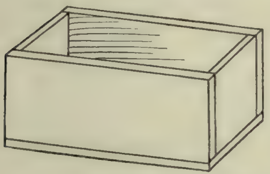
sawed from 1 pc. $1\frac{1}{4}'' \times 3''$—$16'$ #1 Pine @ \$45 per M. How much does the ladder cost if the labor equals the cost of material?

18. Sawhorse. The various parts of the sawhorse require a piece of lumber $1\frac{1}{4}'' \times 4'' \times 14'$ #1 Pine @ \$45 per M. How much more or less does it cost to make it of a piece $2'' \times 4''$—$16'$ #1 Hemlock @ \$27.50 per M?



SAWHORSE

19. Box. The box is made of $\frac{3}{4}''$ material. If constructed as shown in the drawing, will the ends be equal to the width of the box? Will the sides be equal to the length of the box? Will the height of the sides and of the thickness of the bottom equal the height of the box? The outside dimensions of the box are $20''$ wide by $32''$ long by $12''$ high. Make a list showing the number and exact finished sizes of parts. What are the inside dimensions of the box? If, with allowance for waste, 16 board feet are needed to make the box, how much will the lumber cost at \$50 per M?



BOX

PLASTERING

Plastering is usually done in 2-coat and 3-coat work, and is measured by the square yard.

Lathing is usually estimated by the thousand laths.

1. The following is an estimated cost of 100 sq. yd. of lath and plaster for 2-coat work:

1500 laths @ \$4.75 per M	
10 lb. nails @ \$3.20 per cwt.	
Labor, putting on laths	\$4.50
10 bu. lime @ \$.45 per bushel	
$\frac{3}{4}$ bu., or 6 lb., of hair, @ \$.04 per pound	
1 load of sand	\$1.75
Plasterer, $2\frac{1}{2}$ da., @ \$5.00	
Helper, $2\frac{1}{2}$ da., @ \$1.50	
Cartage	\$1.00

Find the cost per square yard.

2. For lathing and plastering 100 sq. yd., 3-coat work, the following estimate may be used:

1500 laths @ \$4.75 per M	
10 lb. nails @ \$3.20 per hundredweight	
Labor, putting on laths	\$4.50
13 bu. lime @ \$.45 per bushel	
8 lb. hair @ \$.04	
$1\frac{1}{2}$ loads plastering sand @ \$1.70	
1 bbl. plaster Paris	\$1.60
Plasterer, $3\frac{1}{2}$ da., @ \$5.00	
Helper, $3\frac{1}{2}$ da., @ \$1.50	
Cartage	\$1.00

Find the cost per square yard.

3. At \$.36 per square yard for 2-coat work and \$.46 per square yard for 3-coat work, find the cost of lathing and

plastering a room 18 ft. × 21 ft. and 9 ft. high, 2-coat work; 3-coat work, no allowance being made for openings.

SUGGESTION. Entire length of walls = 2(18 ft. + 21 ft.).
Area of ceiling = (18 × 21) sq. ft.

4. A room is 12′ 6″ by 18′ 6″ and 12′ high. It has 2 doors, 3′ 6″ by 8′, and 4 windows, 3′ by 7′. Dimensions of doors and windows include casings. If an allowance of $\frac{1}{2}$ the area of the openings is deducted from the entire surface, what is the cost of plastering the room at \$.46 per square yard?

5. A closet is 9′ by 3′ 6″ and 10′ high. What is the cost of plastering it at \$.40 per square yard? In estimating cost of plastering closets, no allowance is made for openings. The actual area of walls and ceiling is increased by one half.

QUESTIONS ON UNITS OF MEASUREMENT IN SURFACES

1. A book is 7 in. wide and 9 in. long. What is the unit used in expressing its surface?

2. If the floor of the schoolroom is 30 ft. long, and 20 ft. wide in what units should you express its area?

3. A field is 80 rd. long and 40 rd. wide. Its area is found in what units? These units may be changed to what other units?

4. A wall is 9 ft. high and 16 ft. long. What is its area? What is the unit of area? To what other units of area may it be changed?

5. If a sheet of paper is 26 in. by 23 in., what is its area? If its area in square feet had been called for, how many operations would be necessary to find it? What are they?

6. The area of a window is 15 sq. ft. Its width is 30 in. What is its length in inches? in feet?

7. It requires 30 sq. yd. of carpet to cover a floor. The floor is 15 ft. wide. How long is it?

8. A rectangular field contains 20 acres. The fence is 40 rd. long at one end. How many rods of fence are required to inclose the field?

Problems without Numbers

9. If the dimensions of a surface are given in inches and its area is to be expressed in square feet, what must be done?

10. Make and solve a problem to illustrate how you would apply your answer to problem 9.

11. If the dimensions of a wall are given in feet, and its area in square yards is asked for, how is it found?

12. Make and solve a problem to illustrate how you would apply your answer to problem 11.

13. If the dimensions of a wall are given in inches, and its area in square yards is asked for, how many operations are necessary? What are they?

14. Make and solve a problem, using in the solution the operations named in your answer to problem 13.

15. What unit is used in giving the area of a farm? of a carpet? of the surface of a table? Give examples.

16. May the area be given in more than one kind of surface units? Illustrate.

17. If you know the area of a blackboard in square feet and its width in feet, how can you find its length in feet?

18. If the area of a rectangular surface is given in square units, and the length of one side, in linear units of a different name, what must be done in order to find the other side?

MEASURE OF VOLUME, OR CUBIC MEASURE

Table

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord (Cd.)

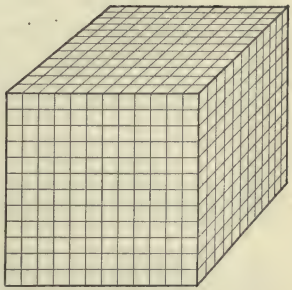
A **solid body** has three dimensions, length, breadth, and thickness. It is also called a **volume**.

A **line** has but one dimension, length; a **surface** has two dimensions, length and width.

A **rectangular solid** is a solid bounded by six rectangular surfaces; its corners are square.

The surfaces of a solid are called **faces**.

The lines in which the faces meet are called **edges**.



How many edges has a crayon box? how many faces? how many corners?

A **cube** is a rectangular solid all of whose faces are square.

A **cubic inch** is a cube whose edges are each one inch long. What is the area of each face? What is the entire area of an inch cube?

A **cubic foot** is a cube whose edges are each one foot long. What is the area of each face? What is the entire area of a cube whose edges are each one foot?

What is a cubic yard? What is the area of each face? What is the area of the top and bottom of a cube whose edges are each one yard long? What is the area of the six faces of such a cube?

PROBLEMS

Oral

1. How many cubic feet are there in a block of wood 1 ft. long, 1 ft. wide, and 1 ft. high? in a block of wood 2 ft. long, 1 ft. wide, and 1 ft. high?

2. How many cubic feet are there in a block of wood 3 ft. long, 1 ft. wide, and 1 ft. high? in a block 3 ft. long, 2 ft. wide, and 1 ft. high? 3 ft. long, 3 ft. wide, and 1 ft. high?

3. If the block of wood is 3 ft. long, 3 ft. wide, and 2 ft. high, how many cubic feet does it contain? if it is 3 ft. long, 3 ft. wide, and 3 ft. high?

The number of cubic feet in each case equals the product of what three numbers?

4. A solid 1 in. long, 1 in. wide, and 1 in. high is a cubic inch. How many cubic inches are there in a solid 12 in. long, 1 in. wide, and 1 in. high? Make a drawing showing such a solid, marked off into cubic inches.

5. How many cubic inches are there in a solid 12 in. long, 2 in. wide, and 1 in. high? Make a drawing showing such a solid, marked off into cubic inches.

6. If a solid is 12 in. long, 12 in. wide, and 1 in. high, how many cubic inches does it contain, or what is its volume?

7. If a solid is 12 in. long, 12 in. wide, and 2 in. high, how many cubic inches are there in its volume?

8. If a solid is 12 in. long, 12 in. wide, and 12 in. high, how many cubic inches does it contain? Such a solid being 1 ft. square is a **cubic foot**. The number expressing its volume in cubic inches is the product of what three numbers?

9. How many cubic inches are there in a brick 8 in. long, 4 in. wide, and 2 in. thick?

Solution. If the brick were 1 in. long, 1 in. wide, and 1 in. thick, it would contain 1 cu. in.; if it were 8 in. long, 1 in. wide, and 1 in. thick, it would contain 8 cu. in.; if it were 8 in. long, 4 in. wide, and 1 in. thick, it would contain 32 cu. in. Since it is 8 in. long, 4 in. wide, and 2 in. thick, it contains 64 cu. in.

The operations performed in the analysis are multiplying 1 cu. in. by 8; multiplying the product, 8 cu. in., by 4; multiplying this product, 32 cu. in., by 2; the result is 64 cu. in.

What is the unit of measurement?

In finding the volume of the brick without going through the analysis, what is to be done?

Notice that the dimensions are given in linear inches; the volume is given in cubic inches. If the dimensions had been linear feet, the volume would be expressed in what units?

10. A pile of 2-foot wood is 10 ft. long, 4 ft. high. How many cubic feet are there in the pile? Analyze. Solve without analyzing.

11. How many cubic feet are there in a block of granite 8 ft. long, 3 ft. wide, and 4 ft. high? Analyze. Solve without analyzing.

12. How many cubic feet are there in $\frac{1}{2}$ of a cord of wood? in $\frac{1}{4}$ of a cord?

13. The volume of a brick is 64 cu. in. It is 8 in. long and 2 in. thick. How wide is it?

14. Having the number of cubic units in a rectangular solid, and the number of linear units in its length and thickness, how can you find its width?

15. Having the volume of a rectangular solid and any two dimensions given, how can you find the other dimension?

Written

16. Measure, or estimate, the length, width, and height of your schoolroom. How many cubic feet of air does it contain? How many cubic feet of air are there for each pupil?

17. How many square feet of floor space are there for each pupil in the room?

18. A cement foundation for a machine is 3 ft. wide, 6 ft. long, and 4 ft. deep. How many cubic feet of cement are there in the foundation?

19. A bin 8 ft. long, 6 ft. wide, and 4 ft. high is filled with wheat. How many bushels of wheat does the bin contain?

Note. Count $1\frac{1}{4}$ cu. ft. to 1 bu.

20. In excavating for a cellar, 24 ft. long, 18 ft. wide, dirt is removed to a depth of 6 ft. How many cubic feet of dirt are taken out? how many cubic yards?

21. In a cellar 24 ft. long and 18 ft. wide, the concrete floor is 6 in. thick. How many cubic yards of concrete are there in the floor?

22. If the concrete in a 6-in. floor of a cellar 24 ft. long and 18 ft. wide is a mixture of sand and Portland cement in the proportion of 4 to 1, how many cubic yards of cement are used? Omit fraction in the final result.

23. 1 cu. ft. equals $7\frac{1}{2}$ gal., approximately. How many gallons of water will a tank hold that is 4 ft. long, 3 ft. wide, and $2\frac{1}{2}$ ft. deep?

24. The product of three numbers is 56. One of the numbers is 2, another is 4. What is the third number?

25. Having given the product of three numbers and two of the numbers, how can you find the third number?

26. A tank is to be constructed so as to have a capacity of 30 cu. ft. Its length is to be 4 ft. and its width 3 ft. What is its depth?

27. If the width and depth of the tank mentioned in problem 26 had been given, how would its length be found?

28. The capacity of a box is 27,648 cu. in. The box is 4 ft. long and 2 ft. wide. What is its depth?

In this problem, what is the unit of volume? What is the linear unit in the dimensions given? What is the unit of volume corresponding to this linear unit? Express the volume given in terms of the unit of volume corresponding to the linear unit. State the problem, giving the dimensions and the volume in terms of the new unit, and then solve.

29. In problem 28 what linear unit corresponds to the unit of volume given? Express the given dimensions in terms of this linear unit. State the problem, using the new linear units, and then solve it.

30. If the length, breadth, and thickness of a rectangular solid are given, how can you find its volume?

NOTE. The dimensions given must always have the same linear unit before you multiply.

31. The dimensions of a box are 2 ft. by 3 ft. by 18 in. What different linear units are used in expressing these dimensions? Give the three dimensions, using 1 ft. as the linear unit. Give the three dimensions, using 1 in. as the linear unit. If the dimensions are given in feet, what is the unit of volume? If the dimensions are given in inches, what is the unit of volume?

32. How many bushels of grain can be stored in an elevator $8' \times 10' \times 60'$?

33. A farmer plows to a depth of 6 in. How many cubic feet of soil does he stir up per acre by the plowing?

MASONRY AND BRICKWORK

Bricks are usually $8\frac{1}{4}'' \times 4'' \times 2\frac{1}{4}''$, but the size varies with locality.

In estimating amount of work, masons measure the length of walls on the outside, thus counting the corners twice. This is done on account of the greater labor of construction. Usually, except by special contract, no allowance is made for doors and windows.

In estimating amount of material, the corners are measured twice, but allowance for doors and windows must be made.

Brickwork is usually estimated by the **thousand.**

Stone work is estimated by the **perch** or the **cord.**

Found by Experience

7 bricks will cover 1 sq. ft. of surface in a 4 in. wall, — $\frac{1}{4}''$ joints.

3 bu. of lime equal 1 bbl.

1 cu. yd. of sand equals 1 load.

1 bbl. of lime and $\frac{1}{2}$ load of sand will lay 1000 bricks.

One mason will lay about 1600 bricks per day of 8 hr.

One helper is usually required to each mason. In some kinds of mason work one helper will serve two masons.

$24\frac{3}{4}$ cu. ft. of stone make 1 perch.

One cord of stone (128 cu. ft.) will lay 100 cu. ft.

$1\frac{1}{2}$ bbl. of lime and 1 load of sand will lay a cord of stone.

It requires 1600 laths to cover 100 sq. yd. of wall surface.

$2\frac{1}{2}$ bbl. of lime, 1 load of sand, and 2 bu. of hair will cover 100 sq. yd. of plastering.

One mason will lay a cord of stone per day of 8 hr.

Lime costs 35¢ per bushel or \$1.05 per barrel.

Sand costs \$1 per load of 1 cu. yd.

Masons' wages are \$5 per day of 8 hours.

Helpers' wages are \$2 per day of 8 hours.

A 4″ wall is 1 brick thick	A 21″ wall is 5 bricks thick
A 9″ wall is 2 bricks thick	A 25″ wall is 6 bricks thick
A 13″ wall is 3 bricks thick	A 29″ wall is 7 bricks thick
A 17″ wall is 4 bricks thick	A 34″ wall is 8 bricks thick

NOTE. These costs of materials and labor should be used as a basis in the following problems, although they vary in different parts of the country and at different times. Any part of a brick is counted as a whole brick.

PROBLEMS

WRITTEN

1. How many bricks would be required to build a 4″ wall, 15′ 6″ long and 7′ 8″ high? (7 bricks cover 1 sq. ft. of surface.)

2. If a wall 15′ 6″ long and 7′ 8″ high is 9″ or two bricks thick, how many bricks are required?

3. How many bricks are required for a wall 24′ long and 11′ 6″ high, if built 13″ thick? if built 17″ thick?

4. At 1600 bricks per day how long would it take one mason and a helper to build a 13″ wall 40′ long and 10′ high?

5. How many masons and helpers would be required to do the work in $2\frac{5}{8}$ days?

6. How much lime and sand to the nearest tenth of a barrel and load is required for a 13″ wall 40′ long and 10′ high?

7. If masons receive \$5.00 per day and helpers receive \$2.00 per day, what is the cost of labor for building a wall that requires $5\frac{1}{4}$ days of work for mason and helper?

8. If lime costs \$1.05 per barrel and sand \$1.00 per load, find the cost of the mortar for a 13″ wall 40′ long and 10′ high.

9. Estimating bricks to cost \$8.50 per M, and the labor ($5\frac{1}{4}$ days for mason and helper), sand, and the lime as in the table, find the total cost of building a 13″ wall 40′ long and 10′ high.

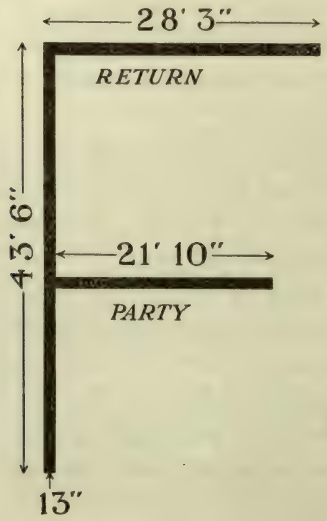
10. A 9″ wall, 31′ 4″ long, and 10′ 9″ high contains a door 3′ 6″ × 7′ 0″ and two windows each 2′ 10″ × 5′ 8″. How many bricks does it take to build the wall?

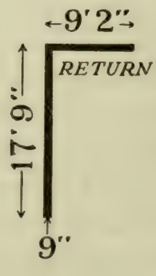
11. How many bricks are required to construct the walls of a building 35′ × 65′ × 28′ high, outside measurements, 2 bricks thick? The walls include 16 windows, 3′ × 6′, and 4 doors, $3\frac{1}{2}$′ × $6\frac{1}{2}$′.

NOTE. A 2-brick wall contains as many bricks as 2 4″ walls and measures $\frac{2}{3}$′ thick.

12. A wall 17′ 9″ long has a return 9′ 2″ long and is 11′ high. For how many bricks will the mason charge if the wall is 9″ in thickness?

13. Estimate the number of bricks required to build a wall 13″ thick, 43′ 6″ long, having a return 28′ 3″ long and also a party wall 21′ 10″ long, if all are 9′ 4″ high.





14. How many bricks are required to build a house 30′ 6″ × 64′ 6″, 34′ high, if the walls are 3 bricks thick and 450 sq. ft. are allowed for openings?

15. How much sand and lime, to the nearest tenth of a barrel and load, is required in the construction of the walls of a house that calls for 126,210 bricks? What is the cost, if lime is worth $.75 per barrel and sand $1.50 per load?

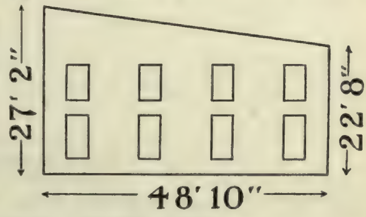
16. How many bricks are required to build a cesspool 12′ × 8′ × 16′, using 4″ walls?

17. How many bricks are required to build a cesspool 24′ × 16′ × 32′, using 4″ walls? Why is answer not twice that in the previous problem?

18. How many cords of stone are required to lay the foundation of the walls for a house 30′ × 65′, the walls to be 8′ 6″ high and 18″ thick? The outside dimensions are used, thus measuring the corners twice.

19. For how many perch would a mason charge for the foundation walls of a house 30′ × 65′, the walls to be 8′ 6″ high and 18″ thick? ($24\frac{3}{4}$ cu. ft. = 1 perch of stone.)

20. How many bricks will be required to build a 3-brick wall like the sketch, 48′ 10″ long, 27′ 2″ high on front end, and 22′ 8″ high at the rear, making allowance for 4 windows, 3′ 6″ × 8′ 6″, and 4 windows, 3′ 6″ × 6′?



NOTE. Find the average height.

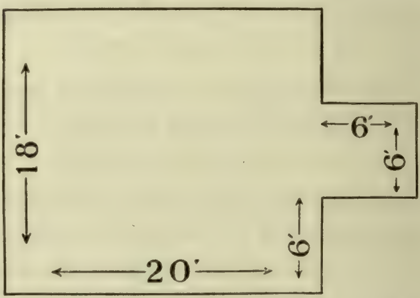
21. How many cords of stone are required to construct a retaining wall 5′ 8″ high and 24′ 6″ long, the wall being 2′ thick at base and 16″ thick at top?

22. How many cords of stone are required to build a reservoir $32' \times 58'$ and $16'$ deep (outside dimensions) the walls averaging $28''$ in thickness?

23. How many barrels of water would a reservoir contain whose inside dimensions are $20' \times 65' \times 35'$, allowing $7\frac{1}{2}$ gal. to 1 cu. ft.?

24. How many bricks are required to lay a sidewalk $303'$ long and $7\frac{1}{2}'$ wide, when the bricks are laid flatwise? when laid on edge?

NOTE. 39 bricks laid flat will cover 1 sq. yd. 71 bricks laid on edge will cover 1 sq. yd.



25. A contractor excavates a cellar 6 ft. deep having the accompanying outline and dimensions. How many cubic yards of material are taken out?

TANKS

1. With sheet lead $10'$ wide what is the shortest length piece that can be ordered to line an open tank $6'$ long, $3'$ wide, and $4'$ high with the least waste of lead? Make diagram.

2. Using sheet lead weighing $3\frac{1}{2}$ lb. per square foot, what is the weight of lead for above tank?

3. What are the cubical contents of such a tank in cubic feet?

4. Allowing $7\frac{1}{2}$ gal. of water to 1 cu. ft., what is the weight of the water when the tank is full? (1 gal. of water weighs $8\frac{1}{3}$ lb.)

5. How many gallons of water will the tank hold?

6. If a tank 4 ft. high is filled with water, what is the pressure per square inch at the ground line? A column of water 1 ft. high exerts a pressure of .433 lb. per square inch.

7. With a pressure of 4.33 lb. per square inch at ground line, what is the height of water in a tank?

Problems without Numbers

1. If you have the dimensions of a stone wall, its length and height in feet, and its thickness in inches, how can you find its volume in cubic feet?

2. What different linear units are used in expressing the dimensions? What must be done before finding the product of the three dimensions?

3. If the dimensions of a rectangular solid are given in yards and feet, what must be done in order to find the volume in cubic feet? Give an example to illustrate.

4. If the volume in cubic feet is known, how can you find the volume in cubic yards? Give an example.

5. If the volume in cubic yards is known, how can you find the volume in cubic feet? Give an example.

6. In finding the number of square feet in the surface of a rectangular floor, what dimensions should be given, and in terms of what linear unit? Give an example to illustrate.

7. If one dimension of a rectangular floor is given in feet and the other dimension in feet and inches, how can you find the area in square feet? Make and solve an example to illustrate.

8. If the area of a rectangular surface is given in square yards and one dimension is given in yards, how can you find the other dimension? What is its unit? Illustrate by an example.

REDUCTION OF DENOMINATE NUMBERS

Study Recitation

Reduction of denominate numbers is the process of changing the denomination without changing the value.

Reduction descending is changing to a number of lower denomination.

1. Reduce 8 gal. 3 qt. 1 pt. to pints.

$$\begin{array}{lll} \text{gal.} & \text{qt.} & \text{pt.} \\ 8 & 3 & 1 \\ \underline{4} & & \\ 32 & & \\ \underline{3} & & \\ 35 \text{ (qt.)} & & \\ \underline{2} & & \\ 70 & & \\ \underline{1} & & \\ 71 \text{ (pt.)} & & \end{array}$$

As 1 gal. = 4 qt., 8 gal. = 8 × 4 qt., or 32 qt. 32 qt. + 3 qt. = 35 qt. As 1 qt. = 2 pt., 35 qt. = 35 × 2 pt., or 70 pt; 70 pt. + 1 pt. = 71 pt.

In practice the work is done as shown at the left, since either factor may be used as multiplier if the numbers are regarded as abstract.

Multiply the given number of units of the highest denomination by the number of units of the next lower denomination it takes to make one of the highest denomination, and to the product add the number of units of the lower denomination, if any. Proceed in like manner with this result and each successive result thus obtained, until the number is reduced to the denomination required.

2. Reduce 11 rd. 4 yd. to yards.

$$\begin{array}{ll} \text{rd.} & \text{yd.} \\ 11 & 4 \\ \underline{5\frac{1}{2}} & \\ 5\frac{1}{2} & \\ \underline{55} & \\ 60\frac{1}{2} \text{ (yd.)} & \\ \underline{4} & \\ 64\frac{1}{2} \text{ (yd.)} & \end{array}$$

If it is required to reduce the $\frac{1}{2}$ yd. to lower denominations, it may be reduced by multiplication as though it were a whole number.

$$\begin{array}{l} \frac{1}{2} \\ \underline{3} \\ 1\frac{1}{2} \text{ (ft.)} \end{array}$$

$\frac{1}{2}$ ft. = $\frac{1}{2}$ × 12 in. = 6 in.

The entire result is, 64 yd. 1 ft. 6 in.

Reduce to the next lower unit, or to the lower of the two units mentioned:

	a.	*b.*	*c.*	*d.*	*e.*
3.	5 bu.	6 cwt.	5 score	7 yd.	5 cu. yd.
4.	4 pt.	10 lb.	10 gross	6 ft.	10 cu. ft.
5.	8 qt.	5 yr.	6 doz.	2 A.	5 cd.
6.	6 gal.	10 wk.	6 quires	12 sq. rd.	2 ft. 3 in.
7.	6 T.	10 da.	2 mi.	5 sq. yd.	2 hr. 20 min.
8.	2 L. T.	12 min.	4 rods	10 sq. ft.	12 sq. rd. 6 sq. yd.

9. How many sheets of paper are there in $2\frac{1}{2}$ quires?

10. If from a gross of pencils $7\frac{1}{2}$ doz. are sold, how many pencils are left?

11. A horse is 14 hands in height. What is the difference between his height and yours?

12. A rug is 2 yd. wide and 3 yd. long. How many square yards are there in the surface of the rug? how many square feet?

13. A grocer buys 5 bu. of apples at $1 per bushel and sells them at 30 ¢ per peck. How much does he gain?

Written

Change the numbers in column *a* to the denominations in column *b*:

	a. Number	*b.* Denomination		*a.* Number	*b.* Denomination
14.	25 da. 6 hr.	hours	**20.**	12 hr. 35 min. 18 sec.	seconds
15.	2 hr. 30 min.	minutes	**21.**	15 lb. 8 oz.	ounces
16.	15 min. 20 sec.	seconds	**22.**	7 lb. 8 oz.	ounces
17.	52 wk. 1 da.	days	**23.**	8 T. 750 lb.	pounds
18.	24 wk. 6 da.	days	**24.**	11 da. 14 hr.	hours
19.	5 da. 15 hr. 20 min.	minutes	**25.**	4 T. 500 lb.	pounds

	a. Number	b. Denomination		a. Number	b. Denomination
26.	18 ft. 6 in.	inches	**32.**	320 rd. 3 yd.	yards
27.	20 yd. 2 ft.	feet	**33.**	12 rd. 8 ft.	feet
28.	16 rd. 2 yd.	yards	**34.**	24 rd. 12 ft.	feet
29.	28 rd. 5 yd.	yards	**35.**	7 rd. 10 ft.	feet
30.	9 rd. 2 yd.	yards, feet, inches	**36.**	27 rd. 8 ft.	feet
31.	11 rd. 4 yd.	yards, feet, inches	**37.**	9 rd. 7 ft.	feet, inches

Reduction ascending is changing to a number of higher denomination.

Study Recitation

1. Reduce 277 pt. to higher units.

```
2)277 (pt.)
8)138 (qt.) 1 (pt.)
4) 17 (pk.) 2 (qt.)
    4 (bu.) 1 (pk.)
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Since 2 pt. = 1 qt., 277 pt. = 138 qt. 1 pt.
Since 8 qt. = 1 pk., 138 qt. = 17 pk. 2 qt.
Since 4 pk. = 1 bu., 17 pk. = 4 bu. 1 pk.
The last quotient and the several remainders give the answer, 4 bu. 1 pk. 2 qt. 1 pt.

Divide the given number by the number of units it takes of its denomination to make one unit of the next higher denomination, and place the remainder, if any, at the right. Divide the quotient thus obtained and each succeeding quotient in the same manner.

The last quotient, with the several remainders annexed in proper order, will be the answer required.

Reduce to the next higher unit:

	a.	b.	c.	d.
2.	8 pt.	32 oz.	21 da.	180 sec.
3.	16 dry quarts	400 lb.	48 hr.	2 score
4.	8 bu.	4000 lb.	24 mo.	24 doz.
5.	16 liquid quarts	4480 lb.	180 min.	60 quires

6. Reduce 28 in. to feet and inches.

7. Reduce 22 ft. to yards and feet.

8. Reduce 36 qt. to pecks and quarts.

9. Reduce 130 min. to hours and minutes.

10. A piece of board 8 ft. long is cut into 6 in. pieces for an exercise in a manual training class. How many such pieces are cut from the board?

11. A box 6 ft. long, 3 ft. wide, and 8 ft. high, inside measure, is filled with sand. How many cubic feet of sand does it hold?

12. A floor is 12 ft. by 15 ft. How many square yards of carpet are required to cover it?

13. A piece of land is 16 rd. wide and 20 rd. long. How many acres are there in the piece?

14. A room is 12 ft. long, 10 ft. wide, and 9 ft. high. How many square yards are there in the four walls?

15. A farmer sells 5000 lb. of hay at $16 per ton. How much does he receive for the hay?

16. What is the cost of 24 qt. of gasoline at 20¢ per gallon?

17. What is the value of 300 lb. of wheat at 90¢ per bushel?

Written

18. In 57 pt., how many gallons and pints are there?

19. In 235 in., how many yards, feet, and inches are there?

20. In 2475 min., how many days, hours, and minutes are there?

21. Reduce 475 cu. ft. to cubic yards and cubic feet.

22. Reduce 240 qt. to bushels and pecks.

23. Reduce 168 ft. to rods and yards.

24. Reduce 20,460 ft. to miles and rods.

25. Reduce 2560 lb. to hundredweight and pounds.

26. Reduce 75 oz. to pounds and ounces.

27. In 778 pt. how many bushels are there?

28. A farmer's load of hay weighs 2430 lb. How much does he receive for it at $16 per ton?

29. A coal merchant delivers a load of coal weighing 4300 lb. How much is the load worth at $8 per ton?

30. A pile of wood 44 ft. long, 6 ft. high, and 4 ft. wide, is sold for $6 a cord. How much is paid for it?

31. A field 80 rd. long and 60 rd. wide yields 15 bu. of wheat per acre. If the wheat is sold at $1 a bushel, what is its value?

32. What is the cost of painting one side of a tight board fence 60 ft. long and 6 ft. high, if two coats of paint are put on, at a cost of 18¢ per square yard?

33. What is the cost of tinting the walls of a room $13\frac{1}{2}$ ft. by 18 ft., if the room is 9 ft. high, at a cost of 7¢ per square yard?

34. What is the cost of tinting the ceiling of the same room, at 7¢ per square yard?

35. The two sides of a roof are to be shingled. Each side is 15 ft. wide and 40 ft. long. If it takes 900 shingles to cover one square (100 sq. ft.), how many shingles are required to cover the roof?

A denominate fraction expressed either as a **common fraction** or as a **decimal** is reduced to lower denominations in the same way as integral denominate numbers are reduced, viz. by multiplication.

Oral and Written

1. What part of a peck is $\frac{5}{12}$ of a bushel? Or, reduce $\frac{5}{12}$ of a bushel to pecks.

NOTE. Any number of bushels is reduced to pecks by multiplying by 4. Any fraction of a bushel is reduced to pecks in the same way.

SOLUTION. $\frac{5}{12} \times 4 = \frac{5}{3} = 1\frac{2}{3}$. The answer is $1\frac{2}{3}$ pk.

2. Reduce $\frac{2}{3}$ of a peck to quarts. How are pecks reduced to quarts? What is the result?

3. .75 of a foot equals how many inches? How are feet reduced to inches? Apply the process.

4. Reduce $\frac{7}{8}$ of a gallon to lower denominations.

SOLUTION. $\frac{7}{8} \times 4 = 3\frac{1}{2}$, the number of quarts. $\frac{1}{2}$ qt. reduced to pints $= \frac{1}{2} \times 2 = 1$, the number of pints. The answer is 3 qt. 1 pt.

5. Reduce $\frac{4}{5}$ of a yard to lower denominations.

SOLUTION. $\frac{4}{5} \times 3 = 2\frac{2}{5}$, the number of feet. $\frac{2}{5} \times 12 = 4\frac{4}{5}$, the number of inches. *Ans.* 2 ft. $4\frac{4}{5}$ in.

6. Reduce $\frac{7}{8}$ of a day to hours.

7. In $\frac{3}{4}$ of a cord, how many cubic feet are there?

8. Reduce $\frac{1}{8}$ of a barrel to lower denominations.

9. Reduce $\frac{7}{8}$ of a yard to lower denominations.

10. Reduce $\frac{3}{4}$ of a gallon to quarts.

11. Reduce .7 of a pound to ounces.

$$\begin{array}{r} 16 \\ .7 \\ \hline 11.2 \text{ (oz.)} \end{array}$$

In .7 of a pound there are .7 of 16 oz., or 11.2 oz.

12. What decimal of an hour is .07 of a day? .15 of a day?

13. What decimal of a quart is .05 of a peck? .45 of a peck?

14. Reduce $\frac{3}{4}$ of a quart to the fraction of a gallon. How are quarts reduced to gallons?

SOLUTION. $\frac{3}{4} \div 4 = \frac{3}{16}$, the fraction of a gallon.

15. What part of a gallon is $\frac{3}{5}$ of a pint?

SOLUTION. 1 gal. = 8 pt. $\frac{3}{5} \div 8 = \frac{3}{40}$. Hence, $\frac{3}{5}$ pt. = $\frac{3}{40}$ gal.

16. How can you reduce days to weeks? What decimal of a week is .21 of a day?

17. 30.8 rd. is what decimal of a mile?

18. What part of a gallon is 2 qt. 1 pt.?

SOLUTION

2 qt. 1 pt. = 5 pt.
1 gal. = 8 pt.

The question now becomes, 5 pt. is what part of 8 pt.? *Ans.* $\frac{5}{8}$.

19. What part of a day is 12 hr. 30 min.?

SUGGESTION

12 hr. 30 min. = how many minutes?
1 day = how many minutes?
How may the question now be stated?

20. What part of 2 bu. is 2 pk. 2 qt.?

21. What part of a ton of hay is 1800 lb.? what decimal?

Reduce first to a fraction; then to a decimal of the next higher unit:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
22.	7 oz.	240 rd.	1900 lb.	2 ft.	40 min.	5 in.
23.	120 rd.	192 rd.	1600 lb.	8 hr.	40 sec.	750 lb.

ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

Study Recitation

In compound numbers, as in simple numbers, the numbers to be added must be of the *same denomination.*

In adding simple numbers, the sum of the numbers of any denomination is divided by ten, the quotient is added to the next higher denomination, and the remainder is written under the numbers added. In adding compound numbers, the sum of the numbers of any denomination is divided by the number of units of that denomination it takes to make one of the next higher denomination; the remainder, if any, is written under the numbers added, as in simple numbers, and the quotient is added to the number of the next higher denomination, as in simple numbers.

Add:

1.

wk.	da.	hr.
7	6	12
5	4	13
3	5	10
17	2	11

Adding the hours gives the sum 35 hr. Dividing 35, the number of hours, by 24, to reduce to days, gives 1 da. and the remainder 11 hr. Write the remainder, 11, under the hours added. Add the 1 da. to the numbers representing days; the sum is 16 da. Dividing 16, the number of days, by 7, to reduce to weeks, gives 2 wk., with 2 da. remainder. Write the 2 under the days added. Add the 2 wk. to the numbers representing weeks; the sum is 17 wk. The entire sum is 17 wk. 2 da. 11 hr.

2.

rd.	yd.	ft.	in.
8	5		
3	2		
4	3		
16	4	1	6

Adding the yards, gives the sum, 10 yd. Dividing 10, the number of yards, by $5\frac{1}{2}$, to reduce to rods, gives 1 rd., with $4\frac{1}{2}$ yd. remainder. Write the $4\frac{1}{2}$ yd. under the numbers added. Add the 1 rd. to the rods; the sum is 16 rd. The entire sum is 16 rd. $4\frac{1}{2}$ yd.

The $\frac{1}{2}$ yd. may be reduced to lower denominations by multiplication. $\frac{1}{2}$ yd. $= 1\frac{1}{2}$ ft. $\frac{1}{2}$ ft. $= 6$ in. Instead of writing $4\frac{1}{2}$ yd., the sum may be written 4 yd. 1 ft. 6 in., the entire sum being 16 rd. 4 yd. 1 ft. 6 in.

Add:

	rd.	ft.
3.	5	14
	2	13
	8	10
	17	4

Adding the feet gives the sum, 37 ft. Divide 37, the number of feet, by $16\frac{1}{2}$, the number of feet in a rod, by reducing $16\frac{1}{2}$ to $\frac{33}{2}$ and 37 to $\frac{74}{2}$. Dividing $\frac{74}{2}$ the number of feet, by $\frac{33}{2}$ gives 2, the number of rods, and $\frac{8}{2}$ ft., or 4 ft., remaining. Write the 4 ft. under the number of feet added, and add the 2 rd. to the rods. The entire sum is 17 rd. 4 ft.

		bu.	pk.	qt.
4.	From	8	2	5
	take	5	3	7
		2	2	6

7 qt. cannot be taken from 5 qt. Take 1 pk. from the 2 pk. and reduce it to 8 quarts. 8 qt. + 5 qt. = 13 qt. 7 qt. from 13 qt. = 6 qt. 3 pk. cannot be taken from the 1 pk. remaining. Take 1 bu. from 8 bu. and reduce it to 4 pk. 4 pk. + 1 pk. = 5 pk. 5 pk. – 3 pk. = 2 pk. 5 bu. from the 7 bu. remaining equals 2 bu. The entire answer is 2 bu. 2 pk. 6 qt.

Written

Add:

5.

mi.	rd.	yd.
2	120	5
2	250	3
1	17	1

6.

mi.	rd.	ft.
1	180	15
2	120	13
3	240	9

7.

bu.	pk.	qt.
8	3	5
25	2	7
16	3	6

8.

T.	lb.
8	420
3	965
4	868

9.

hr.	min.
16	45
12	30
20	12

10.

sq. yd.	sq. ft.
3	7
4	5
7	8

11.

cu. ft.	cu. in.
8	1200
15	1440
19	758

Prove the correctness of answer in each case:

12.

	hr.	min.
From	13	15
take	8	50

13.

	mi.	rd.
From	1	180
take		260

14.

	doz.	units
From	$8\frac{1}{2}$	
take	4	3

15.

	bu.	pk.	qt.
From	9	3	2
take	4	2	6

16.

	gal.	qt.
From	8	2
take	2	3

17.

	yd.	ft.	in.
From	9	2	7
take	2	2	9

Finding the difference between dates.

In finding the difference between two dates, treat the number of the year as so many years, the number of the month as so many months, and the day of the month as so many days. In subtracting, count 30 days to the month and 12 months to the year.

1. Find the difference in time between Sept. 8, 1914, and Nov. 21, 1909.

yr.	mo.	da.
1914	9	8
1909	11	21
4	9	17

The later date is written as 1914 yr. 9 mo. 8 da. The earlier date is written as 1909 yr. 11 mo. 21 da. The subtraction is performed in the same way as in other cases of subtraction in compound numbers.

2. A man who was born Nov. 3, 1844, died Jan. 18, 1914. At what age did he die?

3. If you have a brother who is 4 yr. 6 mo. 10 da. older than you are, what was the date of his birth?

4. A man gave a note Aug. 10, 1904, and paid it June 7, 1906. How long did the note run?

Subtract each date from the date above it:

5. 1914, May 12
1913, Sept. 16

6. 1912, Nov. 4
1911, Dec. 5

7. 1911, Aug. 6
1909, Dec. 9

8. 1914, Jan. 4
1908, April 6

9. 1914, Jan. 1
1906, May 8

10. 1913, Feb. 2
1905, Oct. 5

MULTIPLICATION AND DIVISION OF DENOMINATE NUMBERS

Study Recitation

1. Multiply 3 ft. 8 in. by 4.

ft.	in.
3	8
	4
14	8

4×8 in. = 32 in. = 2 ft. 8 in. Write the 8 in. under the denomination inch, and add the 2 ft. to the product of 4×3 ft. Write the sum, 14 ft., under the denomination feet. The product is 14 ft. 8 in.

2. Multiply 2 rd. 12 ft. 4 in. by 4.

rd.	ft.	in.
2	12	4
		4
10	16	4

4×4 in. = 16 in. = 1 ft. 4 in. Write the 4 in. under inches, and add the 1 ft. to the product of 4×12 ft. The sum is 49 ft. Divide 49 ft. by $16\frac{1}{2}$, the number of feet in a rod, by reducing both to half feet.

$16\frac{1}{2}$ ft. = $\frac{33}{2}$ ft. 49 ft. = $\frac{98}{2}$ ft.

$\frac{33}{2}$ ft. is contained in $\frac{98}{2}$ ft. 2 times and $\frac{32}{2}$ ft. remaining. $\frac{32}{2}$ ft. = 16 ft. Write the remainder, 16 ft., under the denomination feet, and add 2 rd. to the product of 4×2 rd. The sum is 10 rd. The entire product is 10 rd. 16 ft. 4 in.

3. Multiply 7 rd. 11 ft. 6 in. by 15.

4. What is the product of 25 bu. 3 pk. by 14?

5. Divide 15 bu. 3 pk. by 4.

	bu.	pk.	qt.
4)	15	3	
	3	3	6

$\frac{1}{4}$ of 15 bu. = 3 bu., with 3 bu. remaining undivided. 3 bu. = 12 pk. 12 pk. + 3 pk. = 15 pk. $\frac{1}{4}$ of 15 pk. = 3 pk., with 3 pk. remaining undivided. 3 pk. = 24 qt. $\frac{1}{4}$ of 24 qt. = 6 qt. The quotient is 3 bu. 3 pk. 6 qt.

Written

6. Divide 115 rd. 2 yd. 1 ft. 6 in. by 15.

7. A man has a farm of 160 A. 100 sq. rd., which he wishes to divide into 6 equal lots. What is the area of each lot?

8. A man walks 3 mi. 70 rd. the first hour, 4 mi. 10 rd. the second hour, and 3 mi. 50 rd. the third hour. What is the average distance that he walks each hour?

9. How many pieces of board 1 ft. 3 in. long can be sawed from a board 16 ft. 6 in. long, if $1\frac{1}{2}$ in. are wasted in sawing? How much of the board will be left? Reduce both dimensions to inches and divide.

10. If $12\frac{1}{2}$ lb. of spices are put into 8 packages of equal size, what is the weight of each package?

11. Make a rule for multiplication of compound denominate numbers; for division of compound denominate numbers.

To the Teacher. These exercises need not be given unless further practice is deemed necessary.

Multiply or divide as indicated:

12. 6 × 1 pk. 3 qt.	**26.** 54 mi. 222 rd. 3 yd. ÷ 9.
13. 5 × 2 bu. 3 pk.	**27.** 93 gal. ÷ 8.
14. 12 × 2 lb. 5 oz.	**28.** 349 bu. 4 qt. ÷ 14.
15. 10 × 1 T. 250 lb.	**29.** 1 mi. 122 rd. 1 yd. ÷ 12.
16. 24 × 5 yd. 2 ft.	**30.** 3 gal. 1 pt. ÷ 5.
17. 3 × 5 yd. 1 ft. 6 in.	**31.** 12 hr. 30 min. ÷ 10.
18. 4 × 16 ft. 6 in.	**32.** 5 mi. 240 rd. ÷ 8.
19. 12 × 4 rd. 4 ft.	**33.** 14 ft. 8 in. ÷ 11.
20. 15 × 3 qt. 1 pt.	**34.** 10 yd. 2 ft. ÷ 8.
21. 6 × 4 gal. 3 qt.	**35.** 2 T. 1600 lb. ÷ 4.
22. 8 × 2 mi. 180 rd.	**36.** 3 cu. yd. 3 cu. ft. ÷ 6.
23. 4 × 3 sq. yd. 7 sq. ft.	**37.** 6 rd. 9 ft. ÷ 12.
24. 8 × 1 cu. yd. 5 cu. ft.	**38.** 8 ft. 4 in. ÷ 2 ft. 1 in.
25. 5 × 4 hr. 40 min.	**39.** 1 bbl. 3 qt. ÷ 10 gal. 3 qt.

In examples 38, 39, reduce the dividend and divisor to the same denomination before dividing.

PROBLEMS

Written

1. If a locomotive consumes an average of 93 lb. 6 oz. of coal per mile, how many tons will it consume on a run of 168 mi.?

2. If each of a span of horses eats 1 pk. 6 qt. of oats a day, and oats are worth 48¢ per bushel, what is the cost of oats fed to a span of horses during the month of December?

3. From an alfalfa field of 15 acres, 3 crops were harvested in one season. The average yield per acre for each crop was 2850 lb. How much was the season's yield worth at $14.50 per ton?

4. A side of bacon weighs 8 lb. 12 oz. A ham weighs 13 lb. 4 oz. What is the cost of both, if bacon is worth 24¢ per pound, and ham, 20¢ per pound?

5. If 1000 shingles are required to cover 100 sq. ft. of roof, how many shingles are required to shingle the two sides of a roof if each side is 40′ by 18′?

6. If shingles are bought in bunches, each bunch containing 250 shingles, how many bunches must be bought for the roof in problem 5? How much are they worth at $3.50 per M?

7. A man owned 32 acres of land. He bought another tract 80 rd. long by 60 rd. wide. He then sold 15 A. 60 sq. rd. How much land had he left?

8. If a ton of timothy hay occupies 500 cu. ft. in a mow, how many tons will a mow hold whose dimensions are 40′ × 18′ × 15′?

9. A dealer buys ½ T. of cheese at $250 per ton. If he sells it at an average price of 16½¢ per pound, what is his gain?

10. A housekeeper can buy ice at 50 ¢ per C, paying at the end of each month. Paying in advance, she can buy a book of tickets entitling her to 1000 lb. for \$4. If she buys two books at the same time, they cost her \$7. If she uses $2\frac{1}{2}$ T. of ice during the year, what is the cost by each plan of payment?

11. A farmer sold a load of corn weighing 2748 lb., and a load of wheat weighing 2560 lb. For the wheat he received \$.76 a bushel, and for the corn \$.55 a bushel. How much did he receive for both?

12. An ocean steamer during a voyage of 3025 mi. burns coal at the rate of 1 T. 95 lb. for every 25 mi. How many tons of coal are used up in the voyage?

13. What is the cost of the following coal bill, at \$7.50 per ton: 3650 lb., 2720 lb., 5000 lb., 1960 lb., 2750 lb.?

14. A machinist ships twelve machines weighing 3 T. 750 lb. each. How much does he pay for freight at 22 ¢ per hundredweight?

15. A milkman sold 25 gal. 3 qt. of cream on Monday, 20 gal. 2 qt. on Tuesday, and 21 gal. 1 qt. on Wednesday, at \$1.20 per gallon. How much did he receive for the cream?

16. For each of the remaining 4 days of the week, his sales averaged the same as for each of the first 3 days. How many gallons did he sell during the 4 days?

17. A milkman owns a cow that gives an average of 4 gal. 3 qt. of milk daily during the month of June. How much does he receive for the milk if it is sold for \$.07 per quart?

18. A mail carrier travels 92 mi. 80 rd. in delivering mail for 1 week of 6 days. If he makes 2 deliveries daily, how far does he travel in making each delivery?

19. A bicycle wheel 7 ft. 4 in. in circumference makes 720 revolutions in 2 minutes. At what rate per hour does the wheel revolve?

NOTE. The *circumference* is the distance around the wheel.

20. A dealer sold 420 T. 1250 lb. of coal at $5.50 per ton, and a number of tons of other kinds at $7.50 per ton. In all, he received $4780. How many tons of the second kind did he sell?

21. How many steps of 33 inches each must be taken in walking 4 rods?

22. How many steps of 30 in. each must be taken in walking 2 mi. 300 rd.?

23. How many times will a bicycle wheel 88 in. in circumference turn in going one mile?

24. How many times will a bicycle wheel 88 in. in circumference turn in going $12\frac{3}{8}$ miles?

25. How many strips of carpet 30 in. wide are required to cover a floor 20 ft. × 25 ft., if the strips run lengthwise? crosswise?

26. How many strips of carpet 30 in. wide are required to cover a floor 18 ft. × 22 ft., if the strips run lengthwise? crosswise?

27. In carpeting the floor in problem 26, if the strips are laid lengthwise, when seven strips have been laid, what is the width of the portion of the floor not carpeted?

In buying carpet what must be bought to cover the remaining portion of the floor? Draw a figure on the board representing the floor on the scale of 1 inch to 1 foot. Draw lines showing the lines of joining of the strips.

28. How many square rods are there in a piece of land 24 rd. long and 12 rd. wide?

29. In 288 sq. rd., how many acres are there?

30. What is the cost of making a macadam road 1 mi. long and 12 ft. wide, at 23¢ per square yard?

31. How much would it cost to dig a cellar 18 ft. long, 15 ft. wide, and 6 ft. deep, at 50¢ per cubic yard?

32. One quart, dry measure, contains $67\frac{1}{5}$ cu. in. One quart, liquid measure, contains $57\frac{3}{4}$ cu. in. One bushel equals 32 qt., dry measure. How many more cubic inches are there in 1 bu. than in 8 gal.?

33. How much can a man earn in 5 days digging a ditch, at the rate of $12\frac{1}{2}$¢ per running foot, if he digs 4 rd. per day?

34. A man wishes to pack 42 bu. of apples in barrels. If each barrel contains 3 bu. 2 pk., how many barrels will it take?

35. If you study 45 min. at home each day your school is in session, how many hours of home study will you do during the school year?

36. If you devote 15 min. a day, each day in the year, to reading good literature, how many hours' reading will you do in 1 year? Omit fractions. See how many pages of a book worth reading you can read in 1 hr., and find the number of pages you can read in 1 yr., reading 15 min. a day.

37. How many feet of wire are required for a 4-wire fence around a field 30 rd. 12 ft. long by 20 rd. 8 ft. in width?

38. Sound travels at the rate of 1120 ft. per second. If the time between a flash of lightning and the sound of thunder is 12 sec., how far distant is the storm? There are 5280 ft. in a mile. Give answer in miles and feet.

39. A coal dealer delivers 4 loads of coal to a customer. The loads weigh, respectively, 2 T. 315 lb., 2 T. 340 lb., 1 T. 1950 lb., 2 T. 325 lb. What is the value of the coal at $7 per ton?

40. How many acres are there in a farm 134 rd. long and 52.5 rd. wide?

41. From a lot containing 15 acres of land, a man sold 1400 sq. rd. How many acres did he have left?

42. Draw a rectangle 6 in. by 8 in. to represent a flower bed 12 ft. by 16 ft. Draw lines to represent a walk 2 ft. wide around the bed. What is the length of the walk on two sides of the bed? What is the length of the remaining parts of the walk on the two ends?

43. How many square feet are there in the flower bed mentioned in problem 42? in the walk?

44. A lot 120 ft. long and 80 ft. wide is surrounded by a sidewalk 8 ft. wide. What will be the cost of the sidewalk at $1.50 per square yard?

45. Draw a rectangle on a scale of 1 in. to 8 ft. to represent the lot. Add lines to show the outer boundary of the sidewalk. Add four lines dividing the sidewalk into four rectangles.

46. A door 7 ft. high and 3 ft. wide has a 6 in. casing around it. How many square inches are there in the surface of the casing?

47. Make a drawing on a scale of 1 in. to 1 ft. to represent the door and casing whose dimensions are given in problem 46. If the stock for the casing is cut from two boards 8″ wide, one 10′ long, and the other 12′ long, how many board feet of 2″ lumber are required? How many board feet are wasted?

48. If a rug 12 ft. long and 10 ft. wide is laid on a floor so as to leave a margin of 2 ft. in width all around the rug, what is the area of the floor? What is the area of the part not covered by the rug? Draw a plan of the floor and the rug on a scale of 1 in. to 1 ft.

49. From a lot of land 80 rd. square, 80 sq. rd. were sold. What was the value of the remainder at $50 per acre?

50. A pasteboard box is 8 in. long, 6 in. wide, and 4 in. high. How many square inches of pasteboard will it take to make the sides of the box? to make the ends? to make the top and bottom? How much more than a square foot of pasteboard will be required?

51. A box is 8 in. long, 6 in. wide, and 4 in. high. What is the length and the width of one piece of paper that will just cover the sides and the ends of the box? Draw a rectangle on a scale of 1 in. to 8 in., to represent the piece of paper. Divide the figure by three lines drawn across it, making four rectangles to represent the sides and ends of the box.

52. Draw a rectangle on the same scale to represent the top and the bottom of the box. Write the correct dimensions on each rectangle drawn. Find the area of the paper necessary to cover the box.

53. A room is 18 ft. long, 15 ft. wide, and 9 ft. high. How many square yards are there in the walls, floor, and ceiling of the room?

54. Draw a rectangle on a scale of 1 in. to 6 ft. to represent the four walls of the room described in problem 53, if placed end to end. Draw another rectangle to represent the floor and the ceiling.

55. A city lot is 30 ft. front by 60 ft. deep. If it sells at $5 per square foot, how much is the lot worth? If it sells at $75 per front foot, how much is it worth?

56. A garden 40 ft. by 60 ft. has a gravel walk 4 ft. wide around it. How many square feet are there in the walk? Illustrate by a diagram.

57. A room is 15 ft. by 18 ft. How many square yards of plastering are there in the ceiling? How many yards of carpeting $\frac{3}{4}$ of a yard wide will it take to cover the floor? Which way should the carpet be laid?

58. A wall is 20 ft. long and 10 ft. 6 in. high, with an 18-in. baseboard. How much will it cost to plaster the wall at 12¢ a square yard? If this wall is one side of a room 16 ft. wide, how much will it cost to plaster the four walls and the ceiling at the same rate, making no allowance for doors or windows?

59. How many tiles 6 in. by 4 in. will it take to cover the floor of a hall 8 ft. by 6 ft.?

60. How much glass is there in 12 windows of 8 panes each, if each pane contains 1 sq. ft. 48 sq. in.?

61. How many feet of lumber are there in ten boards 14′ long, 6″ wide, and 1″ thick?

62. How many feet of lumber are there in nine joists 18′ long, 8″ wide, and 2″ thick?

63. Find the cost at \$24 per M of 17 boards 16′ long, 10″ wide, and 1″ thick; of 8 scantlings 14′ long, 4″ wide, and 2″ thick; and of 18 joists 12′ long, 6″ wide, and $2\frac{1}{2}$″ thick. Put this in the form of a receipted bill.

64. How much will it cost for the flooring for a floor 21 ft. by 16 ft. at \$25 per M, allowing nothing for waste?

The flooring is supposed to be 6 in. in width, and the strips are tongued and grooved.

If the strips are tongued and grooved, the number of board feet of lumber required is $1\frac{1}{12}$ times the number of square feet in the surface of the floor.

65. If a thousand shingles are needed to cover 100 sq. ft. of roof, how many thousand shingles are required to cover a gable roof 50 ft. long, each of the two sides being $22\frac{1}{2}$ ft. wide? What will be the cost of the shingles at $4 per M?

66. Measure a door in the schoolroom and the width of its casing, and find the number of square feet in the surface of the door and in the surface of the casing.

67. What will be the cost of cork matting to cover the floor of your schoolroom at 90¢ per square yard?

68. A room 20 ft. by 18 ft. is to be covered with carpet 27 in. wide, worth $1.50 per yard. After the first strip, it is necessary to cut off 9 in. from each of the remaining strips in order to match the figures. How much will it cost to carpet the room?

69. How many square feet are there in a floor 14 ft. 7 in. wide and 19 ft. 4 in. long?

70. At $12\frac{1}{2}$¢ a square yard, how much will it cost to sod a lawn 40 ft. long by 36 ft. wide?

71. A string 40 ft. long was used to outline a square flower bed, the string reaching entirely around it. What were the dimensions of the bed? What was its area? If the bed had the same area and was 20 ft. long, what length of string would be required to go around it? Draw figures on a scale of 1 in. to 10 ft. for both beds.

72. The distance between two cities is 55 miles. How many times will an engine wheel 18 ft. 6 in. in circumference turn in making the trip?

73. From a cask of oil containing 33 gal., 18 gal. 3 qt. were drawn. How much remained?

74. How many baskets, each holding 3 pk., can be filled with 12 bu. of apples?

75. How many ties 8 in. wide will be laid on $20\frac{2}{3}$ ft. of a railroad, if the ties are laid 2 ft. from center to center? Draw a figure on a scale of 1 in. to 2 ft. showing that your answer is correct. How many ties will there be on $24\frac{2}{3}$ ft. of the road?

76. At $.40 per tie, how much will it cost for 2640 railroad ties for one mile of a road?

77. How many barrels, each holding 2 bu. 3 pk., will it take to hold 100 bu. of apples? How many pecks will there be in the partly filled barrel?

78. If it requires 12 qt. of oats per day to feed a horse, how many days will 6 bu. 3 pk. last a span of horses?

79. If a train travels 85 miles in 2 hr. 30 min., what is its rate per hour?

80. At $7 a cord, what is the cost of a load of wood 12 ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. high?

81. A farmer sells 3 loads of hay, each containing 1785 lb. If he sells it at $7 per ton, how much does he receive for it?

82. If 12 equal bins hold 430 bu. 2 pk. of wheat, how much wheat is there in each bin?

83. How many bushels of wheat are there in 12 bins, each containing 124 bu. 3 pk. 5 qt.?

84. Reduce 2 pk. 4 qt. to the decimal of a bushel.

85. If 9 bu. 2 pk. of potatoes cost $3.80, what is the average cost per bushel? per peck?

86. If a school uses 24 crayons a week, it will use how many gross of crayons in 36 weeks?

87. If a dealer buys pencils at $3.50 per gross and sells them at 5¢ apiece, how much does he gain on each gross? on 30 dozen?

88. A tank 2 ft. wide, 4 ft. high, and 6 ft. long will contain how many gallons? how many bushels?

89. A cube is 4 in. square. If all the sides are covered with paper, how many square inches of paper are required?

90. How many cubic yards of earth will be taken out in digging a cellar 20 ft. long, 12 ft. wide, and 5 ft. 4 in. deep?

91. How many bushels of oats does a bin 8 ft. by 6 ft. by 5 ft. contain? How much do they weigh?

92. During a heavy rain, 2 in. of water fell. How many gallons of water fell on a garden 60 ft. by 80 ft.?

93. If a schoolroom is 24 ft. wide, 36 ft. long, and 10 ft. high, how many cubic feet of space will there be to each of 40 pupils?

94. How many feet of boards are required for a coal bin in the corner of a cellar, so constructed that the walls of the cellar form two sides of the bin, and the floor of the cellar the bottom of the bin, the bin being 12 ft. long, 6 ft. wide, and 8 ft. high? (Boards 1 in. thick.)

95. A cask holds 52 gallons of water. The contents will fill how many bottles, if it requires 5 bottles to hold one gallon of water?

96. Sound travels 1120 ft. per second. A hunter sees the flash of a gun at a distance, and 4 seconds later hears the report. What fraction of a mile distant is he from the gun?

97. When shingles are laid 4 in. to the weather, how many shingles will cover the two sides of a roof, each side being 40 ft. by 20 ft.?

NOTE. Shingles are 16 in. long and of different widths, but in reckoning, each width of 4 in. is considered one shingle, and a bundle of shingles is considered as made up of 250 shingles 16 in. long and 4 in. wide. If a shingle is laid 4 in. to the weather, then each shingle will cover 16 sq. in. of space.

98. If a shingle is laid $\frac{1}{4}$ to the weather, how many shingles does it take to cover a square foot? When the length and width of a roof are given, and shingles are laid $\frac{1}{4}$ to the weather, what is the process of finding the number of shingles required to cover the roof? How can you find the number of bundles of shingles required to cover the roof?

NOTE. To the exact number of shingles required to cover 1 square, or 100 sq. ft., about 100 are added to allow for waste in laying.

REVIEW

1. State the difference between a simple denominate number and a compound denominate number. How may the latter be changed to the former?

2. Define quantity; measure. What is meant by measuring a quantity?

3. Define surface; angle; right angle. Give an example of each.

4. What is the difference between a square and a rectangle?

5. How many dimensions has a line? a square? a rectangular solid?

6. What is a cube? a rectangular solid? Give an example of a rectangular solid that is not a cube.

7. 2 in. is what fraction of 1 ft.? what decimal of 1 ft.?

8. 3 qt. is what fraction of 1 gal.? what decimal of 1 gal.?

9. 25 min. is what fraction of 1 hr.? what decimal of 1 hr.?

10. Give each of the tables of denominate numbers, its use, and the abbreviation used for each denominate unit.

Problems without Numbers

11. How can you find the area of a rectangle when its length and breadth are known? Illustrate by an example.

12. When one dimension and the area of a rectangle are given, how can you find the other dimension? Illustrate by an example.

13. How can you find the volume of any rectangular solid? Illustrate by an example.

14. Having two dimensions and the volume of a rectangular solid, how can you find the other dimension? Illustrate by an example.

15. The dimensions of a square, A, are one half the dimensions of a square, B; the surface of A is what part of the surface of B? Illustrate by a diagram.

16. If the dimensions of a square, A, are one third the dimensions of a square, B, the area of B is how many times the area of A? Illustrate by a diagram.

17. Having given the dimensions of a rectangular room in feet, how can you find the area of the four walls, making but one multiplication? making two multiplications? making four multiplications? What other operation is necessary in each case? Illustrate by an example.

18. Having given any number of units of one denomination, how can you find what fraction they are of one unit of the next higher denomination? Illustrate by original examples.

19. Having given any number of units of one denomination, how can you find what decimal they are of one unit of the next higher denomination?

20. Give original examples under each table, to illustrate reduction ascending, reduction descending.

RATIO AND PROPORTION

RATIO

Study Recitation

The **ratio** of one number to another of the same kind is the quotient arising from the division of the first number by the second.

The ratio of 4 to 2 is $4 \div 2$, or 2; the ratio of 10 to 2 is $10 \div 2$, or 5; the ratio of 5 to 3 is $5 \div 3$, or $\frac{5}{3}$; the ratio of 7 to 9 is $7 \div 9$, or $\frac{7}{9}$; the ratio of $\frac{5}{6}$ to $\frac{2}{3}$ is $\frac{5}{6} \div \frac{2}{3}$, or $\frac{5}{4}$.

The ratio of two numbers may be expressed by writing a colon between them; as,

$6 : 9$, read, the ratio of 6 to 9.
$4 : 2$, read, the ratio of 4 to 2.
$\frac{1}{3} : \frac{3}{4}$, read, the ratio of $\frac{1}{3}$ to $\frac{3}{4}$.

The **antecedent** is the first term of a ratio.

The **consequent** is the second term of a ratio.

A ratio may always be expressed as a fraction, the numerator being the antecedent, and the denominator, the consequent.

The ratio of $6 : 7 = \frac{6}{7}$ $\begin{matrix}\text{Antecedent.}\\ \text{Consequent.}\end{matrix}$

A fraction is the ratio of the numerator to the denominator. Since every ratio may be regarded as a fraction, the laws governing the changes of one or both terms of a fraction by multiplication or division apply to changes in the terms of a ratio.

Multiplying the (numerator) of a (fraction) multiplies the (fraction). Dividing the (numerator) of a (fraction) divides the (fraction). Multiplying the (denominator) of a (fraction) divides the (fraction). Dividing the (denominator) of a (fraction) multiplies the (fraction).

Multiplying or dividing both (numerator) and (denominator) of a (fraction) by the same number does not change the value of the (fraction).

In the foregoing statements, substitute the word "antecedent" for the word "numerator," the word "consequent" for the word "denominator," and the word "ratio" for the word "fraction," and give each statement as thus changed.

Multiplying or dividing both terms of a ratio by the same number does not change its value.

EXAMPLES AND PROBLEMS

Oral

Find the ratio of:

	a.	*b.*	*c.*	*d.*
1.	16 to 8	8 to 16	.25 to 1	1 to .25
2.	15 to 5	5 to 15	.50 to 1	1 to .50
3.	3 to 4	4 to 3	$.37\frac{1}{2}$ to 1	1 to $.37\frac{1}{2}$
4.	$\frac{1}{4}$ to $\frac{3}{4}$	$\frac{3}{4}$ to $\frac{1}{4}$	$.33\frac{1}{3}$ to 1	1 to $.33\frac{1}{3}$
5.	$\frac{2}{5}$ to $\frac{4}{5}$	$\frac{4}{5}$ to $\frac{2}{5}$	$.62\frac{1}{2}$ to 1	1 to $.62\frac{1}{2}$
6.	1 ft. to 1 in.	1 in. to 1 ft.	1 pt. to 1 qt.	1 qt. to 1 pt.
7.	1 qt. to 1 pt.	1 pt. to 1 qt.	\$5 : \$15	\$15 to \$5

8. John has 8 cents and James has 4 cents. What is the ratio of John's money to James's money?

The question, "What is the ratio of John's money to James's money?" means the same as the question, "John's money is how many times James's money?" *Ans.* 8 : 4, or $\frac{8}{4}$, or 2.

9. A can do a piece of work in 3 days. B can do the same piece of work in 5 days. What is the ratio of the time it takes A to the time it takes B?

Ans. $\frac{3}{5}$. The ratio expressed by a fraction is always abstract. It tells how many times or what part of a time the antecedent contains the consequent. The result is the quotient, or the ratio.

10. In problem 9, what is the ratio of the time it takes B to the time it takes A? How else may the question be asked?

11. In one bin there are 40 bushels of wheat; in a second bin there are 20 bushels of wheat. What is the ratio of the quantity of wheat in the first bin to the quantity in the second bin?

12. If a third bin contains 30 bushels, how many bushels must the fourth bin contain in order that the ratio of the quantity of wheat in the third bin to the quantity in the fourth bin shall be the same as the ratio of the quantity in the first bin to the quantity in the second bin?

13. Can you find the ratio of $3 to 6 days?

NOTE. There can be no ratio between dollars and days, because dollars cannot be measured by days. The unit of the first number is $1; the unit of the second number is 1 day. *Like numbers* have the same unit; as, $3 and $6; 4 days and 6 days; 8 hours and 4 hours. A ratio can exist only between two numbers having the *same unit.*

14. What is the ratio of 12 days to 3 days?

15. What is the ratio of 18 to 2?

16. What is the ratio of 4 to 4? What is the ratio of any number to itself? $4 : 4 = 7 : 7 = 8 : 8$, etc.

17. Is the ratio of a number to a larger number greater or less than 1? Illustrate.

Is the ratio of a number to a smaller number greater or less than 1? Illustrate.

Two ratios cannot be equal when one of them is greater than 1, and the other less than 1, or equal to 1.

WRITTEN

18. What is the ratio of 18 to 24?

19. What is the ratio of 25 to 6?

Express the following ratios as fractions, and tell which are greater than 1 and which are less than 1:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
20.	3 : 4	8 : 5	12 : 35	13 : 14	17 : 24	15 : 48
21.	25 : 55	24 : 32	16 : 50	18 : 40	15 : 65	65 : 15
22.	$\frac{2}{3} : \frac{4}{5}$	$\frac{1}{2} : \frac{1}{4}$	$2\frac{1}{2} : 1\frac{2}{3}$	$\frac{3}{5} : 2\frac{3}{8}$	$\frac{3}{4} : 2\frac{1}{8}$	$\frac{5}{6} : 1\frac{4}{5}$
23.	$3\frac{1}{3} : 6\frac{2}{3}$	$4\frac{1}{2} : 9$	$8 : 2\frac{2}{3}$	$3 : 4\frac{1}{2}$	$2\frac{1}{2} : 3\frac{1}{2}$	$5\frac{1}{3} : 8$

Find the value of the following ratios:

24. 2 yd. 1 ft. : 3 yd. 1 ft.

25. 10 gal. 1 qt. : 5 gal. 1 pt.

26. 1 lb. 1 oz. : 3 lb. 3 oz.

27. 4 pk. 1 qt. : 1 pk. $\frac{1}{2}$ pt.

SIMPLE PROPORTION

STUDY RECITATION

A **proportion** is an expression of equality between ratios. It may be written in three ways:

(*a*) $12 : 6 = 8 : 4$, read, the ratio of 12 to 6 equals the ratio of 8 to 4.

(*b*) $12 : 6 :: 8 : 4$, read, 12 is to 6 as 8 is to 4.

(*c*) $\frac{12}{6} = \frac{8}{4}$, read, twelve sixths equals eight fourths.

The first and fourth terms of a proportion are the **extremes**; the second and third terms are the **means**.

The first and third terms of a proportion are the **antecedents**; the second and fourth terms are the **consequents**.

The fourth term of a proportion is a **fourth proportional** to the other three terms.

When the second and third terms of a proportion are the same number, this number is a **mean proportional** to the other two. In $4 : 8 = 8 : 16$, 8 is a mean proportional to 4 and 16.

In every proportion, the product of the means equals the product of the extremes.

This is the principle upon which the solution of problems in proportion depends.

If three terms of a proportion are given, either two extremes and a mean, or two means and an extreme are given.

The product of the extremes divided by the given mean equals the other mean.

The product of the means divided by the given extreme equals the other extreme.

Study Recitation

Find the missing term in each of the following proportions :

1.	**2.**	**3.**	**4.**
$12 : 5 = 24 : ?$	$12 : 5 = ? : 10$	$12 : ? = 24 : 10$	$? : 5 = 24 : 10$

Solution

$\frac{5 \times \overset{2}{\cancel{24}}}{\cancel{12}} = 10$, *Ans.* $\frac{12 \times \overset{2}{\cancel{10}}}{\cancel{5}} = 24$, *Ans.* $\frac{\cancel{12} \times \overset{5}{\cancel{10}}}{\underset{2}{\cancel{24}}} = 5$, *Ans.* $\frac{\cancel{5} \times \overset{12}{\cancel{24}}}{\underset{2}{\cancel{10}}} = 12$, *Ans.*

Solve:

5. $3 : 6 = 8 : ?$	**9.** $5 : ? = 6 : 12$.	**13.** $? : 9 = 8 : 18$.
6. $3 : 8 = ? : 16$.	**10.** $? : 5 = 10 : 10$.	**14.** $14 : ? = 7 : 10$.
7. $4 : 7 = 8 : ?$	**11.** $4 : ? = 8 : 10$.	**15.** $3 : 9 = ? : 15$.
8. $2 : 5 = ? : 20$.	**12.** $3 : 8 = 6 : ?$	**16.** $3 : ? = 4 : 16$.

PROBLEMS

WRITTEN

1. If a man earns \$12 in 6 days, how many dollars can he earn in 28 days?

SOLUTION. As the answer is to be dollars, make 12 the third term. If a man earns \$12 in 6 days, can he earn a greater or a less number of dollars in 28 days? *Ans.* A greater number. Therefore 28, the larger of the two numbers representing days, is the second term of the proportion and 6 is the first term.

The proportion is $6 : 28 = 12 : ?$ $\quad \frac{28 \times \overset{2}{\cancel{12}}}{\cancel{6}} = 56.$ *Ans.* \$56.

Most problems in proportion can be more easily solved by the fractional method or by analysis.

Fractions. 28 days is $\frac{28}{6}$, or $\frac{14}{3}$, of 6 days. Therefore the man will earn $\frac{14}{\cancel{3}}$ of $\$\overset{4}{\cancel{12}}$, or \$56.

Analysis. If a man earns \$12 in 6 days, he earns $\frac{1}{6}$ of \$12, or \$2, in 1 day. He earns $28 \times \$2$, or \$56, in 28 days.

TO THE TEACHER. The teacher who wishes to teach at this time "The Use of Symbols," pp. 387 to 399, can make use of this method as well.

2. If the interest on a note for 7 months is \$35, what is the interest on the same note for 15 months?

(*a*) *Proportion.* $7 : 15 = 35 : ?$ $\quad 15 \times \frac{\overset{5}{\cancel{35}}}{\cancel{7}} = 75.$ *Ans.* \$75.

(*b*) *Fractions.* $\frac{15}{\cancel{7}}$ of $\$\overset{5}{\cancel{35}} = \$75$.

(*c*) *Analysis.* Interest for 1 month = \$5.
Interest for 15 months = $15 \times \$5$, or \$75.

3. How many days will it take 24 men to do a piece of work that can be done by 9 men in 4 days?

(*a*) *Proportion.* $24 : 9 = 4 : ?$ $\quad \frac{\cancel{4} \times \overset{3}{\cancel{9}}}{\underset{\underset{2}{\cancel{6}}}{\cancel{24}}} = 1\frac{1}{2}.$ *Ans.* $1\frac{1}{2}$ da.

NOTE. 24 men take *less* time than 9 men. As the answer is to be less than 4 da., make the smaller number of men (9) the second term.

(*b*) *Fractions.* $\frac{\overset{3}{\cancel{9}}}{\underset{2}{\underset{\cancel{8}}{\cancel{24}}}}$ of $\cancel{4}$ da. $=\frac{3}{2}$ da. $=1\frac{1}{2}$ da.

(*c*) *Analysis.* If 9 men take 4 da., 1 man takes 9×4 da., or 36 da., and 24 men take $\frac{1}{24}$ of 36 da., or $1\frac{1}{2}$ da.

4. How many bushels of wheat can be bought for \$60, if 12 bushels can be bought for \$6?

(*a*) *Proportion.* \$6 : \$60 = 12 : ? $\frac{12\times \overset{10}{\cancel{60}}}{\cancel{6}}=120.$ *Ans.* 120 bu.

(*b*) *Fractions.* $\frac{\overset{10}{\cancel{60}}}{\cancel{6}}$ of 12 bu. = 120 bu.

(*c*) *Analysis with 1 bu. as unit.* If 12 bu. cost \$6, 1 bu. costs $\frac{1}{12}$ of \$6, or \$.50. For \$60 as many bushels can be bought as \$.50 is contained times in \$60, or 120 bu.

(*d*) *Analysis with \$1 as unit.* If for \$6, 12 bu. can be bought, for \$1, $\frac{1}{6}$ of 12 bu., or 2 bu., can be bought. For \$60, 60×2 bu., or 120 bu. can be bought.

Solve the following examples by the method that is easiest for you:

5. How many bushels of wheat can be bought for \$72, if 6 bushels can be bought for \$3?

6. Thirty men can do a piece of work in 6 days. At the same rate, how long will it take 18 men to do the same work?

7. If 5 men can do a piece of work in 20 days, how many men can do the work in $12\frac{1}{2}$ days?

8. A piece of lead pipe $5\frac{1}{2}$ ft. long weighs 11 pounds. What is the weight of a piece 17 ft. in length?

9. How much wheat can be bought for \$150, if 20 bushels can be bought for \$16?

10. A contractor requires 80 days to build a house when his men work 10 hours a day. How many days will he require when his men work 8 hours a day?

11. A freight train runs 20 miles an hour, and a passenger train on the same road runs 35 miles an hour. If the passenger train makes a trip between two points in 4 hours, how many hours will it require for the freight train to make the same trip?

12. The rates of two steamboats are as 9 to 12. If the faster boat makes a trip in 6 hours, how long will it take the slower boat to make the same trip?

13. A contractor agrees to do a piece of work in 40 days, and employs 20 men upon it. The work is half finished at the end of 30 days. How many additional workmen must he employ in order to finish the work on time?

SUGGESTION. Before beginning the solution, change the statement of the problem: At the end of 30 days as much work remains to be done as has already been done.

The problem may then be stated: If 20 men do a piece of work in 30 days, how many men will it require to do an equal piece of work in 10 days?

14. If a loaf of bread costs 4¢ when flour is \$6 a barrel, how much should it cost when flour is \$$7\frac{1}{2}$ a barrel?

15. If 4 men or 6 boys can do a piece of work in 12 days, in how many days will 6 men and 3 boys do the same piece of work?

16. If 4 farm hands can build a fence in 3 days, how many farm hands would it take to build a similar fence in $\frac{1}{2}$ a day?

PROPORTIONAL PARTS

1. Divide the number 12 into two parts in the ratio of 2 to 4.

Solution. For every 2 units in the smaller part, there are 4 units in the larger part or 6 units in both parts. The smaller part is $\frac{2}{6}$ of the whole, or $\frac{2}{6}$ of 12, or 4. The larger part is $\frac{4}{6}$ of the whole, or $\frac{4}{6}$ of 12, or 8.

Test. $4 + 8 = 12$. $4:8 = 2:4$.

2. Divide the number 24 into two parts whose ratio equals the ratio of 3 to 5.

3. Separate 36 into two parts whose ratio equals the ratio of 5 to 7.

4. Separate 15 into two parts whose ratio equals the ratio of $\frac{1}{2}$ to $\frac{1}{3}$.

Suggestion. Before beginning the analysis, reduce $\frac{1}{2}$ and $\frac{1}{3}$ to 6ths. The ratio then becomes $\frac{3}{6}:\frac{2}{6}$, which is the same ratio as $3:2$, and the requirement is to separate 15 into two parts whose ratio equals the ratio of 3 to 2.

5. 20 pieces of stock for woodwork are divided between James and John so that the number of pieces James has is to the number John has as 2 is to 3. How many has each?

6. If John earns $\frac{3}{4}$ as much per day as Henry, what is the ratio of John's earnings to Henry's earnings? What is the ratio of Henry's earnings to John's earnings?

7. Divide 40 pieces of stock between James and John so that the number James has is $\frac{3}{5}$ the number John has.

8. Two men are 30 miles apart and are traveling toward each other. One travels 2 miles an hour, and the other 3 miles an hour. How far will each have traveled when they meet? In how many hours will they meet?

9. A garden 18 ft. long is divided into 3 beds whose lengths are to each other as 2, 3, and 4. What is the length of each bed?

PERCENTAGE

Study Recitation

1. What is $\frac{1}{2}$ of 24? $\frac{1}{4}$ of 48? $\frac{2}{5}$ of 35? $\frac{3}{4}$ of 32?

Finding each result is finding a fractional part of a number.

2. What is $\frac{50}{100}$ of 24? $\frac{25}{100}$ of 48? $\frac{40}{100}$ of 35? $\frac{75}{100}$ of 32?

Finding each result is finding a number of hundredths of a number. Notice that the fractions in example 1 are reduced to hundredths in each case in example 2.

3. What is .5 of 24? .25 of 48? .4 of 35? .75 of 32?

Finding each result is finding a decimal part of a number. Notice that the fractions in example 1 are changed to decimals in example 3, and that these decimals equal the fractions in example 2. Thus, $.5 = \frac{5}{10}$; $.25 = \frac{25}{100}$, etc.

Per cent means **per hundred** or **hundredths**. The sign for it is %.

Percentage is the part of arithmetic that treats of per cent.

$$\frac{1}{2} = \frac{50}{100} = .50 = 50\,\%$$
$$\frac{1}{4} = \frac{25}{100} = .25 = 25\,\%$$
$$\frac{2}{5} = \frac{40}{100} = .40 = 40\,\%$$
$$\frac{3}{4} = \frac{75}{100} = .75 = 75\,\%$$

7 % means .07, or $\frac{7}{100}$

30 % means .30, or $\frac{30}{100}$, or $\frac{3}{10}$

$12\frac{1}{2}$ % means $.12\frac{1}{2}$, or .125, or $\dfrac{12\frac{1}{2}}{100}$, or $\dfrac{1}{8}$

Write as per cent:

4. .04 .06 .07 .10 .25 .48 .96 .68 .33

Write as decimals:

5. 2 % 5 % 25 % 30 % 75 % $87\frac{1}{2}$ % $33\frac{1}{3}$ % $12\frac{1}{2}$ %

Note. $87\frac{1}{2}\% = .87\frac{1}{2} = .875$; $33\frac{1}{3}\% = .33\frac{1}{3}$.

Learn the following table:

$\frac{1}{2} = 50\%$	$\frac{2}{5} = 40\%$	$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{1}{20} = 5\%$
$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{3}{5} = 60\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{1}{25} = 4\%$
$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{4}{5} = 80\%$	$\frac{7}{8} = 87\frac{1}{2}\%$	$\frac{5}{4} = 125\%$
$\frac{1}{4} = 25\%$	$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{1}{10} = 10\%$	$\frac{9}{8} = 112\frac{1}{2}\%$
$\frac{3}{4} = 75\%$	$\frac{5}{6} = 83\frac{1}{3}\%$	$\frac{1}{12} = 8\frac{1}{3}\%$	$\frac{4}{3} = 133\frac{1}{3}\%$
$\frac{1}{5} = 20\%$	$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{1}{16} = 6\frac{1}{4}\%$	$\frac{5}{3} = 166\frac{2}{3}\%$

NOTE. $\frac{1}{2}\%$ means $\frac{1}{2}$ of 1%, or $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$; $\frac{1}{8}\%$ means $\frac{1}{8}$ of 1%, or $\frac{1}{800}$.

Tell what per cents correspond to the fractions given below, and what fractions in their lowest terms correspond to the per cents and decimals given below:

6. $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{3}{5}, \frac{4}{5}, \frac{3}{10}, \frac{1}{10}, \frac{1}{8}$.

7. $\frac{3}{20}, \frac{7}{20}, \frac{9}{20}, \frac{11}{20}, \frac{3}{25}, \frac{6}{25}, \frac{9}{25}, \frac{8}{25}, \frac{11}{25}, \frac{12}{25}$.

8. 25%, 10%, 20%, 75%, 80%, 60%, 40%, 30%, 50%, 70%.

9. .75, .20, .50, .60, .10, .25, .80, .70, .30, .40, .90, .35.

To express per cent as a common fraction, write the number of per cent as the numerator and 100 *as the denominator. Reduce this fraction to its lowest terms.*

Thus, $25\% = \frac{25}{100} = \frac{1}{4}$. $\qquad 37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{75}{200} = \frac{3}{8}$.

To express per cent as a decimal, remove the sign % and place a decimal point two places to the left.

Thus, $25\% = .25$; $125\% = 1.25$; $2\frac{1}{2}\% = .02\frac{1}{2} = .025$; $15.5\% = .155$.

To express a common fraction as per cent, change it to hundredths and use the sign % instead of the denominator 100.

Thus, $\frac{1}{4} = \frac{25}{100} = 25\%$. $\frac{3}{8} = \frac{37\frac{1}{2}}{100} = 37\frac{1}{2}\%$.

To express a decimal as per cent, omit the decimal point and write the sign %.

Thus, $.25 = 25\%$; $.33\frac{1}{3} = 33\frac{1}{3}\%$; $1.75 = 175\%$; $1.87\frac{1}{2} = 187\frac{1}{2}\%$; $1.625 = 162\frac{1}{2}\%$.

Express as common fractions, integers, or mixed numbers:

	a.	*b.*	*c.*	*d.*	*e.*
10.	2%	$33\frac{1}{3}\%$	125%	$112\frac{1}{2}\%$	$\frac{1}{2}\%$
11.	5%	$66\frac{2}{3}\%$	120%	$137\frac{1}{2}\%$	$\frac{3}{4}\%$
12.	7%	$6\frac{1}{4}\%$	100%	$162\frac{1}{2}\%$	$\frac{1}{5}\%$
13.	10%	$8\frac{1}{3}\%$	200%	$166\frac{2}{3}\%$	$\frac{5}{8}\%$
14.	12%	$12\frac{1}{2}\%$	175%	$187\frac{1}{2}\%$	$\frac{1}{10}\%$
15.	15%	$16\frac{2}{3}\%$	105%	$233\frac{1}{3}\%$	.5%
16.	20%	$83\frac{1}{3}\%$	150%	$116\frac{2}{3}\%$	.05%
17.	25%	$37\frac{1}{2}\%$	165%	$133\frac{1}{3}\%$	.005%
18.	29%	$87\frac{1}{2}\%$	300%	$108\frac{1}{3}\%$	.25%
19.	34%	$62\frac{1}{2}\%$	275%	$162\frac{1}{2}\%$	.025%

Express the above per cents as decimals.

Express first as per cents; then as decimals:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
20.	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{7}{25}$	$\frac{7}{4}$	$3\frac{1}{8}$	20
21.	$\frac{5}{8}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{9}{8}$	$4\frac{1}{4}$	30
22.	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{11}{50}$	$\frac{4}{3}$	$2\frac{1}{5}$	25
23.	$\frac{1}{16}$	$\frac{9}{10}$	$\frac{1}{40}$	$\frac{7}{3}$	$5\frac{1}{3}$	$33\frac{1}{3}$
24.	$\frac{5}{16}$	$\frac{3}{20}$	$\frac{3}{50}$	$\frac{6}{5}$	$3\frac{3}{4}$	$12\frac{1}{2}$
25.	$\frac{1}{3}$	$\frac{7}{50}$	$\frac{5}{12}$	$\frac{17}{16}$	$4\frac{7}{8}$	$37\frac{1}{2}$

Express as per cents:

26.	.25	1.25	2.50	$1.12\frac{1}{2}$	$1.62\frac{1}{2}$
27.	$.37\frac{1}{2}$	1.20	3.75	$2.37\frac{1}{2}$	$1.66\frac{2}{3}$
28.	$.03\frac{3}{4}$	1.50	4.80	$3.62\frac{1}{2}$	$1.87\frac{1}{2}$
29.	$.33\frac{1}{3}$	1.60	5.90	$1.16\frac{2}{3}$	$2.06\frac{2}{3}$
30.	$.04\frac{1}{2}$	1.75	6.40	$1.33\frac{1}{3}$	$1.05\frac{1}{4}$

31. Find $\frac{1}{2}$ of each of the following numbers:

12, 18, 25, 30, 62, 48, 256, 312.

32. Find 50% of each of the above numbers by multiplying them by .50.

33. Find 25% of each of the following numbers by multiplying them by .25.

16, 24, 60, 180, 254, 82, 96, 480, 320, 1600, 5600, 3840.

34. Prove your work by finding $\frac{1}{4}$ of each of these numbers.

35. 7% of a number equals what part of it? what decimal of it? Find $\frac{7}{100}$ of 365. Find .07 of 365 by using the decimal. Which is the better way?

36. Find 25% of 36 by using the per cent as a fraction in its lowest terms. Find 25% of 36 by multiplying by the decimal .25. Which is the better way and why?

To find any per cent of a number.

Study Recitation

1. Find 10% of 30, or $\frac{1}{10}$ of 30.

Solution. $10\% = \frac{1}{10}$; $\frac{1}{10}$ of $30 = 3$.

Every example in which it is required to *find a per cent of a number* may be changed to an example in common fractions, or in decimals.

2. Find $37\frac{1}{3}\%$ of 240.

$$\begin{array}{r} 240 \\ \underline{.37\frac{1}{3}} \\ 80 \\ 16\,80 \\ \underline{72\,0} \\ 89.60 \end{array}$$

3. Find $37\frac{1}{2}\%$ of 240.

$37\frac{1}{2}\% = \frac{3}{8}$; $\frac{3}{8}$ of $240 = 90$.

In example 2, change $37\frac{1}{3}\%$ to a decimal and multiply.

In example 3, change $37\frac{1}{2}\%$ to $\frac{3}{8}$; $\frac{3}{8}$ of 240 may be found mentally.

4. Find $\frac{3}{4}\%$ of \$480. **5.** Find $.05\%$ of \$6244.

$\$480$ $.00\frac{3}{4}$ $\$3.60$	In example 4, change $\frac{3}{4}\%$ to $.00\frac{3}{4}$; then multiply. In example 5, change $.05\%$ to $.0005$; then multiply.	$\$6244$ $.0005$ $\$3.1220$

Reduce the given per cent to a fraction in its lowest terms and find this fractional part of the number. Or,

Multiply the number by the given per cent expressed as a decimal.

Change the following examples in percentage to examples in common fractions, and to examples in multiplication of decimals. Fill the blanks, and solve:

Percentage	Common Fractions	Multiplication of Decimals
6. Find 75% of 48.	Find $\frac{3}{4}$ of 48.	Find .75 of 48.
7. Find $66\frac{2}{3}\%$ of 21.	Find —— of 21.	Find —— of 21.
8. Find 10% of 40.	Find —— of 40.	Find —— of 40.
9. Find 10% of 25.	Find —— of 25.	Find —— of 25.
10. Find 25% of 18.	Find —— of 18.	Find —— of 18.
11. Find 40% of 30.	Find —— of 30.	Find —— of 30.

Solve, changing the given per cents to common fractions:

12. 5% of 40; 60; 80; 100; 140; 120; 240; 600; 500.

13. 4% of 50; 100; 200; 75; 125; 150; 600; 700; 800.

14. $12\frac{1}{2}\%$ of 24; 32; 40; 64; 80; 56; 48; 72; 88; 96; 144.

15. $33\frac{1}{3}\%$ of 18; 9; 12; 15; 39; 45; 27; 30; 36; 42; 51.

16. $66\frac{2}{3}\%$ of 21; 12; 15; 30; 24; 27; 18; 33; 36; 42; 54.

17. 20% of 25; 10; 15; 55; 35; 50; 20; 30; 40; 45; 125.

18. 75% of 24; 8; 12; 16; 28; 36; 40; 44; 48; 100.

19. $37\frac{1}{2}\%$ of 32; 40; 56; 64; 80; 72; 48; 88; 96; 24; 104.

Written

Solve, using the given per cents as decimals:

20. 7% of 28; 39; 251; 175; 600; 500; 400; 300; 1000.

21. 12% of 36; 49; 87; 120; 140; 160; 180; 240; 300.

22. 6% of 18; 36; 45.6; 27.3; 5.5; 4.6; 7.2; 900; 500.

23. 8% of 2.7; 3.5; 2.78; 15.6; 3.4; 5.5; 6.5; 750; 800.

24. 3% of 150; 48; $273\frac{1}{3}$; $28\frac{2}{3}$; 5.4; 6.5; 7.8; 940; 870.

25. $3\frac{1}{2}$% of 48; 24; 36; 125; 27; 3.5; 6.7; 5.4; 800; 700.

26. $7\frac{1}{2}$% of 25; 40; 20; 35; 426; 800; 640; 528; 416.

27. 15% of $80.

28. $8\frac{1}{3}$% of $36.

29. $37\frac{1}{2}$% of 48 lb.

30. 50% of 12 oz.

31. $12\frac{1}{2}$% of 84 da.

32. 60% of $18.

33. 75% of 12 mo.

34. $\frac{1}{2}$% of $400.

First find 1% of $400, and then $\frac{1}{2}$%.

35. 100% of 12 eggs.

36. $\frac{2}{3}$% of $1200.

37. 200% of 6 cents.

38. $6\frac{1}{4}$% of 64.

39. $1\frac{1}{2}$% of $100.

40. $2\frac{1}{2}$% of 200.

41. 90% of 81.

42. $\frac{5}{7}$% of 91.

43. 4% of $250.

44. 10% of 87.50 acres.

45. 150% of 12 eggs.

46. $137\frac{1}{2}$% of $800.

47. $133\frac{1}{3}$% of $25.

Work by both rules, and decide which one is the better:

48. $12\frac{1}{2}$% of 24; 56; 83; 128; 250; 300; 600; 900.

49. 25% of 16; 28; 34; 46; 142; 200; 300; 400.

50. 120% of 15; 35; 36; 48; 27; 100; 150; 300.

51. 175% of .28; 18; 19; 42; 126; 400; 800; 900.

Find first 25%; then 50%; then $37\frac{1}{2}$%; then $112\frac{1}{2}$%; then 5% of:

	a.	*b.*	*c.*	*d.*	*e.*
52.	$200	15620	154 mi.	$400.80	16000
53.	$316	17890	264 yd.	$316.40	18000
54.	$2480	44000	344 gal.	$9480.48	24000

Find first 2%; then $3\frac{1}{2}$%; then $5\frac{3}{4}$%; then 6% of:

55.	1640	15780	$125.50	$2340.90	$7080.25
56.	2970	20000	$116.80	$3500.40	$2040.50
57.	8341	50000	$240.54	$6000.70	$9854.75

Find first $37\frac{1}{2}$%; then $137\frac{1}{2}$%; then $133\frac{1}{3}$%; then $\frac{1}{4}$%; then .05% of:

58.	2030	18500	$640.50	$6500.50	$10000
59.	3050	17800	$730.25	$9250.75	$25000
60.	6070	19400	$825.16	$8045.60	$80000

PROBLEMS

Oral

1. From a tank containing 50 gal. of vinegar, 10% leaked out. How many gallons were lost? How many were left in the cask?

Solution. 10% of a number is $\frac{1}{10}$ of it; $\frac{1}{10}$ of 50 gal. is 5 gal., which leaked out. There were left 50 gal. less 5 gal., or 45 gal.

2. In an orchard there are 85 trees. If 20% of them are apple trees, and the others are pear trees, how many trees of each kind are there in the orchard?

3. A man who has an income of $6770 a year pays an income tax of 1% on the part of his income that exceeds $4000. What is the tax?

4. Find the income tax of a woman who pays 1 % on her income in excess of \$3000, if she has a salary of \$2000, receives \$1000 a year rent, and \$500 interest on bonds.

5. A merchant bought cloth at 80 ¢ a yard, and sold it for 25 % more than it cost. For how much more than the cost did he sell it? What was the selling price per yard?

6. A piece of cloth that cost \$1.25 a yard was sold at a loss of 20 %. How much less than the cost was received for it? What was the selling price?

7. If a clothing dealer pays \$15 apiece for overcoats, and sells them at a gain of 20 %, what is the selling price? How much does he make on one coat? on 1 dozen coats?

8. A farmer tests 80 kernels of corn to see how many will grow. 12½ % fail to grow. How many kernels grow? How many fail to grow?

9. There are 40 words in a spelling lesson. If a boy misses 10 % of them, how many does he spell correctly?

10. If I fail to work 5 % of the problems in my arithmetic lesson, and there are 20 problems in the lesson, how many do I work? What per cent of them do I work? How many do I fail to work?

11. A farmer harvested 250 bu. of potatoes, but 10 % of them were spoiled by freezing. How many bushels were spoiled? How many bushels were not damaged?

12. If a man buys a horse for \$88, and sells it so as to gain 12½ %, for how much does he sell the horse? How much does he gain?

13. A man sold ⅓ of his farm. What per cent of it did he sell? How many hundredths of it did he sell? How many hundredths of the farm had he left? What per cent of the farm had he left?

14. A pupil solved correctly $\frac{3}{4}$ of the problems in his lesson. What part of the problems in his lesson had he still to solve? What per cent of the problems did he solve correctly? What per cent of the problems had he still to solve?

15. A man has 30 tons of coal and sells 10 % of it at \$4 a ton. How much does he receive for what he sells? How much is the rest of the coal worth at the same rate?

16. About $16\frac{2}{3}$ % of a pound of butter is salt and water. How many pounds of butter can be made from 190 lb. of butter fat?

HINT. $16\frac{2}{3}\ \% = \frac{1}{6}$. $\frac{5}{6}$ of 1 lb. of butter is butter fat. 190 lb. $\div \frac{5}{6} =$ 228 lb. *Ans.*

17. A cow whose milk tested 4 % gave 4800 lb. of milk a year. How many pounds of fat did the cow produce?

NOTE. A test of 4 % means 4 % of butter fat.

18. If $\frac{1}{4}$ of a box of lemons is sold, and $\frac{1}{3}$ of the remainder is unsalable, what per cent of the lemons is not salable?

WRITTEN

19. A real estate dealer bought 1275 acres of land, and sold $66\frac{2}{3}$ % of it to one man, and 20 % of the remainder to another man. How many acres did each purchase, and how many remained?

20. If bread made from a sack of flour weighs $33\frac{1}{3}$ % more than the flour, what is the weight of the bread made from a sack of flour weighing 50 lb.?

21. Under the 1913 Federal Reserve Act, country banks must hold 12 % of their demand deposits in reserve. If the demand deposits of the Farmers' Bank amount to \$185,465.17, how much must be reserved?

Farm Problems

22. A cow gave 22 lb. of milk at night and 19 lb. in the morning. The test showed the evening's milk to contain 4.2 % of butter fat and the morning's milk 4.3 %. How much butter fat did the cow produce at the two milkings?

23. Mr. A took 62 lb. of cream testing 28 % to the creamery. What was its value at 35 cents per pound for butter fat?

24. A farmer delivered the following amounts of cream to a creamery: Monday, 196 lb.; Tuesday, 92 lb.; Wednesday, 99 lb.; Thursday, 102 lb.; Friday, 93 lb.; and Saturday, 95 lb. The average test for the week was 32.5 %, and butter fat was worth $.35 a pound. What was the amount of his cream check for the week? If this was the income from 25 cows, what was the average per cow?

25. Corn silage contains the following digestible food materials: protein, 1.2 %; carbohydrates, 14 %; and fat, 0.7 %. How many pounds of each are there in a ton (2000 pounds) of silage?

26. There is no perfect method for the separation of fat from milk, so there is some left in skim milk. The following are the amounts under varying conditions:

Shallow setting	0.8 per cent
Deep setting	0.2 per cent
Separator	0.05 per cent

What would be the loss with 1000 lb. of milk under each system?

27. A farmer has a herd of 20 cows that average 6000 lb. of milk per year. What would be the loss in pounds of butter fat per year under each system? Find the value of the butter fat lost in each case if the average price is $.30 per pound.

28. A farmer tested the skim milk from his separator and found it to contain .13 % of butter fat. The separator was adjusted, and a second test showed .05 % of butter fat. If the separator had been out of adjustment 30 days, how great was his loss with a herd of 20 cows averaging 25 pounds of milk per day, butter fat being valued at 28 cents per pound?

29. The average composition of butter is:

Fat	83 per cent
Water	13 per cent
Casein	1 per cent
Salt	3 per cent

The output of a creamery for one week was 50 tubs, of 60 lb. each, of butter having the average composition. How much was fat? water? casein? salt?

30. According to the reports of the American Guernsey Cattle Club, Anton's Patience produced during the month of July, 1913, 114,570 lb. of milk testing 4.97 %. Find the total pounds of butter fat produced and the average daily production of milk and butter fat.

31. The average production of 326 cows in the Lyndeboro (Mass.) Cow Test Association for a period of one year was 6463.2 lb. of milk testing 3.76 % fat. What was the average production of butter fat? the total for all herds?

The following is the record of a cow by periods, each four weeks in length. Determine for each period the number of pounds of butter fat produced and the profit above the cost of feed:

	Periods	Pounds of Milk	Test	Value Butter	Cost of Stall Feed
32.	1	650.4	3.8 %	$8.69	$5.32
33.	2	631.2	3.8	8.10	5.32
34.	3	631.1	3.9	8.61	3.78
35.	4	705.7	3.5	8.61	4.97

	Periods	Pounds of Milk	Test	Value Butter	Cost of Stall Feed
36.	5	646.4	4.0%	$7.50	$3.99
37.	6	615.7	4.0	7.17	Pasture
38.	7	490.9	4.2	6.00	Pasture
39.	8	275.0	4.2	3.62	Pasture
40.	9	55.8	4.2	.81	Pasture
41.	10	Dry			
42.	11	826.7	3.8	11.35	3.77
43.	12	867.3	3.7	13.06	5.85
44.	13	851.7	4.0	15.11	6.20

In the following, how many pounds of butter fat were produced by all the mature cows? by the four-year-olds? the three-year-olds? the two-year-olds?

	Pounds Milk	Per Cent	Pounds Fat
45. 389 mature cows average	10492	3.83	?
46. 131 four-year-olds average	9487	3.95	?
47. 178 three-year-olds average	8981	3.94	?
48. 300 two-year-olds average	7761	4.03	?

49. One cow in a herd gave in the morning 25 lb. of milk testing 4 % and in the evening 30 lb. testing 3.8 %. What was the average per cent of butter fat that day? the total weight of the butter fat?

50. Another cow in the same herd gave a total of 65 lb. of milk that day, with an average test of 3.1 %. Which produced the greater amount of butter fat?

51. At $.25 a pound, find the amount received during May for butter fat from the cows in examples 49 and 50.

Find the number of pounds of fat in each of the following:

52. 50 lb. of milk, 5.0 % fat.

53. 40 lb. of milk, 4.8 % fat.

54. 25 lb. of milk, 4.5 % fat.

55. 80 lb. of milk, 4.0 % fat.

56. 75 lb. of milk, 4.4 % fat. **57.** 45 lb. of milk, 3.5 % fat.

58. Separator cream usually contains from 15 % to 35 % fat. Which is the more profitable to sell, 1000 lb. of cream at $.15 a pound, or butter fat at $.25 a pound, if the cream contains 25 % butter fat?

The following are approximate percentages of plant-food constituents in fresh manure from various farm animals. How many pounds of nitrogen, phosphoric acid, and potash are there in one ton of each manure, and what is the value of 1 ton of each, if nitrogen is worth $.15 per pound and phosphoric acid and potash are each worth $.05 per pound?

	Per Cent of Nitrogen	Per Cent of Phosphoric Acid	Per Cent of Potash
59. Horse	0.58	0.28	0.53
60. Cow	0.44	0.16	0.40
61. Pig	0.45	0.19	0.60
62. Sheep	0.83	0.23	0.67
63. Poultry	1.60	1.50	0.80

64. The following has been suggested as a good ration for a dairy cow weighing 1000 lb. and giving 15 lb. of milk per day: 10 lb. of a mixture of one third each of corn meal, ground oats and bran; 35 lb. of corn silage; 15 lb. of medium red clover. How many pounds of each would be required to feed a herd of 50 cows per day?

What would be the number of pounds of dry matter and the fertilizing constituents fed a herd of 50 cows per day, basing computations upon the following table:

	Per Cent of Dry Matter	Per Cent of Fertilizers		
		Nitrogen	Phosphoric Acid	Potash
65. Corn meal	87.1	1.58	.63	.40
66. Oats	88.9	1.86	.77	.59
67. Bran	88.3	2.67	2.89	1.61
68. Silage	22.0	.28	.11	.37
69. Clover	88.7	2.07	.38	2.20

70. If 80 % of the fertilizing constituents of the feed for the 50 cows in examples 65 to 69 are recovered in the manure, what would be the amount of nitrogen, phosphoric acid, and potash in the manure for a period of 365 days?

71. The surface 8 inches of Sheboygan County (Wisconsin) peat soil weighed about 350,000 pounds per acre. It contained: nitrogen 3.39 %, phosphoric acid .157 %, and potash .485 %. How many pounds of each were there in an acre of this surface soil?

To find what per cent one number is of another.

STUDY RECITATION

1. What *fractional part* of 10 is 8? What *decimal part?*

SOLUTION. 8 is $\frac{8}{10}$, or $\frac{4}{5}$, of 10; 8 is $\frac{8}{10}$, or .8, of 10.

What per cent of 10 is 8?

SOLUTION. 8 is $\frac{8}{10}$ of 10; $\frac{8}{10} = .80$, or 80 %.

2. 7 is what per cent of 14?

SOLUTION. 7 is $\frac{7}{14}$, or $\frac{1}{2}$, of 14. $\frac{1}{2} = 50$ %.

Notice that this is a special case of the problem *To find what fractional part one number is of another*. See p. 136.

What per cent of:

3. 25 is 5?	**6.** \$48 is \$6?	**9.** 10 is 50?	**12.** 1 is $\frac{1}{2}$?
4. 60 is 2?	**7.** \$54 is \$9?	**10.** 12 is 48?	**13.** $\frac{3}{4}$ is $\frac{1}{4}$?
5. 72 is 9?	**8.** \$42 is \$7?	**11.** 16 is 64?	**14.** $\frac{4}{5}$ is $\frac{1}{5}$?

SOLUTION TO EX. 9. 50 is $\frac{50}{10}$, or $\frac{500}{100}$, or 500 %, of 10.

SOLUTION TO EX. 13. To find what per cent $\frac{1}{4}$ is of $\frac{3}{4}$, divide $\frac{1}{4}$ by $\frac{3}{4}$.
$\frac{1}{4} \div \frac{3}{4} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3} = 33\frac{1}{3}$ %. Or, $\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3} = \frac{1}{3} = 33\frac{1}{3}$ %.

15. What per cent of $2460 is $123?

$$\begin{array}{r} .05 = 5\% \\ \$2460\overline{)\$123.00} \\ 123\ 00 \\ \hline \end{array}$$

$123 is $\frac{123}{2460}$ of $2460. Divide $123 by $2460. The result is .05, or 5%.

Use the number following "what per cent of" as divisor and the other number as dividend, and express the quotient as per cent.

Written

What per cent of:

16. 480 is 180?	**20.** $1600 is $600?	**24.** 500 is 600?
17. 1452 is 252?	**21.** $167 is $110.22?	**25.** 400 is 1200?
18. 2848 is 356?	**22.** $360 is $240?	**26.** 350 is 750?
19. 1989 is 221?	**23.** $185 is $111?	**27.** 600 is 720?

PROBLEMS

Oral

1. What per cent of a quart is a pint?

2. What per cent of a yard is a foot?

3. A man earns $30 a month. His expenses for the year are $240. What per cent of his earnings does he save?

4. A boy earns $6 a week, and pays $4 a week for living expenses. What per cent of his weekly wages does he save?

5. If cloth is bought at 50¢ a yard and sold at 60¢ a yard, the gain is what per cent of the cost? The selling price is what per cent of the cost?

6. If a milkman sells milk at 7¢ a quart, which costs 5¢, what per cent of the cost does he gain?

7. In a field of 20 acres there are 15 acres of corn. What per cent of the field is corn?

8. From a pile of wood containing 24 cords, 20 cords are sold. What per cent of the wood is sold? What per cent remains unsold?

9. 8 hours is what per cent of a day? 15 seconds is what per cent of a minute?

10. 1 ounce is what per cent of a pound?

Written

The following is the equipment for four tables in a domestic science kitchen:

Iron ware	$2.00	Wooden ware	$ 2.50
Tin ware	3.50	Granite and enamel ware	14.00
Cutlery	5.50	Earthenware and glassware	6.50
Wire ware	3.00	Silverware	9.00
		Total	$46.00

What per cent of the total expense is for

11. Iron ware?
12. Tin ware?
13. Cutlery?
14. Wire ware?
15. Wooden ware?
16. Granite and enamel ware?
17. Earthenware and glassware?
18. Silverware?

Analyses of several brown silt loams found in Illinois showed the following weights of plant food constituents in 2,000,000 lb. of soil:

Soil	Nitrogen	Phosphoric Acid	Potash
19. 1	4370	1170	32240
20. 2	4840	1200	32940
21. 3	4290	1190	35340
22. 4	4910	1220	32960
23. 5	5050	1190	36250
24. 6	6750	1410	45020

Find the per cent of nitrogen, phosphoric acid, and potash in each soil.

25. A prize cow yielded, during one year, 27404.4 lb. of milk that contained 1058.34 lb. of butter fat. What was her test?

26. Another cow yielded, during one year, 26458.5 lb. of milk that contained 1058.34 lb. of butter fat. What was her test?

27. The record for Guernseys is held by Spottswood Daisy Pearl. In one year she gave 18602.75 lb. of milk which contained 957.38 lb. of butter fat. What was her average test?

28. Eminent's Bess holds the world's record for Jersey cows. During the year closing Sept. 11, 1913, she produced 18782.88 lb. of milk which contained 1132.75 lb. of butter fat. What was the average per cent of butter fat in her milk?

29–32. In examples 32–35 on p. 293, find the percentage of profit above the cost of feed for each period.

To find a number when a certain per cent of it is given.

Study Recitation

1. 9 is $\frac{1}{4}$ of what number? 9 is .25 of what number? 9 is 25% of what number?

Notice that this is a special case of the problem *To find a number when a fractional part of it is given.* See p. 140.

2. 12 is $\frac{1}{5}$ of what number? 20% of what number?

3. 16 is $\frac{1}{2}$ of what number? 50% of what number?

4. 80 is $\frac{2}{5}$ of what number?

Solution. If 80 is $\frac{2}{5}$ of a number, $\frac{1}{5}$ of it $= \frac{1}{2}$ of 80, or 40; $\frac{5}{5}$ of it $= 5 \times 40$, or 200. Or, $80 \div \frac{2}{5} = \overset{40}{\cancel{80}} \times \frac{5}{\cancel{2}} = 200$.

5. 80 is 40% of what number?

SOLUTION. 1% of the number = $\frac{1}{40}$ of 80, or 2; 100% of the number = 100 × 2, or 200. Or, 80 ÷ .40 = 200.

6. 14 is 3% of what number?

SOLUTION. Substituting $\frac{3}{100}$ for 3%, we have,

14 is $\frac{3}{100}$ of what number?

If 14 is $\frac{3}{100}$ of some number, $\frac{1}{100}$ of that number = $\frac{1}{3}$ of 14, or $\frac{14}{3}$, and $\frac{100}{100}$ of the number = 100 × $\frac{14}{3}$, or $\frac{1400}{3}$, or $466\frac{2}{3}$.

ORAL

Change the following examples in percentage to examples in fractions. Reduce any number of hundredths to lowest terms:

PERCENTAGE	COMMON FRACTIONS
7. 8 is 25% of what number?	Changed, becomes 8 is $\frac{1}{4}$ of what number?
8. 24 is $33\frac{1}{3}$% of what number?	Changed, becomes 24 is $\frac{1}{3}$ of what number?
9. 12 is $66\frac{2}{3}$% of what number?	Changed, becomes 12 is $\frac{2}{3}$ of what number?
10. 35 is 50% of what number?	Changed, becomes ____________
11. 16 is 80% of what number?	Changed, becomes ____________
12. $5\frac{1}{2}$ is 25% of what number?	Changed, becomes ____________
13. 7 is 20% of what number?	Changed, becomes ____________
14. 9 is 75% of what number?	Changed, becomes ____________

Written

When a certain per cent of a number is given, the number may also be found without reducing the per cent to a fraction.

15. 21 is 7 % of what number?

$21 \div 7 = 3.$ $100 \times 3 = 300.$ Or,

$$.07.\overline{)21.00.}\quad 3\,00$$

If 21 is 7 % of a number, 1 % of the number is $\frac{1}{7}$ of 21, or 3. 100 % of the number is 100 times 3, or 300.

I. *To find a number when a certain per cent of it is given, divide the number expressing the part by the rate per cent used as a whole number, and multiply the quotient by* 100. *Or,*

II. *Divide the number expressing the part by the rate per cent used as a decimal.*

The second rule is more convenient when the number expressing the rate per cent will not exactly divide the number expressing the part.

16. 25 is 8 % of what number?

$$8\overline{)25}\quad 3\tfrac{1}{8}$$

$$\begin{array}{r} 100 \\ 3\frac{1}{8} \\ \hline 12\frac{1}{2} \\ 300 \\ \hline 312\frac{1}{2}, \text{ or} \\ 312.5 \end{array}$$

Or,

$$.08.\overline{)25.00.0}\quad 3\,12.5$$

17. 24 is 12 % of what number?

Analysis. If 24 is 12 % of a number, 1 % of the number is $\frac{1}{12}$ of 24, or 2. 100 % of the number is 100×2, or 200.

18. 28 is 7 % of what number? 4 %? 10 %? 14 %? 5 %?

19. 18 is 9 % of what number? 3 %? 6 %? 10 %? 2 %?

20. 24 is 6 % of what number? 12 %? 4 %? 8 %? 3 %?

21. 30 is 30 % of what number? 10 %? 15 %? 5 %?

22. 75 is 75 % of what number? 25 %? 5 %? 15 %?

23. 60 is 10 % of what number? 6 %? 5 %? 15 %?

24. 100 is 25 % of what number? 50 %? 10 %? $12\frac{1}{2}$ %?

25. 33 is 11 % of what number? 3 %? 10 %?

26. 40 is 10 % of what number? 5 %? 8 %? 20 %?

27. 21 is 3 % of what number? 7 %? 10 %? $12\frac{1}{2}$ %?

28. 45 is 5 % of what number? 9 %? 10 %? $33\frac{1}{3}$ %?

The above analysis requires the application of the first rule.

Written

Solve the following examples by the second rule:

29. 15 is 7 % of what number?

$$\begin{array}{r} 2\,14\frac{2}{7} \\ ,07.\overline{)15,00.} \end{array}$$

30. 19 is 7 % of what number?

31. 256 is 16 % of what number? 8 %? 10 %?

32. 250 is 12 % of what number? 15 %? 18 %?

33. 12.5 is 4 % of what number? 1 %? 10 %?

34. 3.25 is 6 % of what number? 10 %? $12\frac{1}{2}$ %?

35. 46.8 is 8 % of what number? 16 %? 20 %?

Give the numbers of which the number at the left in each line is a given per cent:

36. \$18.50 is 20 %, $33\frac{1}{3}$ %, $66\frac{2}{3}$ %, of what number?

SOLUTION. \$18.50 is 20 %, or $\frac{1}{5}$, of \$92.50.

37. \$12.36 is $8\frac{1}{3}$ %, 75 %, 40 %, 60 %, 80 %, $37\frac{1}{2}$ %, of what number?

38. \$24.30 is $66\frac{2}{3}$ %, $33\frac{1}{3}$ %, $62\frac{1}{2}$ %, $16\frac{2}{3}$ %, $12\frac{1}{2}$ %, of what number?

39. $\frac{3}{4}$ is 20 %, 25 %, 30 %, $37\frac{1}{2}$ %, 10 %, $12\frac{1}{2}$ %, of what number?

Finding a number when a number increased or decreased by a certain per cent of itself is given.

Study Recitation

If a number is *increased* by $\frac{1}{4}$ of itself, it equals $1\frac{1}{4}$ times the original number, or $\frac{5}{4}$ of it.

If a number is *decreased* by $\frac{1}{4}$ of itself, it equals $\frac{3}{4}$ of the original number.

1. *a.* If $\frac{5}{4}$ of a number is 25, what is the number?

b. If 125 % of a number is 25, what is the number?

c. If a number increased by 25 % of itself equals 25, what is the number?

2. *a.* If $\frac{3}{4}$ of a number is 18, what is the number?

b. If 75 % of a number is 18, what is the number?

c. If a number decreased by 25 % of itself equals 18, what is the number?

3. What number increased by 20 % of itself equals 18? 24? 30? 60? 90?

4. What number decreased by $33\frac{1}{3}$ % of itself equals 4? 8? 12? 16? 20? 24?

5. What number increased by $12\frac{1}{2}$ % of itself equals 18? 27? 45? 36? 54? 63? 72? 81?

6. What number increased by 6 % of itself equals 318?

$$100\% + 6\% = 106\%$$

$$\begin{array}{r} 3\,00 \\ 1.06.\overline{)318.00.} \end{array}$$

Since 318 equals 1.06 times the number, the number equals 318 ÷ 1.06.

Divide the number expressing the part by 100 % plus the given rate per cent.

7. What number decreased by 5 % of itself equals 380?

$100\% - 5\% = 95\%$

$$\begin{array}{r} 4\,00 \\ .95.\overline{)380.00.} \end{array}$$

Since 380 equals .95 of the number, the number equals 380 ÷ .95, or 400.

Divide the number expressing the part by 100 % *minus the given rate per cent.*

WRITTEN

What number *increased* by

8. 18 % of itself = 472?
9. $12\frac{1}{2}$ % of itself = 810?
10. 35 % of itself = $ 1080?
11. 16 % of itself = $ 3480?
12. 15 % of itself = $ 2300?
13. 20 % of itself = $ 1800?
14. 25 % of itself = $ 2500?
15. $37\frac{1}{2}$ % of itself = $ 2200?
16. 40 % of itself = $ 1400?
17. $62\frac{1}{2}$ % of itself = $ 2600?
18. $87\frac{1}{2}$ % of itself = $ 3000?
19. 5 % of itself = $ 2100?
20. 6 % of itself = $ 3180?
21. 10 % of itself = $ 3300?
22. 15 % of itself = $ 3450
23. 18 % of itself = $ 2360?
24. 32 % of itself = $ 2640?
25. 14 % of itself = 2280?
26. 16 % of itself = 3480?
27. 19 % of itself = 2380?

What number *decreased* by

28. 18 % of itself = 410?
29. $12\frac{1}{2}$ % of itself = 700?
30. 35 % of itself = $ 2600?
31. 16 % of itself = $ 7560?
32. 15 % of itself = $ 6800?
33. 25 % of itself = $ 6000?
34. 20 % of itself = $ 1800?
35. $37\frac{1}{2}$ % of itself = $ 5500?
36. 5 % of itself = $ 3800?
37. 7 % of itself = $ 1860?
38. 8 % of itself = $ 1840?
39. 9 % of itself = $ 9100?
40. 10 % of itself = $ 1800?
41. 11 % of itself = $ 8900?
42. 28 % of itself = $ 7200?
43. 50 % of itself = $ 1500?
44. 60 % of itself = $ 4000?
45. 35 % of itself = 6500?
46. 40 % of itself = 6000?
47. 18 % of itself = 8200?

PROFIT AND LOSS

The application of percentage to (1) profit and loss, (2) trade discount, (3) commission, (4) insurance, (5) taxes, and (6) duties involves nothing new except terms and business usages.

The **cost** of anything is the amount paid for it by the buyer.

The **selling price** is the amount received by the seller.

The **gain** or **profit** is the excess of the selling price over the cost.

The **loss** is the excess of the cost over the selling price.

The **list price** of an article is the amount asked for it.

The gain or loss per cent is always reckoned on the cost price. The cost is 100% of itself.

Study Recitation

1. A merchant bought a coat for \$15 and sold it for \$20. How much did he gain? What per cent did he gain?

2. A merchant bought a coat for \$15 and sold it for \$10. How much did he lose? What per cent did he lose?

3. A merchant gained \$5 by selling a coat for \$20. What was the cost?

4. A merchant lost \$5 by selling a coat for \$20. What was the cost?

5. I bought a horse for \$120 and sold it at a profit of 25%. Find the gain and the selling price.

Analysis by Direct Statement

25% of the cost = $\frac{1}{4}$ of it.

$\frac{1}{4}$ of \$120 = \$30, gain.

\$120 + \$30 = \$150, selling price.

Analysis by Question and Answer

What is 25% of \$120? *Ans.* \$30. \$30 = gain. If a horse was bought for \$120 and sold at a profit of \$30, what was the selling price? *Ans.* \$150.

6. I bought a horse for \$120 and sold it for \$150. Find the gain per cent.

ANALYSIS BY DIRECT STATEMENT

\$150 − \$120 = \$30, gain.
\$30 is $\frac{30}{120}$, or $\frac{1}{4}$, of \$120.
$\frac{1}{4}$ of 100% = 25%, gain.

ANALYSIS BY QUESTION AND ANSWER

If a horse is bought for \$120 and sold for \$150, what is the gain? *Ans.* \$30. \$30 is what per cent of \$120? *Ans.* 25%.

7. I sold a horse for \$150, thereby gaining 25%. Find the cost.

ANALYSIS BY DIRECT STATEMENT

100% of the cost + 25% of it = 125% of the cost = selling price, \$150.
125% of the cost = $\frac{5}{4}$ of it.
$\frac{5}{4}$ of the cost = \$150.
$\frac{4}{4}$ of the cost = $4 \times \frac{1}{5}$ of \$150, or \$120, cost.

ANALYSIS BY QUESTION AND ANSWER

If a horse is sold at a gain of 25%, for what per cent of the cost is it sold? *Ans.* 125%. 125% of a number is what fraction of it? *Ans.* $\frac{5}{4}$ of it. \$150 is $\frac{5}{4}$ of what number? *Ans.* \$120. \$120 = cost.

TO THE TEACHER. Lead pupils to see what is given and what is asked for in each of the three problems above, and how each problem may be derived from either of the others. Require pupils to give original problems like any one of the three, and then to derive from these original problems two others like those above.

PROBLEMS

ORAL

Each of the following problems may be changed so as to become a problem under one or more of the three general problems in percentage, decimals, or fractions.

1. A lot costing \$1600 is sold at a profit of $12\frac{1}{2}$%. What is the gain? the selling price?

2. A 50-cent knife is sold for 70 cents. Find the gain per cent.

3. If 100% is gained by selling a lace collar for \$1.50, how much would be gained by selling it for \$1?

4. A jeweler sold a watch for \$32, thereby gaining $33\frac{1}{3}\%$. Find the gain and the cost.

5. By selling corn at 54 cents a bushel, I lost 10%. Find the cost and the loss.

6. What per cent do I lose when I buy at 20 cents and sell at 10 cents? What per cent do I gain when I buy at 10 cents and sell at 20 cents?

7. A 60-acre farm costing \$4800 is sold at a gain of $12\frac{1}{2}\%$. Find the selling price.

8. A merchant buys pianos at \$250 apiece. How much must he charge for them in order to make a profit of 20%?

9. What must be the list price of a watch costing \$36 in order that a profit of 25% may be made on it?

10. What is the per cent of profit on apples bought for \$1.50 a bushel and sold for \$2.40 a bushel?

Written

11. For how much must I sell apples costing \$2.40 a barrel to gain $37\frac{1}{2}\%$?

12. I lose 30% in selling a suit for \$21.70. Find the cost.

13. I lose 10% in selling an article for \$15.30. What would have been the selling price had I gained 10%?

14. A house costing \$3750 is sold for \$3975. What is the gain per cent?

15. An article costing \$7.50 is sold at a loss of 20%. The purchaser sells it for \$10. What is his gain per cent?

16. A piano costing \$150 is sold at a profit of 20%. The retail merchant who buys the piano pays \$5 freight, and sells it at a profit of 25%. Find the selling price.

17. A merchant sells carpet at \$.75 a yard, and gains 25%. What would be the gain if the carpet were sold at \$.85 a yard?

18. By selling an article for \$7.20 I gain $12\frac{1}{2}$%. What per cent should I gain by selling it for \$7.50?

19. A dealer sells two wagons for \$75 each. On one he gains 25%, and on the other he loses 25%. Does he gain or lose by the transaction, and how much?

ANALYSIS BY QUESTION AND ANSWER

A gain or a loss of 25% is what part of the cost? *Ans.* $\frac{1}{4}$ the cost. If a gain of $\frac{1}{4}$ the cost is made by selling a wagon for \$75, \$75 is what fraction of the cost? *Ans.* $\frac{5}{4}$ of the cost. \$75 is $\frac{5}{4}$ of what sum? *Ans.* \$60. \$60 = cost.

If a loss of $\frac{1}{4}$ the cost results from selling a wagon for \$75, \$75 is what fraction of the cost? *Ans.* $\frac{3}{4}$ of the cost. \$75 is $\frac{3}{4}$ of what sum? *Ans.* \$100, or cost. (\$100 + \$60), cost, − \$150, selling price, = \$10, loss.

20. I sell two machines at \$150 each. On the first I gain 25%, and on the second I lose 20%. Do I gain or lose, and how much?

21. A boy buys 6 apples for a dime, and sells them at the rate of 2 for 5¢. What is his gain per cent?

22. A boy buys apples at the rate of 5 for 5¢, and sells them at the rate of 3 for 5¢. What is his per cent of gain?

23. A boy buys 120 oranges at the rate of 4 for 10¢, and sells them at the rate of 3 for 10¢. What is his per cent of gain or loss? What is his per cent of gain or loss if 10% of the oranges are unsalable?

24. A farmer sells 75 sheep for what 100 cost. Find his gain per cent.

SUGGESTION. 75 sheep cost $\frac{3}{4}$ as much as 100 sheep; therefore he gains $\frac{1}{3}$ of the cost.

25. A merchant buys bankrupt stock at 45¢ on the dollar, and sells it at 10% below the original list price. What is his gain per cent? What is meant by "buying at 45¢ on the dollar"?

26. An article costing $10 is sold for $8.50. What per cent is lost? Another article costing $8.50 is sold for $10. What per cent is gained?

27. Mr. Lewis bought 12,000 bushels of wheat at $.60 a bushel, and 6500 bushels of corn at $.55 a bushel. In selling, he gained $16\frac{2}{3}$% on the wheat and lost 20% on the corn. What per cent did he gain or lose on the whole transaction?

28. At what price must goods costing $40 be listed so that if they are sold at $16\frac{2}{3}$% below the list price a profit of 25% may be realized?

Suggestion. At what price must goods costing $40 be sold to realize a profit of 25%? *Ans.* $50. If $50 is received for goods sold at $16\frac{2}{3}$% below the list price, what is the list price? $50 is $\frac{5}{6}$ of what number?

29. A quantity of wheat which cost $1.16 per bushel was sold at a loss of 25%. How many bushels were sold if the total loss was $37.70?

Suggestion. (*a*) What is the loss on 1 bushel? Then $37.70 is the loss on how many bushels?

(*b*) If $37.70 is $\frac{1}{4}$ of the total cost, what is the total cost? Then how many bushels at $1.16 can be bought for this amount?

30. Fifty pounds of soap are purchased at 5 cents a pound. In a damp cellar it absorbs water equal to 10% of its weight, and is then sold at 6 cents a pound. What is the gain? the gain per cent?

31. I sell $\frac{3}{5}$ of an article for what $\frac{3}{4}$ of it costs. Find the gain per cent.

Suggestion. I sell $\frac{1}{5}$ of the article for $\frac{1}{4}$ of the cost and $\frac{5}{5}$ of it for $\frac{5}{4}$ of the cost.

COMMERCIAL OR TRADE DISCOUNT

Study Recitation

Manufacturers and merchants frequently deduct a certain per cent from the list price of goods when selling them or offering them for sale. This deduction is called a **discount.**

Find the selling price of the following goods offered at a bargain sale:

		Regular Price	Discount Sale Price
1.	Cloth	\$1.60 a yd.	$12\frac{1}{2}$%
2.	Gowns	\$35	20%
3.	Handkerchiefs	\$2.00 a doz.	25%
4.	Suits	\$48	$37\frac{1}{2}$%
5.	Umbrellas	\$4.00	30%
6.	Shoes	\$5.00	10%
7.	Stockings	\$1.25	20%
8.	Clocks	\$6.00	$66\frac{2}{3}$%
9.	Blankets	\$8.00	$37\frac{1}{2}$%

Many business houses give two or more discounts off the list price in consideration of payment in cash, changes in market price, or for other reasons.

These deductions are called **trade discounts** or **commercial discounts.**

Note. Discounts made for payments in cash are sometimes called **cash discounts**, and discounts made for payment within a certain time, as 10 days, 1 month, etc., are called **time discounts.**

The **net price** is the list price less the discounts; it is the price the buyer pays.

In trade discounts, the first discount is computed on and deducted from the list price; the second discount is computed on and deducted from the resulting remainder, etc.

10. A piano was listed at \$600 with a discount of $33\frac{1}{3}\%$ and another discount of 10 % for cash. Find the net price.

3)\$600	$33\frac{1}{3}$ of a number = $\frac{1}{3}$ of it.
200	$\frac{1}{3}$ of \$600 = \$200, first discount.
10)400	\$600 − \$200 = \$400, remainder.
40	10 %, or $\frac{1}{10}$, of \$400 = \$40, second discount.
\$360	\$400 − \$40 = \$360, net price. *Ans.*

Or, a discount of $33\frac{1}{3}\%$, or $\frac{1}{3}$, of the cost, leaves $\frac{2}{3}$ of the cost. A further discount of 10 %, or $\frac{1}{10}$, of the remainder, leaves $\frac{9}{10}$ of $\frac{2}{3}$, or $\frac{3}{5}$, of the cost. Net price = $\frac{3}{5}$ of \$600 = \$360.

Written

Find the net price of each of the following sales:

	List Price	Discounts		List Price	Discounts
11.	\$60	25 %, 20 %	**15.**	\$600	$16\frac{2}{3}$ %, 5 %
12.	\$125	20 %, 25 %	**16.**	\$800	$12\frac{1}{2}$ %, 6 %
13.	\$180	$16\frac{2}{3}$ %, 20 %	**17.**	\$840	20 %, $37\frac{1}{2}$ %
14.	\$270	$33\frac{1}{3}$ %, 25 %	**18.**	\$1000	50 %, 10 %

19. The following bill was sold at a discount of 20 %:

INDIANAPOLIS, IND., Sept. 2, 1914.

C. B. BAXTER,

MENOMONIE, WIS.,

Bought of ACME TOOL COMPANY

TERMS: 30 days net; 5 % 10 days.

2 doz. hammers	@ \$ 5.00	\$10.00	
2 doz. handsaws	@ \$12.00	24.00	
		34.00	
Less 20 %		6.80	
			\$27.20
Less 5 %			1.36
Rec'd payment, Acme Tool Company per J.			\$15.84

Mr. Baxter secured the discount of 5 % by paying his bill within 10 days. If he had paid in 30 days, his bill would have been \$27.20.

PROBLEMS

Written

1. Discounts of 20 % and 10 % are allowed on goods listed at $60. Find the discounts and the net price.

2. A gun was listed at $120, with trade discounts of $33\frac{1}{3}$ %, 25 %, and 10 %. What was the selling price?

3. A merchant sold a bill of goods listed at $200 with 20 % and 5 % off for cash. How much did he receive?

4. Which is better for the purchaser, 40 % and 20 % off, or a single discount of 53 %?

5. Find a single discount equivalent to 25 %, 20 %, and 10 % off, on a bill of goods listed at $120.

6. A merchant buys goods at 50 % and 20 % off, and sells them at 10 % and 20 % off the list price. What is his gain per cent?

7. A merchant bought 40 yd. of silk listed at 83¢ a yard, at a discount of 50 % and 10 % off. He sold the silk at 25 % and 10 % off from the list price. What was his profit?

8. What is the net amount of a bill, the list price of which is $500, trade discount 10 %, and 5 % off for cash?

9. A bill of goods amounted to $3300, but the buyer was allowed $16\frac{2}{3}$ % and 10 % off. How much did he pay?

10. G. H. King & Co., of Evansville, Ind., sells to L. N. Reed of Chicago, Ill., on account, 60 days net, 5 % 10 days: 5 sewing machines at $60 each, less 20 % discount; and 1 doz. sewing machines at $50 each, less 25 % discount. Make out, foot, and receipt the bill, and find the net amount if it is paid within 10 days; if it is not paid for 60 days.

11. Construct and solve two other problems involving trade discount.

COMMISSION AND BROKERAGE

An **agent** is a person authorized to buy, sell, or transact business for another.

An agent who buys or sells goods is often called a **commission merchant.**

An agent who buys and sells real estate, bonds, and stocks for others is called a **broker.**

Commission is a per cent paid to an agent for the transaction of business. The commission paid to a broker is often called **brokerage.**

The **proceeds** is the sum left after the commission and other expenses have been deducted.

Business usage or principle.

In selling, commission is computed on the sum received for the goods; in buying, on the sum paid for the goods; in collecting, on the sum collected.

For information concerning the commission, or brokerage, paid a broker for buying or selling stocks and bonds, see p. 376.

Study Recitation

1. An agent sells goods to the amount of \$2450. What is his commission at $2\frac{1}{2}\%$, and what are the proceeds?

Solution

$2\frac{1}{2}\%$ of a number $= .02\frac{1}{2}$ of it.
$\$2450 \times .02\frac{1}{2} = \$61.25$, commission.
$\$2450 - \$61.25 = \$2388.75$, proceeds.

2. If an agent receives \$500 for selling goods at 5 % commission, how much does he sell?

Solution

\$500 is 5% of what number? $\$500 \div .05 = \$10,000$. *Ans.*

PROBLEMS

Oral

1. An agent sold a house for $6000 at 10 % commission. What was his commission?

2. Find the commission at 2 % on 1500 bu. of wheat sold at $1 a bushel.

3. If I receive $1000 for buying goods at 10 % commission, what is the cost of the goods?

4. How much better is a commission of 6 % on $1000 than a commission of 2 % on $1500?

Written

5. An agent collected a $565 debt. What was his commission at 5 %, and how much should he remit to the creditor?

6. Mr. B. sold 16,000 bu. of wheat at $.75 a bushel on a commission of $2\frac{1}{2}$ %. What was his commission?

7. A merchant received $61.25 as his commission for selling goods to the amount of $2450. What was the rate of his commission?

8. A real estate dealer bought 6 lots at $2200 each, and charged 5 % commission. What was his commission?

9. An agent charged 5 % commission for selling a quantity of goods. He received $15 for his services. What was the selling price of the goods? How much did the party receive for whom the goods were sold?

Analyze the problem beginning with: 5 % of the selling price = $15.

10. A commission merchant sold 25 boxes of oranges each containing 12 dozen, at $.25 per dozen. Find his commission at 6 %.

11. Mr. Cook planted 10 acres in celery, paying \$60 per acre for labor, etc., and \$50 per acre for fertilizers. He cut from each acre 27,000 plants, which he sold in crates, each crate containing 5 dozen bunches of 3 plants each at \$3.00 per crate, with a commission of 4% for selling. Find Mr. Cook's profit on the 10 acres of celery.

12. A commission merchant receives turkeys weighing 3140 lb. He sells them at 11¢ a pound. How much is his commission at 5%?

13. If a commission merchant receives $2\frac{1}{2}$% commission for buying goods, how much does each dollar's worth of goods cost his principal, or the party for whom he is acting?

14. A real estate agent is offered a yearly salary of \$1800, or a commission of $2\frac{1}{2}$% on his sales. If his average monthly sales amount to \$4800, which is the better offer for him to accept? What is the difference between the two offers for one year?

15. A commission firm is ordered to buy 5000 bushels of wheat @ 75¢. The commission is $\frac{1}{8}$¢ per bushel. What is the cost of the wheat, including the commission?

16. Find the cost of 500 baskets of peaches at 70¢ a basket, the commission for buying being $4\frac{1}{2}$%.

17. Find the total cost of 1000 bales of cotton weighing 500 lb. each, at \$.08$\frac{3}{4}$ a pound, the freight being \$.50 a hundred pounds, drayage \$70, and commission 3%.

18. An agent for an automobile company received a commission of 10% on his sales. In one month he sold one automobile at \$900 and another at \$1250. How much did he earn? How much did he make above his expenses of \$75?

19. Construct and solve two problems in commission.

INSURANCE

Insurance is security against financial loss on account of destruction or loss of property, health, or life. We shall consider property insurance only.

The **policy** is the contract between the insured and the insurance company.

The **face** of the policy is the amount insured.

The **premium** is the sum paid the insurance company for taking the risk.

The **rate** is a specified sum per $100 of insurance or a certain per cent of the sum insured.

Business usage or principle.

Premiums are computed on the face of the policy, usually at a certain sum per hundred dollars. Thus, a premium of 80¢ per hundred for three years means that the company charges 80¢ for every $100 insurance for three years.

Study Recitation

1. A house valued at $24,000 is insured for three years for $\frac{2}{3}$ of its value at 80¢ a hundred. What is the amount of premium to be paid?

Solution

$\frac{2}{3}$ of $24,000 = $16,000, face of policy.

$$\frac{\$16,000}{100} \times .80 = \$128, \text{ premium.}$$

PROBLEMS

Oral

1. How much must I pay for a policy of $1000 on my furniture at 50¢ a hundred for one year?

2. A ship worth $80,000 was insured for $\frac{7}{8}$ of its value at $1.00 per hundred. Find the premium.

WRITTEN

3. Mr. Brown insured his house for \$3500, his barn for \$850, and his personal property for \$1250, at 90 ¢ a hundred for three years. Find the premium.

4. Find the cost of insuring a piano worth \$440 for full value at $\frac{3}{4}\%$.

5. Find the cost of insuring a house worth \$12,000 for $\frac{3}{5}$ of its value at $\frac{1}{2}\%$.

6. I insured a house worth \$24,800 for $\frac{7}{8}$ of its value at $\frac{7}{8}\%$. Find the premium.

7. A building valued at \$40,000 is insured for three years for $\frac{7}{8}$ of its value, at 80 ¢ a hundred. What is the amount of the premium to be paid?

8. A factory was insured for \$15,000, the rate of premium being \$40 per \$1000. After 3 annual premiums had been paid, a fire occurred. The stock saved was valued at \$4800. How much insurance did the owner realize after deducting the cost of insurance?

9. A company charges a premium of \$39.20 on \$4900. What is the premium per hundred? the rate per cent?

10. An insurance company whose rate is 80 ¢ a hundred charges Mr. Goodyear \$278.20 premium. Find the face of the policy.

11. After three annual payments of \$500 had been made on a factory, the premium being at 2% on $\frac{7}{8}$ of the value of the factory, it was burned. Find the loss to the insurance company. What was the owner's loss by the fire?

12. Construct and solve two problems in insurance.

TAXES

A **tax** is a sum of money assessed on property for public use.

The **assessed valuation** of property is its estimated value for the apportionment of taxes.

The **rate of taxation** is the amount of tax per dollar of assessed valuation. Thus, a rate of .0285 means that \$.0285 are charged on every dollar of assessed valuation.

Principle.

The tax is computed on the assessed valuation.

Study Recitation

1. In a township whose assessed valuation is \$625,000, a tax of \$15,000 is to be raised. What is the rate of taxation, and what is A's tax if his property is assessed at \$10,500?

Solution

\$15,000 is $\frac{15000}{625000}$, or $\frac{3}{125}$, of \$625,000.

$\frac{3}{125}$ of 100% = $2\frac{2}{5}$%, rate of taxation.

$2\frac{2}{5}$% of a number = .024 of it, expressed as the decimal of a dollar.

\$.024 is the rate of taxation in cents and mills per dollar.

\$10,500 × .024 = \$252, A's tax.

PROBLEMS

Oral

1. What is B's tax if the rate of taxation is $1\frac{1}{2}$% and his assessed valuation is \$1000?

2. In a certain city whose assessed valuation is \$1,000,000, a tax of \$10,000 is to be raised. Find the rate of taxation and Mr. Healy's tax on property assessed at \$6000.

3. In a certain township the tax was \$.017 on a dollar. What was the tax on a farm assessed at \$10,000?

Written

4. Which would be more profitable for me, and how much more profitable, — to rent my house for $50 a month, taxes and repairs costing $140 annually; or to sell it for $5000 and invest the money at 5%?

5. In a township whose assessed valuation is $900,000, a tax of $10,000 is to be raised. What is the rate of taxation, expressed in cents and mills?

6. Mr. Field owns property worth $8500. He pays yearly $100 taxes, $50 for repairs, and $30 for other expenses. For how much per month must he rent his property in order to make 4% on his investment?

7. The tax rate in a city is 1.4%. Find the yearly tax on a house and lot valued at $6700 and furniture worth $300.

8. A town has an assessed valuation of $4,500,000 and wishes to levy a tax of $35,000 for bridges. What must be the rate of taxation?

9. Construct and solve two problems in taxes.

FEDERAL INCOME TAX

The income tax law of the United States, passed in 1913, requires that every single person shall pay an annual tax of 1% on his net income in excess of $3000, and married persons living together an annual tax of 1% on their joint income in excess of $4000. When the net income exceeds $20,000, an additional tax, or **surtax**, thereon must be calculated as per schedule below:

1% on amount over $20,000 and not exceeding $50,000.
2% on amount over $50,000 and not exceeding $75,000.
3% on amount over $75,000 and not exceeding $100,000.
4% on amount over $100,000 and not exceeding $250,000.
5% on amount over $250,000 and not exceeding $500,000.
6% on amount over $500,000.

Study Recitation

1. Albert Blake and his wife together have a gross income as follows: Salary, \$5000; interest on bonds, \$1000; rent on house, \$1500. To find their net income, Mr. Blake deducts the following expenses: Taxes, \$200; repairs, \$100; interest on mortgage, \$500. On his net income he is allowed an exemption of \$4000. What is his income tax on the remainder at 1%?

Solution. Gross income	\$7500
General deductions	800
Net income	6700
Exemption	4000
Taxable income on which 1% is to be calculated	\$2700

Income tax, 1% of \$2700 = \$27.

2. Find the income tax of a man living with his wife who has a net income of \$110,000.

Solution. \$110,000 – \$4000 exemption = \$106,000

Normal tax of 1% on \$106,000	=	\$1060
Surtax of 1% on (\$50,000 – \$20,000) \$30,000	=	300
Surtax of 2% on (\$75,000 – \$50,000) \$25,000	=	500
Surtax of 3% on (\$100,000 – \$75,000) \$25,000	=	750
Surtax of 4% on (\$106,000 – \$100,000) \$6000	=	240
		\$2850

Written

Find the Federal income tax of a man whose exemption is \$4000 on a net income of:

3. \$4500 **5.** \$25,000 **7.** \$60,000 **9.** \$120,000 **11.** \$240,000

4. \$6500 **6.** \$80,000 **8.** \$76,000 **10.** \$180,000 **12.** \$510,000

Find the Federal income tax of a woman whose exemption is \$3000 on a net income of:

13. \$4600 **15.** \$15,000 **17.** \$79,000 **19.** \$190,000 **21.** \$250,000

14. \$3500 **16.** \$28,000 **18.** \$58,000 **20.** \$260,000 **22.** \$520,000

DUTIES

Duties or **customs** are taxes levied by the government on imported goods.

An **ad valorem duty** is a tax computed upon the price paid for the goods; as, an ad valorem duty of 75% on the cost.

A **specific duty** is a tax on the quantity of goods without regard to their cost; as, a specific duty of 15 cents a yard, or 25 cents a pound.

An **invoice** is a bill accompanying the imported goods, showing their quantity and value.

A **tariff** is a schedule of duties.

NOTE. Duties are collected at **customhouses** which are found at ports of entry.

Principle.

Ad valorem duties are computed at a certain per cent of the value of the goods, and specific duties at a certain sum per unit of weight, length, etc.

STUDY RECITATION

1. What is the duty, at 30% ad valorem, on 16,500 clocks invoiced at $1.60 each?

SOLUTION

16,500 × $1.60 = $26,400, cost.
30% of a number = .30 of it.
$26,400 × .30 = $7920, duty.

2. What is the duty, at $1.85 a pound, on 45 cases of tobacco of 185 lb. each?

SOLUTION

45 × 185 lb. = 8325 lb.
8325 × $1.85 = $15,401.25

PROBLEMS

Written

1. Find the duty on 5000 bu. of oats and 10,000 bu. of barley, the duty on oats being $.06 per bushel and on barley $.15 per bushel.

2. S. M. Giles of Chicago imported from Geneva 150 watches invoiced at $55 each, and 250 diamonds at $80 each. What was the duty, the rate being 30 % ad valorem on watches and 10 % on diamonds?

3. Find the duty, at 20 % ad valorem, on 2260 lb. of cheese invoiced at 40¢ a pound.

4. What is the duty, at 25 % ad valorem, on blankets valued at $225?

5. Find the duty, at 50 % ad valorem, on a rug valued at $2800.

6. Stewart & Co. imported from England 18 doz. silk umbrellas invoiced at $2 each. How much must they pay for the invoice, including ad valorem duty at 35 %?

7. Find the duty, at 25 % ad valorem, on an invoice of $300 worth of blankets.

8. What is the duty, at 30 % ad valorem, on 500 dozen table knives valued at $7 a dozen?

9. J. Ransom & Co. imported from Havana 75 hhd. of molasses, at $5 a hogshead; and 800 boxes of cigars (each box weighing 1 lb. and containing 100 cigars), at $8 per box. 7 % was allowed for leakage on the molasses. Find the duty at 15 % ad valorem on the molasses, and $4.50 per pound plus 25 % ad valorem on the cigars.

10. Construct and solve two problems in customs and duties.

REVIEW

1. Define per cent; cost; selling price; profit; loss; trade discount; cash discount; net price; commission; agent; proceeds; insurance; policy; premium; tax; assessed valuation; specific duty; ad valorem duty; invoice; tariff.

2. Give the rules for expressing per cents as common fractions and common fractions as per cents.

Problems without Numbers

3. How can you find any per cent of a number?

4. How can you find what per cent one number is of another?

5. How can you find a number when a certain per cent of it is given?

6. How can you find a number when the number increased or decreased by a certain per cent of itself is given?

7. If you know the cost and the selling price how can you find the gain or the loss per cent?

8. If you know the selling price and the per cent of gain or loss how can you find the cost?

9. If you know the gain and the per cent of gain how can you find the cost? the selling price?

10. If you know the list price and several rates of discount, how can you find the net price?

11. If you know the amount of a sale and the rate of commission, how can you find the amount of commission?

12. If you know the amount insured and the rate of insurance, how can you find the premium?

13. If you know the assessed valuation and the rate of taxation, how can you find the amount of tax?

14. If you know the amount of an invoice and the rate ad valorem, how can you find the amount of duty?

GENERAL REVIEW

Written

Time yourself with these exercises. Then work them again, trying to beat your record.

Write from dictation, add, and test results.

	a.	*b.*	*c.*	*d.*
1.	978609	825499	$5438.12	$5382.49
	183214	943888	9321.42	3892.38
	536189	797677	4432.65	4781.44
	741062	684369	7567.83	1672.55
	387418	432408	6098.74	2563.66
2.	$38\frac{5}{8}$	$23\frac{3}{10}$	$87\frac{5}{8}$	$42\frac{5}{6}$
	$32\frac{3}{4}$	$44\frac{1}{15}$	$60\frac{5}{16}$	$25\frac{2}{3}$
3.	1.2475	32.08	384.9877	990.09
	43.8054	1.043	84.456	43.
	56.0205	942.987	469.234	674.0004
	75.7189	853.897	794.0089	.0005
4.	3 ft. 9 in.	5 pk. 7 qt.	5 lb. 4 oz.	4 doz. 3
	5 ft. 6 in.	6 pk. 3 qt.	3 lb. 3 oz.	5 doz. 9

Subtract and test:

5.	874321	453621	$9008.95	$1945.72
	595698	321069	5123.48	1134.54
6.	$56\frac{1}{4}$	$39\frac{2}{3}$	$74\frac{5}{8}$	$31\frac{3}{15}$
	$40\frac{1}{8}$	$18\frac{5}{6}$	$62\frac{3}{16}$	$15\frac{1}{5}$
7.	987.04	87.0802	518.	418.056
	34.5109	49.34	.0762	219.8

	a.	*b.*	*c.*
8.	16 yd. 2 ft.	20 mi. 8 rd.	15 wk. 2 da.
	12 yd. 2 ft.	19 mi. 16 rd.	8 wk. 5 da.

Find the products, canceling when possible:

	a.	*b.*	*c.*	*d.*
9.	953×8742	$\frac{3}{4} \times \frac{5}{8}$	5.06×3.087	$215 \times \$64.18$
10.	278×6209	$\frac{5}{16} \times \frac{5}{6}$	$.034 \times 1.0275$	$23\frac{1}{3} \times 14\frac{2}{5}$
11.	$1650 \times \$328.56$	$25\frac{1}{4} \times 13\frac{1}{3}$	$.07 \times .0057$	1.009×25.08

Find the quotients and remainders. Test answers.

	a.	*b.*	*c.*	*d.*
12.	$80565 \div 615$	$\frac{5}{8} \div \frac{3}{4}$	$.875 \div .025$	4 ft. 6 in. $\div 4$
13.	$908800 \div 568$	$\frac{3}{5} \div \frac{3}{4}$	$38.844 \div 16.6$	4 gal. 1 qt. $\div 8$
14.	$\$6318.90 \div 900$	$15\frac{5}{6} \div 3\frac{1}{6}$	$388.44 \div 2.34$	$.50463 \div .089$

15. Find the ratio of 12 to 48; of 48 to 12; of $\frac{3}{4}$ to $\frac{3}{8}$.

16. Find the ratio of 1 in. to 5 ft.; 5 ft. to 1 in.

Reduce to improper fractions, whole, or mixed numbers.

	a.	*b.*	*c.*	*d.*	*e.*
17.	$2\frac{1}{4}$	$3\frac{5}{8}$	$7\frac{1}{2}$	$5\frac{3}{5}$	$6\frac{3}{4}$
18.	$\frac{25}{3}$	$\frac{14}{7}$	$\frac{16}{3}$	$\frac{24}{4}$	$\frac{19}{5}$

19. Reduce to lowest terms: $\frac{15}{45}$; $\frac{20}{30}$; $\frac{16}{36}$; $\frac{15}{90}$; $\frac{27}{36}$; $\frac{32}{40}$.

20. Reduce to fractions having the l.c.d.: $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$; $\frac{3}{4}, \frac{3}{8}, \frac{3}{16}$; $\frac{2}{5}, \frac{3}{10}, \frac{1}{15}$; $\frac{3}{5}, \frac{1}{8}, \frac{1}{4}$.

Reduce common fractions to decimals and to per cents; and decimals and per cents to common fractions.

	a.	*b.*	*c.*	*d.*	*e.*
21.	$\frac{3}{4}$	$\frac{3}{16}$	$\frac{5}{6}$	$\frac{7}{12}$	$\frac{4}{25}$
22.	.30	$.37\frac{1}{2}$	$.16\frac{2}{3}\%$	2.25	3.75%
23.	.75	$.87\frac{1}{2}$	$.83\frac{1}{3}\%$	1.50	$3.37\frac{1}{2}$

Reduce the numbers in (1) to the denominations in (2):

	a. (1) Number	*a.* (2) Denomination	*b.* (1) Number	*b.* (2) Denomination	*c.* (1) Number	*c.* (2) Denomination
24.	8 qt.	pints	48 oz.	pounds	36 sq. ft.	sq. yards
25.	5 gal.	quarts	8000 lb.	tons	4 A.	sq. rods
26.	8 qt.	gallons	5 T.	pounds	54 cu. ft.	cu. yards
27.	16 qt.	pecks	24 in.	feet	4 cu. yd.	cu. feet
28.	8 pt.	quarts	3 ft.	inches	180 sec.	minutes

Find the cost of:

29.	164 lb. @ 25¢	18 yd. @ $83\frac{1}{3}$¢	10 yd. @ \$1.50
30.	32 lb. @ $12\frac{1}{2}$¢	24 yd. @ $37\frac{1}{2}$¢	10 lb. @ \$1.25
31.	18 lb. @ $33\frac{1}{3}$¢	48 yd. @ $16\frac{2}{3}$¢	8 gal. @ \$$1.62\frac{1}{2}$

Find the number of yards that can be bought:

	Amount	Price	Amount	Price	Amount	Price
32.	\$2	\$.10	\$20	\$$.12\frac{1}{2}$	\$50	\$$.33\frac{1}{3}$
33.	\$5	\$.50	\$21	\$$.37\frac{1}{2}$	\$26	\$$.66\frac{2}{3}$

	a. 10% of:	*b.* $33\frac{1}{3}$% of:	*c.* 25% of:	*d.* 20% of:	*e.* $133\frac{1}{3}$% of:	*f.* $187\frac{1}{2}$% of:
34.	50	\$240	200 T.	7340	24000	72000
35.	75	\$150	500 cu. in.	6890	36000	96000
36.	100	\$300	700 gal.	8250	87000	80000

Find what per cent the first number is of the second:

	a.	*b.*	*c.*	*d.*
37.	10 of 40	12 of 45	300 of 500	2 in. of 1 ft.
38.	40 of 10	45 of 12	500 of 300	1 oz. of 2 lb.

Of what number is

39.	18, 2%	10, 5%	\$210, $37\frac{1}{2}$%	\$2400, $37\frac{1}{2}$%
40.	24, 3%	16, 4%	\$100, $62\frac{1}{2}$%	\$1000, $16\frac{2}{3}$%

Find the profit or loss:

	a. Cost	*a.* Rate of Gain	*b.* Cost	*b.* Rate of Loss	*c.* Cost	*c.* Rate of Gain
41.	\$100	25 %	\$80	$12\frac{1}{2}$ %	\$500	8 %
42.	\$60	$33\frac{1}{3}$ %	\$200	18 %	\$700	4 %

Find the per cent of profit or loss:

	Cost	Selling Price	Cost	Selling Price	Cost	Selling Price
43.	\$100	\$125	\$50	\$75	\$30	\$25
44.	\$100	\$75	\$30	\$35	\$50	\$40

Find the discount and the net price:

	List Price	Rate of Discount	List Price	Rate of Discount	List Price	Rates of Discount
45.	\$100	25 %	\$300	$16\frac{2}{3}$ %	\$300	$33\frac{1}{3}$ %, 10 %
46.	\$40	$12\frac{1}{2}$ %	\$600	25 %	\$2000	25 %, 5 %

Find the commission:

	Sales	Rate of Com.	Sales	Rate of Com.	Purchase	Rate of Com.
47.	\$5000	20 %	\$1000	15 %	\$900	$1\frac{1}{2}$ %
48.	\$7200	$12\frac{1}{2}$ %	\$2000	18 %	\$8500	4 %

Find the premium on each of the following policies:

	Face of Policy	Rate	Face of Policy	Rate	Face of Policy	Rate
49.	\$1000	20¢ per \$100	\$8000	$\frac{1}{4}$ %	\$20000	$\frac{3}{5}$ %
50.	\$4050	60¢ per \$100	\$1400	$\frac{3}{4}$ %	\$30000	$1\frac{1}{2}$ %

Find the amount of taxes:

	Assessment	Tax Rate	Assessment	Tax Rate	Assessment	Tax Rate
51.	\$2000	\$.03	\$1500	$1\frac{1}{5}$ %	\$8200	$2\frac{1}{4}$ mills
52.	\$3000	\$.001	\$2550	$2\frac{1}{2}$ %	\$3600	$3\frac{1}{2}$ mills

Find the duty on each of the following imports:

	Value	Ad val. Duty	Value	Ad val. Duty	Quantity	Specific Duty
53.	\$1000	20 %	\$20000	25 %	100 lb.	\$1.50 a pound
54.	\$5600	$12\frac{1}{2}$ %	\$16000	$37\frac{1}{2}$ %	300 bu.	\$.05 a bushel

PROBLEMS

Oral

1. Make change from \$5 for a purchase of $3\frac{1}{2}$ yd. ribbon @ 24¢ and $4\frac{1}{4}$ yd. lace @ 80¢.

2. How many acres are there in my farm if it is divided into three fields of 65 acres, 14 acres, and 29 acres?

3. There were 30 pupils in school on Monday. This number was $\frac{2}{3}$ of the total enrollment. How many pupils were enrolled?

4. What number increased by $.37\frac{1}{2}$ of itself equals 440?

5. What number decreased by $.37\frac{1}{2}$ of itself equals 500?

6. How many yards of lace at 8¢ a yard can be bought for 50¢?

7. William is $\frac{2}{7}$ as old as his sister. William's age is 14 years. How old is his sister?

8. A girl saves \$1.50 a week. Her savings are $\frac{3}{4}$ of her wages. How much does she receive per week?

9. A produce dealer sells corn at 60¢ a bushel, and gains $\frac{1}{5}$ of what it cost him. What is the cost per bushel?

10. If \$49 is $87\frac{1}{2}$% of my money, how much money have I?

11. A farmer sold his farm for \$50 an acre, and gained 25% of the cost. How much did it cost him per acre?

12. A second-hand book is sold for 20% less than it cost. If it is sold for 40¢, what is the cost?

13. Oatmeal contains 15.6% protein, and rice, 7.8%. How many more pounds of protein are there in 100 lb. of oatmeal than in 100 lb. of rice?

14. Find the cost of 6 yd. of lining at $\$.12\frac{1}{2}$ a yard.

Written

15. A Camp Fire girl earned \$10 per month. She spent $\frac{1}{4}$ of it for a hat, and $\frac{1}{10}$ of the remainder for gifts, and saved the rest. How much did she save in 5 months?

16. There are 1000 pupils enrolled in school. There are 669 present. How many are absent?

17. I buy 400 apples at the rate of 4 for 1¢ and sell them at the rate of 8 for 5¢. What is the gain?

18. Make out and receipt a bill for:

$5\frac{1}{2}$ yd. velvet @ \$4.80
$2\frac{1}{4}$ yd. lace @ \$.84
$3\frac{1}{4}$ doz. buttons @ \$1.16
2 pairs gloves @ \$1.50
4 yd. lining @ \$.12$\frac{1}{2}$
$4\frac{5}{8}$ yd. silk @ \$1.20

19. After spending .16$\frac{2}{3}$ of his money, Mr. Clark had enough left to buy 18 yd. of linen at 33$\frac{1}{3}$¢ a yard. How much linen could he have bought with all his money?

20. A stenographer's salary is \$1000 a year. She spends $\frac{1}{4}$ of it for board and room, $\frac{1}{25}$ of it for laundry, $\frac{1}{4}$ of it for clothes, and $\frac{1}{10}$ of it for miscellaneous expenses. How much money does she save?

21. A class of 20 girls was provided with material for aprons. Each girl used $\frac{3}{4}$ yd. How much was needed by the class?

22. How much does Mrs. Green save by buying soap for \$2.20 for 50 bars instead of buying it in separate bars at \$.05 a bar?

23. Martha used for a cake 2 cups of sugar @ $2\frac{1}{2}$ ¢, $\frac{1}{4}$ lb. of butter @ 30 ¢, 2 eggs @ $3\frac{1}{2}$ ¢ each, $\frac{1}{2}$ pt. milk @ 8 ¢ a quart, 2 cups of flour worth 3 ¢, and baking powder and salt worth $\frac{1}{2}$ ¢. How much did the cake cost?

24. Find the duty on $4500 worth of goods at $33\frac{1}{3}$ % ad valorem.

25. If a suit was sold at $24.20, which was at a discount of 20 % from the marked price, what was the marked price?

26. A man walked $\frac{5}{12}$ of his journey the first day, $\frac{3}{8}$ of it the next day, and then had 20 miles to travel. How long was the journey?

27. I spent $\frac{1}{4}$ of my money for groceries, 20 % for house rent, $\frac{3}{8}$ for clothing, and the remainder, $133, for sundries. How much had I at first?

28. If a dress sold at $36.90 is sold at a profit of $12\frac{1}{2}$ %, what is the cost of the dress?

29. A piece of property worth $20,000 was taxed $2\frac{1}{2}$ %. What was the amount of the tax?

30. What per cent of 16 is 21? of 21 is 16? What per cent of $\frac{2}{3}$ is $\frac{7}{8}$? of $\frac{7}{8}$ is $\frac{2}{3}$?

31. What per cent of a yard is 2 ft. 9 in.?

32. What per cent of 8 is .375? of 1 ton is 75 lb.? of $27.50 is $12\frac{1}{2}$ cents?

33. A lost $33\frac{1}{3}$ % of his money and then found that $16\frac{2}{3}$ % of the remainder was $30 less than he lost. How much did he lose, and how much had he at first?

34. A merchant's debts are $10,500, and his property amounts to $8400. What per cent of his debts can he pay? How much will Mr. Grey receive, if he has a claim of $750?

35. What is the net price of a bill of goods for \$8000 sold at discounts of 25 % and 10 % ? at one discount of 30 % ?

36. What is the difference between $\frac{1}{3}$ % of \$720 and $33\frac{1}{3}$ % of \$720 ? Write both per cents as mixed decimals of two places.

37. Find the cost of insuring a house worth \$64,000 for $\frac{7}{8}$ of its value at $\frac{1}{2}$ %.

38. Find the commission on a sale of \$4600 worth of goods at $2\frac{3}{4}$ %.

39. If A's money is $33\frac{1}{3}$ % more than B's, what per cent of A's money is B's ?

40. If a man who fails in business owes \$20,000 and has only \$16,000 worth of property, what per cent of his indebtedness can he pay ? If he settles with his creditors at \$.20 on the dollar, how much of his assets will he save ?

41. Mr. Burr delivered 1000 pounds of 3.5 per cent milk at a factory. Find the value of the butter fat at \$.24 a pound.

42. If 92.4 lb. of butter fat were obtained from 2200 lb. of milk, what per cent did the milk test ?

43. If milk that analyzed 4.5 % butter fat yielded 900 lb. of butter fat, how many pounds of milk were there ?

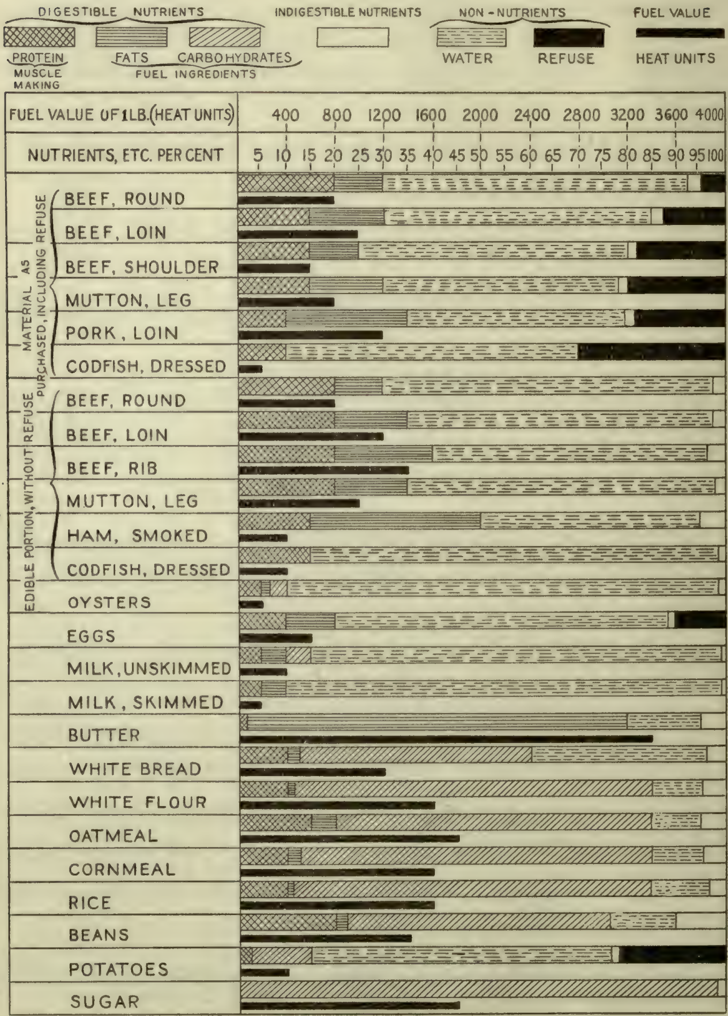
44. A boy who had 240 chickens sold 60 of them. What part remained unsold ? What per cent ?

45. A man paid \$910 for 13 % of a house. What was the value of the house at that rate ?

46. A man spends in one year \$1260, which is $66\frac{2}{3}$ % of his income. What is his income ?

COMPOSITION OF FOOD MATERIALS

Approximate Percents of nutritive ingredients, refuse, and fuel value



Find, from the diagram on the opposite page, the number of pounds each of protein and fats in 10 pounds of :

47. Round beef as purchased, including refuse ; of edible portion without refuse.

48. Loin of beef as purchased, including refuse ; of edible portion without refuse.

49. Shoulder of beef as purchased, including refuse ; of edible portion of rib of beef without refuse.

50. Leg of mutton as purchased, including refuse ; of edible portion without refuse.

51. Loin of pork as purchased, including refuse ; of edible portion of smoked ham without refuse.

52. Dressed codfish as purchased, including refuse ; of edible portion without refuse.

Find the number of pounds of protein in 10 lb. of :

53. Oysters. **54.** Eggs. **55.** Rice. **56.** Beans.

Find the number of pounds of carbohydrates in 10 lb. of :

57. Rice (73 %). **58.** Bread (47 %). **59.** Sugar (98 %).

60. Beans (54 %). **61.** Oatmeal (65 %).

Find the number of pounds of fats in 10 lb. of :

62. Eggs. **63.** Butter (78 %). **64.** Milk. **65.** Oatmeal.

66. The heat units in 1 lb. of round of beef are what per cent of the heat units in 1 lb. of butter ?

67. The heat units in 1 lb. of smoked ham are what per cent of the heat units in 1 lb. of loin of pork ?

68. The heat units in 1 lb. of unskimmed milk are what per cent of the heat units in 1 lb. of eggs ?

69. Make similar problems based on this table.

Farm Problems

70. The world's record for the production of butter fat is held by a Holstein, Banostine Belle de Kol. In one year she produced 27404.4 pounds of milk and 1058.34 pounds of butter fat. What was the average per cent of butter fat?

The Ohio Experiment Station made a study of the losses of manure when exposed in flat piles in a barnyard for a period of 3 months. They obtained the following data. The numbers in each case indicate the pounds of the material in a ton of manure. Find the loss per cent of each constituent.

	Organic Matter	Nitrogen	Phosphoric Acid	Potash
January	**71.** 416.00	**72.** 13.66	**73.** 4.90	**74.** 16.28
April	254.79	8.32	3.73	6.29

75. Mr. Fox put 15 dozen eggs, costing $.35 per dozen, in an incubator. 80 % of the eggs hatched. What was his profit, if he sold the chicks at 8 cents each?

76. What would have been the per cent of profit, if Mr. Fox had raised the chickens at a cost of $.40 each, and sold them at an average of $.60 each?

77. Butter bought for 20 ¢ a pound is sold for 35 ¢ a pound. Allowing 5 % for transportation, what is the gain per cent?

78. A farmer sowed $1\frac{1}{2}$ bu. of oats and harvested 60 bu. What per cent of the seed oats was the yield?

79. At an egg-laying contest, the first place was won by a white Leghorn hen from England that laid 282 eggs, and the second place by a white Leghorn hen from Connecticut that laid 269 eggs. What per cent more eggs did the first hen lay than the second?

80. If the average in this contest was 156 eggs, by what per cent did the first hen exceed the average? the second hen?

81. A farmer was offered two samples of cottonseed meal, the first of which, containing 42 % of protein, cost \$27 per ton, and the second, containing 37 %, cost \$24 per ton. Making no allowance for other ingredients, how much, to the nearest hundredth cent, did the protein cost per pound in each sample?

82. John picked $3\frac{1}{2}$ pt. of berries, Charles 4 pt., and James 3 pt. What per cent of the whole amount did each pick?

83. A man rented a field to a tenant in return for $33\frac{1}{3}$ % of the grain to be raised. The owner of the field sold his share of the grain for 80¢ a bushel, receiving \$240. How many bushels did he have, and how many bushels did the tenant have?

84. Mr. Brown planted 8 acres of his land in celery, setting out 25,000 plants to the acre. The yield was 2000 dozen plants to the acre. What per cent of the plants matured?

85. A garden plot 180 feet square is laid out as follows: $16\frac{2}{3}$ % of it in potatoes, 20 % of the remainder in asparagus, and the remainder in other vegetables. What per cent of the whole field is planted in other vegetables?

At the Buffalo Exposition a six months' dairy cow contest was run with the following results. What was the number of pounds of butter fat produced by each animal? What animals were above the average in production of milk? of butter fat?

Breed	Pounds of Milk per Cow	Average Test
86. Guernsey	5425.5	4.60 %
87. Jersey	5397.4	4.52 %
88. Ayrshire	6599.6	3.60 %

	Breed	Pounds of Milk per Cow	Average Test
89.	Holstein	7852.0	3.25 %
90.	Red Poll	5738.8	3.98 %
91.	Brown Swiss	6178.4	3.98 %
92.	French-Canadian	4932.8	3.99 %
93.	Shorthorn	6377.1	3.57 %
94.	Polled Jersey	4065.7	4.66 %
95.	Dutch Billed	4978.7	3.40 %

A record was kept of a Guernsey cow, Sweet Brier, at the Minnesota Station for eight years with the following results. Find the number of pounds of butter fat produced each year. What was the average number of pounds of milk produced per year? of butter fat?

	Year	Milk	Per Cent of Butter Fat		Year	Milk	Per Cent of Butter Fat
96.	1st	7057.0	5.13	**100.**	5th	6364.6	5.00
97.	2d	7094.5	5.04	**101.**	6th	7594.5	5.35
98.	3d	4744.9	4.89	**102.**	7th	4258.9	5.04
99.	4th	8426.7	4.98	**103.**	8th	4897.2	4.84

104. A farmer raised 540 bu. of oats. He kept 25 % of them and sold the remainder at $1.25 per bushel. How much did he receive for the part sold? How many bushels did he keep?

105. A farmer who had 1000 doz. eggs for sale which cost him $.21 a dozen, lost 10 % of them by breakage. If he sold the rest at $.30 a dozen, what per cent did he gain?

106. If you buy a farmhouse for $9500 and pay $500 for repairs, for how much per month must you rent it to gain 5 % on your investment, after paying 1 % for taxes?

Suggestion. 6 % of the investment will be the yearly rental.

107. If a farmer sold cattle and horses for $9550, thereby gaining 25 %, how much would he have gained by selling them at a profit of 35 %?

SIMPLE INTEREST

If a man rents or hires a piece of land, he agrees to pay for its use a certain sum per year, called rent. If he borrows or hires money, he agrees to pay for its use a certain *per cent* of it for one year.

A borrows \$250 from B for one year, and agrees to pay B the \$250 at the end of the year, and also a sum equal to 6% of the \$250 borrowed. If A borrows the money for more than one year, he pays for its use 6% of it each year, and a fraction of 6% of it for any fraction of a year. He may pay 5% a year, or any other rate that is agreed upon.

The **principal** is the sum of money borrowed.

The **interest** is the sum paid for the use of the principal.

The **rate of interest** is the per cent of the principal paid for its use for *one year*. Thus, the expression, "interest at the rate of 5%," means that the interest for *one year* equals 5% of the principal.

The **amount** is the sum of the principal and the interest.

1. What is meant by interest at 4%? at 5%? 7%? $2\frac{1}{2}$%? $4\frac{1}{2}$%? $3\frac{3}{4}$%?

2. What is the interest on \$100 for one year at 6%? at 5%? $3\frac{1}{2}$%? $6\frac{1}{2}$%? 10%?

3. Find the interest on \$400 for 1 year at 4%; at 5%; 6%; $4\frac{1}{2}$%.

4. Find the interest on \$400 for 2 years at 4%; at 5%; 6%; $4\frac{1}{2}$%.

Study Recitation

1. What is the interest on $250 for one year at the rate of 5%? the amount?

Solution

Interest	$250 × .05 = $12.50
Principal	$250.00
Amount	$262.50

2. C borrows $120 of D for 2 years 6 months at 5%. What is the interest? the amount?

Solution

Principal	$120
Rate	.05
Interest for 1 year	6.00
Number of years	$2\frac{1}{2}$
Interest for $\frac{1}{2}$ year, or 6 months	3.00
Interest for 2 years	12.00
Interest for 2 years 6 months	15.00
Principal	120.00
Amount due at end of 2 years 6 months . .	$135.00

Written

Find the interest and the amount for the given rate and time:

3. $500 for 3 years at 4%.

4. $175 for 3 years at 7%.

5. $125 for 2 years at 6%.

6. $800 for 6 years at 5%.

7. $280 for 1 year 6 months at 6%.

8. $560 for 4 years 4 months at 8%.

9. $600 for 4 months at 6%.

10. $150.50 for 3 years 2 months at 6%.

11. $240 for 1 year 1 month at 6%.

12. $240 for 1 year 5 months at 6%.

In computing interest, 30 days are usually regarded as a month. Since there are 30 days in 1 month, 3 days equal $\frac{1}{10}$ of a month.

The interest for 1 month is $\frac{1}{12}$ of the interest for 1 year. The interest for any number of months is the interest for 1 month multiplied by the number of months.

Any number of months and days may be expressed as months and tenths of a month.

To reduce any number of days to a fraction of a month.

Divide the number of days by 3 and call the result tenths of a month.

1. 3 days = .1 of a month.
2. 6 days = .2 of a month.
3. 12 days = .4 of a month.
4. 1 day = $.0\frac{1}{3}$ of a month.
5. 2 days = $.0\frac{2}{3}$ of a month.
6. 5 days = $.1\frac{2}{3}$ of a month.
7. 10 days = $.3\frac{1}{3}$, or $\frac{1}{3}$ mo.
8. 26 days = $.8\frac{2}{3}$ of a month.
9. 17 days = ? of a month.
10. 29 days = ? of a month.
11. 9 days = ? of a month.
12. 18 days = ? of a month.
13. 11 days = ? of a month.
14. 28 days = ? of a month.
15. 20 days = ? of a month.
16. 23 days = ? of a month.
17. 6 months and 6 days = 6.2 months.
18. 7 months and 16 days = $7.5\frac{1}{3}$ months.
19. 5 months and 13 days = ? months.
20. 4 months and 14 days = ? months
21. 3 months and 20 days = ? months.
22. 4 years, 3 months, and 7 days = 4 years $3.2\frac{1}{3}$ months.
23. 6 years, 7 months, and 13 days = 6 years $7.4\frac{1}{3}$ months.
24. 9 months and 18 days = 9.6 months.
25. 11 months and 23 days = ? months.
26. 3 months and 17 days = ? months.

Study Recitation

1. What is the interest on $150 for 6 yr. 7 mo. 13 da. at 5 % per annum?

Solution

	$150
	.05
Interest on $150 for 1 yr. at 5 %	$7.50
	6
Interest on $150 for 6 yr.	$45.00
	12)$7.50
Interest on $150 for 1 mo.	$ .625
7 mo. and 13 da. = $7.4\frac{1}{3}$ mo.	$7.4\frac{1}{3}$
	$208\frac{1}{3}$
	2500
	4375
Interest on $150 for $7.4\frac{1}{3}$ mo.	$ $4.6458\frac{1}{3}$
Interest on $150 for 6 yr.	$45.00
Interest on $150 for 6 yr. 7 mo. 13 da. . . .	$49.65

NOTE. In final results in interest, count 5 mills or more as an extra cent, and discard less than 5 mills.

2. Find the amount of $250 for 3 yr. 9 mo. 17 da. at 7 %.

Solution

	$250
	.07
Interest on $250 for 1 yr. at 7 %	$17.50
	3
Interest on $250 for 3 yr.	$52.50
	12)$17.50
Interest on $250 for 1 mo.	$1.458+
9 mo. 17 da. = $9.5\frac{2}{3}$ mo.	$9.5\frac{2}{3}$
	972
	7290
	13 122
Interest on $250 for $9.5\frac{2}{3}$ mo.	$ 13.9482+
Interest on $250 for 3 yr.	$ 52.50
Interest on $250 for 3 yr. 9 mo. 17 da.	$ 66.45
Principal	$250.00
Amount of $250 for 3 yr. 9 mo. 17 da. at 7 %	$316.45

Written

Find the interest on:

3. \$2500 for 60 days at 5%.
4. \$125 for 2 years 9 months at 7%.
5. \$900 for 90 days at $6\frac{1}{2}$%.
6. \$600 for 2 years 8 months at 5%.
7. \$850 for 1 year 10 days at 6%.
8. \$625.80 for 8 months 15 days at 5%.

What is the interest and the amount on each of the following at 6%? at 7%? at 8%? at 10%?

9. \$180 for 3 years 9 months 18 days.
10. \$165 for 2 years 7 months 9 days.
11. \$224 for 6 years 2 months 4 days.
12. \$168 for 2 years 11 months 6 days.
13. \$250 for 2 years 5 months 12 days.
14. \$750 for 4 years 6 months.
15. \$4200 for 5 months 60 days.
16. \$75,000 for 1 year 6 months 15 days.

Find the interest on the following at 6%; at 5%; at 4%; at $4\frac{1}{2}$%; at 8%; at 3%.

17. \$423 for 5 mo.
18. \$189 for 3 mo.
19. \$686.50 for 8 mo.
20. \$87.75 for 3 yr. 6 mo.
21. \$457 for 2 yr. 4 mo.
22. \$595 for 15 mo.
23. \$638 for 10 mo.
24. \$742 for 14 mo.
25. \$844 for 9 mo.
26. \$963 for 15 mo.
27. \$738 for 12 mo.
28. \$609 for 6 yr. 5 mo.
29. \$384 for 1 yr. 8 mo.
30. \$489.60 for 2 yr. 3 mo.
31. \$514 for 1 yr. 5 mo.
32. \$840 for 1 yr. 7 mo.

33. $892 for 2 yr. 3 mo.
34. $576 for 5 yr. 6 mo.
35. $731 for 3 yr. 5 mo.
36. $816 for 2 yr. 2 mo.
37. $987 for 5 mo.
38. $3490 for 6 yr. 2 mo.
39. $996 for 7 mo.
40. $684 for 4½ mo.
41. $377 for 3 yr. 1 mo.
42. $468 for 15 da.
43. $559.50 for 5 yr. 8 mo.
44. $743 for 4 yr. 3 mo.
45. $634 for 2 yr. 2 mo.
46. $224 for 8 yr. 5 mo.
47. $313.25 for 2 yr. 2 mo.
48. $428.50 for 6 yr. 4 mo.
49. $530 for 60 days.
50. $748 for 90 days.
51. $857 for 30 days.
52. $9650 for 5 mo.
53. $985.40 for 7 mo.
54. $89 for 8 mo. 10 days.
55. $486 for 6 mo. 6 days.
56. $576.30 for 3 yr. 4 mo.
57. $636.90 for 1 yr. 6 days.
58. $624 for 1 yr. 9 mo. 6 days.
59. $413 for 1 yr. 3 mo. 15 days.
60. $385 for 1 yr. 16 days.

SIX PER CENT METHOD

Study Recitation

Another method of computing interest, called the 6 % method, is frequently employed.

The interest on $1 for 1 year at 6 % = 6 cents.
The interest on $1 for 2 months (⅙ of a year) = 1 cent.
The interest on $1 for 1 month, or 30 days = ½ of 1 cent, or 5 mills.
The interest on $1 for 6 days (⅕ of 30 days) = 1 mill.
The interest on $1 for 1 day = ⅙ of a mill.

1. Find by the 6 % method the interest on $200 for 2 yr. 6 mo. 6 days at 6 %.

Solution

Interest on $1 for 2 yr. at 6 %	$.12
Interest on $1 for 6 mo. at 6 %	.03
Interest on $1 for 6 da. at 6 %	.001
Interest on $1 for 2 yr. 6 mo. 6 da. at 6 %	$.151

The interest on $200 = 200 × $.151, or $30.20.

2. Find the interest on \$500 for 60 days at 6 %.

SOLUTION

Since the interest at 6 % for 60 days is 1 % of the principal, it may be found by moving the decimal point two places to the left. 1 % of \$500 = \$5.

3. What part of 60 days are 30 days? 10 days? 6 days? 120 days or 2 mo.? 90 days or 3 mo.?

4. Since the interest of \$100 for 60 days is \$1, what is the interest of \$100 for 30 days? for 120 days?

To find the interest on \$1 *for any period at* 6 %, *take* 6 *times as many cents as there are years plus* ½ *as many cents as there are months plus* ⅙ *as many mills as there are days.*

To find the interest on any sum, for any time, at 6 %, *multiply the interest on* \$1, *for the given time, at* 6 %, *by the number of dollars.*

To find the interest on any sum for 60 *days at* 6 %, *move the decimal point two places to the left.*

What is the interest on \$1 for the following periods of time at 6 %:

5.	2 years?		8 months?		2 years 8 months?
6.	8 years?		10 months?		8 years 10 months?
7.	3 years?		3 months?		3 years 3 months?
8.	9 years?		7 months?		9 years 7 months?
9.	10 years?		11 months?		10 years 11 months?
10.	18 days?	15 days?	13 days?	23 days?	2 days?
11.	24 days?	21 days?	14 days?	26 days?	5 days?
12.	12 days?	27 days?	16 days?	29 days?	4 days?
13.	1 day?	7 days?	17 days?	11 days?	6 days?
14.	9 days?	8 days?	19 days?	3 days?	22 days?
15.	10 days?	25 days?	60 days?	30 days?	120 days?

16. 4 months 12 days? 5 months 7 days?

17. 7 months 18 days? 3 months 21 days?

18. 9 months 24 days? 8 months 23 days?

19. 10 months 6 days? 7 months 25 days?

20. 10 months 9 days? 9 months 16 days?

21. 2 years 6 months 12 days? 4 years 11 months?

22. 3 years 5 months 18 days? 12 years 7 months 3 days?

23. 4 years 7 months 24 days? 5 years 8 months?

24. 1 year 6 months 7 days? 7 years 9 months 15 days?

25. 5 years 8 months 27 days? 8 years 11 months?

Written

Find the interest on each of the following principals, for the given time, at 6%:

26. \$250 for 3 yr. 6 mo. 18 days.

27. \$175 for 2 yr. 9 mo. 24 days.

28. \$215 for 4 yr. 10 mo. 12 days.

29. \$125 for 6 yr. 6 mo. 9 days.

30. \$120.75 for 3 yr. 7 mo. 18 days.

31. \$118.50 for 4 yr. 8 mo. 15 days.

32. \$500 for 8 yr. 2 mo. 4 days.

33. \$625 for 1 yr. 1 mo. 1 day.

34. \$840 for 1 yr. 6 mo. 15 days.

35. \$920 for 2 yr. 4 mo. 20 days.

36. \$750 for 3 yr. 3 mo. 10 days.

37. \$320 for 4 yr. 9 mo. 6 days.

38. \$1500 for 3 yr. 8 mo. 5 days.

39. \$1600 for 2 yr. 2 mo. 18 days.

40. \$1750 for 1 yr. 5 mo. 15 days.

41. $940.25 for 1 yr. 7 mo. 10 days.

42. $450.62 for 2 yr. 10 mo. 6 days.

43. $830.91 for 3 yr. 11 mo. 5 days.

To find the interest on any sum, for any time, at any rate per cent.

STUDY RECITATION

1. The interest on a given sum of money, for a given time, at 6%, is $24. Find the interest on the same sum, for the same time, at 1%; at 5%; at 7%.

SOLUTION. Interest on the same sum for the same time, at 1%, is $\frac{1}{6}$ of $24, or $4. At 5% it is 5 × $4; at 7%, 7 × $4.

From the interest on any principal at 6%, the interest at other rates may be found by adding or subtracting aliquot parts of the interest at 6%, as follows:

Adding	$\frac{1}{6}$ of itself	for the interest at 7%
Subtracting	$\frac{1}{6}$ of itself	for the interest at 5%
Adding	$\frac{1}{3}$ of itself	for the interest at 8%
Subtracting	$\frac{1}{3}$ of itself	for the interest at 4%
Dividing by	4	for the interest at $1\frac{1}{2}$%
Adding	$\frac{1}{4}$ of itself	for the interest at $7\frac{1}{2}$%
Subtracting	$\frac{1}{4}$ of itself	for the interest at $4\frac{1}{2}$%
Dividing by	2	for the interest at 3%

WRITTEN

2–127. Solve examples 26 to 43 (pp. 344, 345), substituting rates of 3%, 4%, $4\frac{1}{2}$%, 5%, $2\frac{1}{2}$%, $5\frac{1}{2}$%, $3\frac{3}{4}$%.

128. Mr. H. H. Jones borrows $1200 June 5, and pays the debt October 23. What is the interest at $4\frac{1}{2}$%?

129 Find the interest on a debt of $900 from Jan. 5, 1914, to Dec. 30, 1914 at 5%.

130. Find the amount due on a debt of $360 from July 7, 1913, to Jan. 20, 1914 at 6%.

EXACT INTEREST

Exact interest, or **accurate interest**, as it is sometimes called, is found by taking the *exact* number of days between two dates, and reckoning 365 days to a common year, and 366 days to a leap year.

Interest is usually reckoned on the basis of 12 months of 30 days each, or 360 days to the year. If the time for which the principal is on interest is stated in years, or in years and months, the usual method also gives the exact interest; but if the time the principal is on interest is given in days, the usual method does not give the exact interest. If the time the principal is on interest is less than 1 year, and if the dates between which the principal draws interest are given, the exact number of days may be found, but the usual method will not give the exact interest.

The interest on \$1460 for 60 days, at 6%, is found by the usual method to be \$14.60.

This is upon the supposition that 60 days are $\frac{60}{360}$ of a year. But 360 days are exactly $\frac{360}{365}$, or $\frac{72}{73}$, of a common year; and $\frac{360}{366}$, or $\frac{60}{61}$, of a leap year. If a man pays the interest for the use of money for 360 days, when he should pay for the use of the money for 365 days or for 366 days, he pays $\frac{5}{365}$, or $\frac{1}{73}$, too much in the common year, and $\frac{6}{366}$, or $\frac{1}{61}$, too much in the leap year.

To find exact interest when the time is in days, diminish the interest as found in the usual way by $\frac{1}{73}$ of itself if the time is a part of a common year; diminish it by $\frac{1}{61}$ of itself if the time is a part of a leap year.

If the time is given in years, months, and days, find the interest for the years and months by the usual method, and to it add the exact interest for the days.

1. Find by the 6% method the interest on \$7300 for 60 days at 6%.

2. Find the exact interest, in a common year, on \$7300 for 60 days at 6%. By how much does this differ from the answer in example 1?

To find the exact number of days between two dates.

1. How many days are there between May 20 and November 15 of the same year?

May 20 to 31 =	11 days
June	30 days
July	31 days
August	31 days
September	30 days
October	31 days
November	15 days
	179 days

To the number of days remaining in the earliest month given, add the number indicated by the date of the latest month given, and also the number of days in each of the intervening months.

The number of days remaining in May is 11. The latest month named is November, and the date given is 15. The number of days in each of the intervening months, respectively, is 30, 31, 31, 30, 31. The sum of all is 179.

2. Find the time from June 15, 1910 to Sept. 20, 1914.

From June 15, 1910 to June 15, 1914, it is 4 years.

From June 15, 1914 to Sept. 20, 1914, it is 15 days + 31 days + 31 days + 20 days, or 97 days.

The entire time is 4 years 97 days; counting 1 leap year, it is 4 years 98 days.

WRITTEN

3. Find the time from Feb. 1, 1912 to Sept. 4, 1912.

NOTE. Since 1912 was a leap year, count 29 days in February.

4. Find the time from April 3, 1913 to Jan. 5, 1914.

5. Find the exact interest at 6 % on $1200 from Jan. 1, 1913 to June 28, 1913.

The following table is often used for convenience in reckoning time between two dates:

Table of Days between Dates

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan. . .	365	31	59	90	120	151	181	212	243	273	304	334
Feb. . .	334	365	28	59	89	120	150	181	212	242	273	303
Mar. . .	306	337	365	31	61	92	122	153	184	214	245	275
Apr. . .	275	306	334	365	30	61	91	122	153	183	214	244
May . .	245	276	304	335	365	31	61	92	123	153	184	214
June . .	214	245	273	304	333	365	30	61	92	122	153	183
July . .	184	215	243	274	304	335	365	31	62	92	123	153
Aug. . .	153	184	212	243	273	304	334	365	31	61	92	122
Sept. . .	122	153	181	212	242	273	303	334	365	30	61	91
Oct. . .	92	123	151	182	212	243	273	304	335	365	31	61
Nov. . .	61	92	120	151	181	212	242	273	304	334	365	30
Dec. . .	31	62	90	121	151	182	212	243	274	304	335	365

If you want to find the number of days from May 4 to Sept. 4, follow the horizontal line beginning with May in the left-hand column until it crosses the column headed September. The number at the crossing is 123 days. The number of days from May 4 to Sept. 8 would be 4 more, or 127 days.

(If February 29 for leap year comes between the dates, add 1 day.)

PROBLEMS

Written

Find the exact interest due on the following:

1. \$75 for 36 days at 6 %. **3.** \$250 for 24 days at 8 %.

2. \$125 for 66 days at 6 %. **4.** \$325 for 33 days at 7 %.

5. \$115.25 for 80 days at 5 %; at 6 %.

6. \$1000 for 90 days at 4 %; at 3 %.

7. \$3500 for 30 days at 3 %; at $4\frac{1}{2}$ %.

8. \$275 from Jan. 12, 1914 to May 13, 1914, at 6 %.

9. \$315 from Sept. 24 to Dec. 28 of the same year at 7 %.

10. \$425 from April 20, 1911 to Nov. 12, 1914, at 4 %.

PROBLEMS IN INTEREST

Study Recitation

Given the principal, the interest, and the time, to find the rate.

1. At what rate of interest will $100 yield $12 in 2 yr.?

Solution. The interest of $ 100 at 1 % for 2 years is $ 2. If $ 100 in 2 years yields $ 12, the rate per cent equals $12 ÷ $ 2, or 6. *Ans.* 6 %.

Divide the given interest by the interest for the given time at 1 %.

Written

2. At what rate will $ 200 yield $ 36 in 3 years?

3. At what rate will $ 426 yield $53.25 in 2 years 6 months?

4. The interest of $ 560 for 3 years 3 months is $145.60. What is the rate?

5. A principal of $ 260 yields $29.90 in 2 years 3 months 18 days. What is the rate?

6. At what rate will a principal double itself in 10 years? in 20 years? in 25 years?

Suggestion. 100 % ÷ 10 = 10 %.

Given the principal, the interest, and the rate, to find the time.

Study Recitation

1. In what time will $100 gain $12 interest at 6 %?

Solution. The interest on $ 100 at 6 % for 1 year is $ 6. The number of years is $12 ÷ $ 6, or 2. *Ans.* 2 yr.

Divide the given interest by the interest at the given rate for 1 year.

Written

2. In how many years will \$600 gain \$120 at 5%?

3. In how many years will \$700 gain \$56 at 4%?

4. In how many years will \$800 gain \$72 at 3%?

5. In how many years will a principal double itself (gain 100% of itself) at 5%?

SOLUTION. 100% ÷ 5% = 20. *Ans.* 20 years.

6. In how many years will a principal double itself at 6%? 10%? 4%?

Given the interest, the rate, and the time, to find the principal.

Study Recitation

1. What principal will yield \$12 in 2 years at 6%?

SOLUTION. Since \$1 in 2 years at 6% yields \$.12, as many dollars must be invested as \$.12 is contained times in \$12. \$12 ÷ \$.12 = 100. Hence, \$100 is the principal.

Divide the given interest by the interest on \$1 *for the given time at the given rate.*

Written

2. What principal will yield an interest of \$600 in 5 yr. at 6%?

3. What principal will yield an interest of \$300 in 2 yr. at 5%?

4. What principal will yield an interest of \$480 in 4 yr. at 4%?

5. What principal will yield an interest of \$50.40 in 3 yr. at 3%?

6. What principal will yield an interest of \$69.30 in 2 yr. at $4\frac{1}{2}$%?

PROMISSORY NOTES

A **promissory note** is a written promise made by one person to pay to another a specified sum of money on demand or at a definite time.

PROMISSORY NOTE

> $600 Fort Wayne, Ind., June 13, 1914.
>
> _____Three months_____after date___I___promise to pay to
>
> the order of Charles Rice________________________________
>
> Six hundred and $\frac{00}{100}$ ~~~~~~~~~~~~~~~~~~~~ Dollars.
>
> Value received
>
> James Wilson.

The **maker** of the note is the one who signs it; that is, the one who is to pay it.

The **payee** is the person to whom it is payable.

The **face** of a note is the sum named in it.

Essentials of a Promissory Note:

1. It must be signed by the **maker**.
2. It should promise to pay to the **payee** or to his **order**.
3. It should promise to pay a **definite sum of money**, the **face**.
4. It should state the **place** where and the **time** when it is given.
5. It should state **when** the money is to be paid.
6. It should state **with interest** and the **rate**, if it is an interest-bearing note.

The words "for value received" are usually included in a note, but are not essential.

Who is the maker of the note on p. 351? the payee? What is the face of the note? When is it dated? When is it due?

The note on p. 351 is a **time note** because it is payable at a specified time. A **demand note** is one which the maker may be called upon to pay at any time after date. If in the above note you substitute the words "on demand" for "three months after date," it will become a demand note.

A note is said to **mature** at the expiration of the time specified in the note. A note is due at **maturity**.

A **negotiable note** is a note made payable to the "bearer" or to the "order" of the payee. A note is non-negotiable when it is made payable to the payee only. A negotiable note may be transferred from one person to another.

NOTE. In a few states 3 days of grace are allowed after the expiration of the time mentioned in the note. Days of grace are not to be considered in the examples in this book.

STUDY RECITATION

1. $500.00 INDIANAPOLIS, IND., Aug. 14, 1914.

Six months after date, for value received, I promise to pay to J. C. Root, or order, Five Hundred Dollars, with interest at the rate of 6 per cent per annum.

J. H. JOHNSON.

2. $200.00 MILWAUKEE, WIS., Sept. 15, 1914

Thirty days after date, for value received, I promise to pay to John Jones, or bearer, Two Hundred Dollars, with interest at 7%.

HENRY SMITH.

Both of the above notes are *negotiable*. What words should be omitted from each note to make it *non-negotiable?*

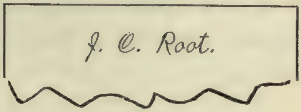
Name the *maker*, the *payee*, and the *holder* of each note.

What is the *face* of each note? What is the date of its *maturity?*

The **indorser** is the person who writes his name on the back of a note to transfer it, if it is made payable to his order, or to guarantee its payment in case the maker fails to pay it when due.

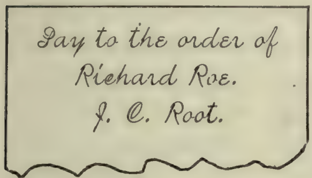
If the holder of the first note on p. 352 wishes to transfer it to John Doe, he must indorse it; that is, write his name on the back of the note. If he writes his name "J. C. Root" and nothing else on the back of the note, it is an **indorsement "in blank,"** and the note is then payable to the holder. The indorsement by J. C. Root renders him liable for the payment in case J. H. Johnson is unable to pay the note.

INDORSEMENT IN BLANK



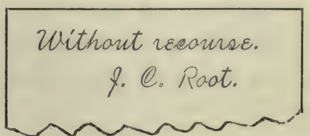
If Mr. Root writes on the back of the note "Pay to the order of Richard Roe, J. C. Root," it is an **indorsement "in full,"** and the note is then payable to Richard Roe, or to the person to whom he may transfer it by his indorsement. The indorser is liable for payment of the note in case the maker fails to pay it.

INDORSEMENT IN FULL



If Mr. Root writes his name on the back of the note, as in either of the above cases, and adds the words "without recourse," it is an **indorsement without recourse**, and the indorser is not liable for payment of the note in case the maker fails to pay it.

QUALIFIED INDORSEMENT



DIFFERENT FORMS OF NOTES

3. $100.25 TERRE HAUTE, IND., June 1, 1914.

Sixty days after date I promise to pay John Jones, or order, One Hundred and $\frac{25}{100}$ dollars, with interest, value received.

A. J. HENRY.

When a note is drawn "with interest," but with no rate specified, the legal rate of the state in which the note is drawn is understood.

4. $150.00 CLEVELAND, O., April 4, 1913.

On demand, for value received, I promise to pay A. H. George, or order, One Hundred Fifty Dollars, with interest at 6%.

A. B. SMITH.

When is the above note due?

If a note makes no mention of interest, no interest can be collected until it matures. If the note is not paid at maturity, it draws legal interest from the date of maturity to the date of payment.

5. $150.00 MILWAUKEE, WIS., May 5, 1914.

Three months from date, I promise to pay John Jones, or order, One Hundred Fifty Dollars, value received.

JAMES JOHNSON.

When did this note mature?

It was not paid until Dec. 5, 1914. For what length of time did it draw interest? why? at what rate? What amount was required to pay the note?

6. $100.00 MADISON, WIS., Feb. 12, 1914.

Six months after date, for value received, we jointly and severally promise to pay James Smith, or order, One Hundred Dollars, with interest at 7%.

JOHN DOE,
RICHARD ROE.

This note is called a **joint and several note**, and its makers, John Doe and Richard Roe, are jointly and singly liable for its payment. If the words "and severally" were omitted, the note would be a **joint note**, and each of the makers would be liable for payment of only one half the amount due at maturity.

A **protest** is a written notice, stating that the maker has failed to pay the note. It is sent to the indorsers by a notary public, in a manner prescribed by law.

A note maturing on a Sunday or on a legal holiday is made payable by law in most states the day before it matures.

7. Write a negotiable note for \$750, with interest at 6 %, making yourself the payee and your teacher the maker. Find the amount due at maturity.

8. Write a negotiable note for \$175.25, payable on demand, with interest at 7 %, making one of your classmates the payee and yourself the maker.

Have the payee fix the time of payment. Find the amount due.

9. Write a negotiable demand note for \$250, without interest, making yourself the payee and John Smith the maker.

Fix the time when payment is demanded. If the note is not paid until 6 months after demand, what amount is due at the time of payment?

10. Write a negotiable note for \$550.25, with interest at 8 %, making yourself the payee and your teacher the maker. Indorse it in blank for transfer. What amount is due at maturity?

11. Write a note from the following data: Negotiable; face, \$2500; maker, J. Smith; payee, yourself; maturity, 1 year 6 months from date; interest, 5 %. Find the amount due at maturity.

12. A 60-day note for \$550, with simple interest at 10%, dated March 1, 1914, was paid at maturity. When was the note due? What was the amount due?

13. A 30-day note for \$1000, without interest, was paid three months after it matured. What was the amount due, the legal rate being 6%?

14. A 60-day note for \$875, with interest at 6%, was paid in ninety days. What was the amount paid, reckoning simple interest? exact interest?

15. A two-months' note for \$1500, with interest at 5%, dated April 1, 1914, was paid at maturity. Find the date of maturity and the amount due.

Note. For partial payments on notes, see Supplement, pp. 483, 484.

MORTGAGES

A **mortgage** is a conveyance of real estate or some interest therein, as security for a loan.

The **mortgagor** is the person who borrows money and gives the mortgage on the property; the **mortgagee** is the person who lends the money on the property as security.

1. Mr. Bent sells a house to Mr. Gordon for \$12,000, receiving \$7000 cash and a mortgage on the house for \$5000 at 5%. What yearly interest must Mr. Gordon pay Mr. Bent on this mortgage?

Solution. 5% of \$5000 = \$250.

2. Mr. Allen invests \$6500 in a guaranteed mortgage at $4\frac{1}{2}$%. How much yearly interest does Mr. Allen receive on this mortgage?

3. Find the interest on a mortgage of \$25,000 at $5\frac{1}{2}$%.

4. Find the semiannual interest on a mortgage for \$12,250 at 5%.

BANKS AND BANKING

A **bank** is an establishment that receives, lends, exchanges, or issues money.

National banks are under the control of the government, and state banks under the control of the state. There are also many private banks.

Federal reserve banks are central banks in the national banking system established for the purpose of enlarging our currency system and lending money to other banks when needed.

Trust companies are also permitted in many states to do a general banking business, though they exercise many other powers as well.

CHECKS

A **check** is a written order by a depositor in a bank, directing the payment of a specified sum of money.

STUB	CHECK
No. 3586	South Bend, Ind., May 1, 1914, No. 3586
Date May 1, '14	First National Bank
Payable to	Pay to the
Frank Arnold	order of Frank Arnold ---------- $500.25
For Mdse.	Five hundred and 25/100 ~~~~~~~~~~ Dollars.
Am't. $500.25	L. A. Clarke.

The terms **maker, payee, face, negotiable**, etc., apply to checks as well as to promissory notes. The various forms of indorsement are also applied in the same manner. The stub that remains in the check book after the check is torn out contains a record of the check.

Deposit Slip

DEPOSITED BY

Charles Spencer

IN THE

First National Bank

South Bend, Ind., Jan. 4, 1914.

Bills	$ 15	
Gold	10	
Silver	13	50
Check on Sec. Nat. Bank	65	14
" " Am. Exch. "	14	33
Total	117	97

If you wish to open an account, you take your money to the bank, and give it with a slip filled out like the one here shown to the "cashier" or "receiving teller." You then receive a book in which the amount is credited to you, and another book of blank checks.

If you wish to draw money for yourself from a bank, you may write a check payable to "cash" which need not be indorsed or a check payable to "self" which must be indorsed.

A check payable "to bearer" may be collected by any one presenting it.

Written

1. A man's balance at the Union Trust Co. was $2314.57. Find his balance after checking out $15.75, $14.83, $72.94, and $83.57. Write the checks and the stubs, choosing your own payees.

2. A lady's balance at the Second National Bank was $653.75 on June 1. On June 3 she drew a check for $54.65 payable to Arthur Hart, and on June 15 she deposited $140

in bills. What was her balance July 1? Write the check, the stub, and the deposit slip.

3. Draw checks, filling in items as you choose, (1) payable to cash; (2) payable to bearer; (3) payable to self; (4) payable to Frank Abbott. Indorse the checks when necessary.

4. George Parker's balance at the bank Jan. 1 was $5000. He deposited on Jan. 18, two $50 bills and checks for $500.75, $625.44, $173.98, $99.58, $44.67, and $74. He drew checks during the month for $264, $783.95, $42.67, and $83.54. What was his balance Feb. 1? Write the checks, the stubs, and the deposit slips, choosing banks and payees.

5. A lady's balance in the bank at the beginning of a month was $584.50. After she had deposited $135.40, $654.78, and $829.40 and drawn checks for $45.64, $15.32, and $1.75, what was her balance? Write the checks, the stubs, and the deposit slips, supplying names, banks, and other items.

6. John Farmer's balance at the Corn Exchange Bank on Feb. 1, 1914 was $1213.56. On Feb. 3 he deposited a check for $145.16, $14.50 in silver, and two $5 gold pieces. On Feb. 4 he drew a check to the order of Thomas Arnold for $19.56. Find his balance on Feb. 5. Write the checks, the stubs, and the deposit slip.

7. Harold Aitken's balance at the New York Produce Exchange Bank on April 1, 1914 was $650.69. On April 2, he drew $17.45; on April 3, $16.50. On April 4, he deposited a check for $110.19 and bills for $17.50. On April 19, he drew a check for $25.60; and on April 30, a check for $17.40. Find his balance on May 1, 1914. Write the checks, the stubs, and the deposit slip.

BANK DISCOUNT

Study Recitation

Bank discount is a sum charged by a bank for payment of a note or a draft before it becomes due.

This sum is the interest, at the bank's rate of discount, on the *amount* of the note or the draft at maturity, from the date the note is discounted to the date it matures.

When a bank lends money to the borrower on his note and takes bank discount, it deducts the interest from the face of the note at the time the note is given, and pays the maker of the note the difference. This difference between the face of the note and the interest is called the *proceeds* of the note.

If A borrows $100 from B and gives B a note for that sum, due in 4 months at 6%, A receives $100, and *4 months afterward* pays B $100 plus the interest on $100 for 4 months at 6%, or $2, making $102. If, however, A gives his note for $100, due in 4 months, to a bank, and the bank discounts it at 6%, the interest on the face of the note for 4 months at 6%, or $2, is *at once* deducted from the face of the note and retained by the bank, and only $98 is paid to A, the maker of the note. This is called the **proceeds** of the note. At maturity, A, the maker of the note, pays the bank $100. The note given to the bank does not specify interest until it matures. If the note is not paid at maturity, it then begins to draw interest. In the first case, A, the maker of the note, receives from B the amount specified in the note, and pays it with interest at maturity. In the second case, the interest on the amount of the note is paid by A in advance to the bank, and the maker, A, receives from the bank the proceeds, or the amount of the note less the interest.

A bank not only discounts notes made by its customers direct to the bank, but it also buys or discounts notes owned by its customers and payable to them. In such cases, the owner of the note indorses it, making it payable to the bank. Such notes may or may not bear interest.

When such a note does not bear interest, the holder receives the face of the note less discount on it, at the bank's rate, from the date it is discounted to the date of maturity.

The **proceeds** of a note is the sum due at maturity less the bank discount.

The **term of discount** is the time from the date of discount to the date of maturity.

1. \$100.00 VINCENNES, IND., Mar. 3, 1914.

Three months from date, for value received, I promise to pay John Adams, or order, One Hundred Dollars.

PETER SMITH.

This note was due 3 months after March 3, 1914, or June 3, 1914; but on April 3 (**date of discount**), the payee, John Adams, needing money, took the note to the First National Bank to have it discounted. He indorsed the note, making it payable to the cashier of the bank. The bank, being asked to cash the note two months before it became due, charged interest on it at its rate of discount, 6%, for the two months. The interest was \$1. This sum was deducted from the \$100, the amount payable to the bank at **maturity** (two months later), by Peter Smith. John Adams received the **proceeds**, \$99.

The transaction was the same as if John Adams had given his own note for \$100 to the bank April 3, payable in two months, without interest, and the bank had discounted it.

If to the above note the words "with interest at 6%" were added, the amount due at maturity would be \$100 plus \$1.50 (the interest for 3 months at 6%), or \$101.50. If John Adams had the note discounted April 3, as above, the bank would compute interest at its rate of discount, as before, on the amount due at maturity, \$101.50. The discount on \$101.50 at 6% for 2 months, the time from the date the note is discounted to maturity, is \$1.02. This is the bank discount. John Adams would receive the proceeds, \$101.50 – \$1.02, or \$100.48.

The bank's rate of discount may be different from the rate of interest specified in the note.

2. \$500.00 MILWAUKEE, WIS., March 10, 1914.

Ninety days after date, for value received, I promise to pay to the order of Richard Roe, Five Hundred Dollars, with interest at 6%.

JOHN DOE.

This note was discounted at a bank, April 3. The bank's rate of discount was 5 %. What were the proceeds?

SOLUTION. This note at maturity, June 8, 1914, would yield the bank \$500 plus the interest on that sum for 90 days at 6 %. The interest is \$7.50. \$500 + \$7.50 = \$507.50.

As the note was brought to the bank for discount on April 3, it had 66 days yet to run to June 8. The bank computed discount on the amount of the note, \$507.50, for 66 days at 5 %. The discount was \$4.65. The proceeds of the note were \$507.50 − \$4.65, or \$502.85.

WRITTEN

Find the bank discount and the proceeds of a note for:

3. \$200, payable in 60 days, discounted at 6 %.

4. \$125, payable in 90 days, discounted at 7 %.

5. \$175, payable in 30 days, discounted at 8 %.

6. \$250, payable in 3 months, discounted at 9 %.

7. \$150, payable in 45 days, discounted at 6 %.

8. \$1000, payable in 30 days, discounted at 5 %.

9. \$1500, payable in 60 days, discounted at $4\frac{1}{2}$ %.

10. \$1600, payable in 90 days, discounted at 6 %.

11. \$2500, payable in 45 days, discounted at $5\frac{1}{2}$ %.

12. \$3000, payable in 2 months, discounted at 6 %.

13. A note for \$200 was given Jan. 5, 1914, payable in 90 days. When did it mature?

14. A note for \$450, bearing 6 % interest, was given Nov. 3, 1910, payable in 3 months. When did it mature? It was discounted at a bank, December 10. The rate of discount was 7 %. For what length of time was it discounted? What was the discount? What were the proceeds?

15. A note for \$600, bearing 6 % interest, was given Oct. 5, 1913, payable in 6 months. It was discounted at a bank Jan. 5, 1914, at 5 %. Find the date of maturity, the discount, and the proceeds.

Find the discount and proceeds of each of the following interest-bearing notes:

	Face of Note	Date	Time	Rate of Interest	Date of Discount	Rate of Discount
16.	$750	Sept. 10	60 days	6%	Oct. 15	5%
17.	$175	Aug. 12	90 days	7%	Oct. 20	6%
18.	$125	June 20	3 months	8%	July 20	6%
19.	$225	June 1	4 months	6%	Aug. 1	4%
20.	$500	Jan. 15	60 days	4%	Feb. 15	6%
21.	$1000	Jan. 5	30 days	5%	Jan. 10	5%
22.	$2500	March 3	60 days	6%	March 4	6%
23.	$1800	April 7	90 days	4%	April 15	6%
24.	$3500	May 6	45 days	5%	May 15	5%
25.	$6000	July 5	60 days	4%	July 15	6%

National banks are required to keep on reserve certain percentages of their deposits. Demand deposits are those payable within 30 days; time deposits, those payable after 30 days or subject to not less than 30 days' notice.

Country banks must keep on reserve 12% of demand and 5% of time deposits; *reserve city banks*, 15% of demand and 5% of time deposits; and *central reserve city banks*, 18% of demand and 5% of time deposits.

26. Find the amount that must be kept on reserve by a country bank having demand deposits of $310,500 and time deposits of $49,260.

27. Find the amount that must be kept on reserve by a reserve city bank having demand deposits of $50,625,200 and time deposits of $9,635,204.

28. Find the amount that must be kept on reserve by a central reserve city bank having demand deposits of $75,632,410 and time deposits of $21,216,308.

SAVINGS BANK ACCOUNTS

A **savings bank** is a bank, under the control of the government, established for the purpose of receiving deposits of money and paying interest thereon.

Interest is usually paid on any number of dollars from $1 to $3000, on condition that such sum has been on deposit for an entire **interest term**, the period between which payments are due, as January 1 and July 1, in some banks; January 1, April 1, July 1, and October 1 in others; or the first of each month.

Each depositor is provided with a **bank book** in which are entered his deposits and the amounts he draws out.

Most savings banks credit interest at the end of every interest term, on the smallest balance on deposit during the entire term. Usually no interest is allowed on cents. Consult a bank in your neighborhood for its custom in this matter.

When interest is due, it may be withdrawn or it may be placed to the credit of the depositor. In the latter case it draws interest the same as other deposits. Savings banks, therefore, pay compound interest.

Compound interest is interest on the amount of the principal and on the interest, if the interest is unpaid when due.

Study Recitation

1. Find the amount and the compound interest of $1000 for 2 yr. 6 mo. at 6% with interest compounded annually.

Solution

Principal	$1000	
Interest for 1st yr. at 6%	60	
Principal at beginning of 2d yr.	$1060	
Interest for 2d yr. at 6%	63.60	
Principal at beginning of 3d yr.	$1123.60	
Interest for 6 mo. at 6%	33.708	
Amount for 2 yr. 6 mo.	$1157.308, or	$1157.31
Less 1st principal		1000
		$ 157.31, Compound int.

NOTE. If interest is compounded semiannually, take ½ the rate and double the number of periods. Thus, 2 yr. at 6% semiannually is the same as 4 yr. at 3% annually.

WRITTEN

2. What is the amount of $3000 for 2 yr. at 6% compounded annually? the compound interest?

3. What is the amount of $4000 for 2 yr. at 4% compounded semiannually? Compound interest is usually reckoned by tables. See p. 481.

4. The following is a statement of Frank Dow's deposits and withdrawals, with interest compounded semiannually at 4% per annum.

STATEMENT

DATE	DEPOSITED		DRAWN OUT		INTEREST		BALANCE	
1913								
Dec. 28	125	00					125	00
1914								
Jan. 1							125	00
March 2			25	00			100	00
July 1					2	00	102	00
July 2	100	00					202	00
1915								
Jan. 1					4	04	206	04

The deposit of Dec. 28, 1913 did not begin to draw interest until January 1, 1914, and the interest for the half year from January 1 to July 1 was computed on the smallest balance for the half year, namely on $100, since $25 was withdrawn before July 1. Money deposited July 2 is allowed to draw interest from July 1, hence the interest due Jan. 1, 1915 is reckoned on $102 + $100, or $202.

NOTE. A fractional part of a cent on the interest is usually dropped.

5. Find the balance due January 1, 1915 on the following, with interest at 4 %, due January 1 and July 1. Arrange the examples as in the form on p. 365.

Deposits		Withdrawals	
Jan. 2, 1914	$1000	April 1, 1914	$ 10
March 1, 1914	200	June 1, 1914	25
July 1, 1914	600		

6. Find the balance due January 1, 1915 on the following account, with interest at $3\frac{1}{2}$ %, due January 1 and July 1.

Deposits		Withdrawals	
July 2, 1914	$500	Oct. 2, 1914	$40
Aug. 1, 1914	50	Oct. 25, 1914	35
Sept. 1, 1914	75		
Oct. 1, 1914	100		

7. Find the balance due January 1, 1915 on the following, with interest at 4 %, due January 1, April 1, July 1, and October 1.

Deposits		Withdrawals	
Jan. 2, 1914	$600	April 1, 1914	$ 50
April 1, 1914	500	Oct. 1, 1914	100
July 1, 1914	400		
Oct. 1, 1914	500		

Note. Take 1 % for each of the four periods.

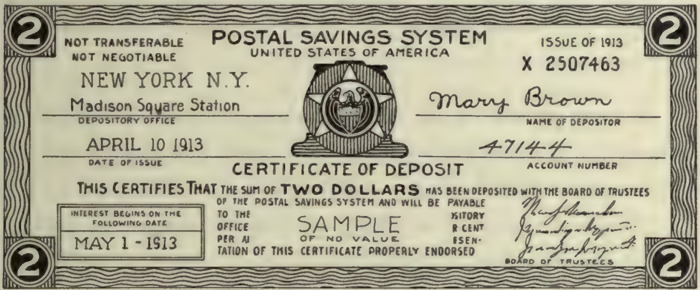
8. Find the balance due on the following, Jan. 1, 1915, with interest at 4 %, due the first of each month.

Deposits		Withdrawals	
Jan. 1	$200	Feb. 2	$100
Apr. 1	100	Aug. 4	100
June 1	100	Nov. 6	100
July 1	100		
Sept. 1	100		
Oct. 1	100		
Dec. 1	100		

POSTAL SAVINGS BANKS

The **postal savings** system was established by the United States government to provide opportunity for depositing small savings, with the security of the government for repayment.

Accounts may be opened with $1 or more by anybody 10 years of age or over. Smaller amounts than $1 may be saved for deposit by the purchase of postal savings cards and stamps.



Deposits are evidenced by **postal savings certificates** issued in denominations of $1, $2, $5, $10, $20, and $50, each bearing the name of the depositor, the number of his account, the date of issue, the name of the depository office, and the date on which interest begins. Not more than $100 may be deposited in one month nor more than $500 all together, exclusive of interest.

Interest is allowed on all deposits at the rate of 2 per cent per annum, computed on each savings certificate separately, and payable annually. No interest is paid on money that remains on deposit for a fraction of a year only. Deposits bear interest from the *first day* of the month next following that in which they are made.

Compound interest is not allowed; but a depositor may withdraw interest payable and include it in a new deposit, which will bear interest at the regular rate.

A depositor is permitted to exchange the whole or any part of his deposits in sums of $20, $40, $60, $80, $100, or multiples of $100 up to and including $500, into United States registered or coupon bonds bearing interest at the rate of $2\frac{1}{2}$ per cent per annum, payable semiannually, and redeemable at the pleasure of the United States after one year from date of issue, both principal and interest payable 20 years from such date in United States gold coin. Such exchange may be made under date of Jan. 1 and July 1 of each year, provided such bonds are then available.

1. If you deposit $20 in a postal savings bank on Jan. 30, 1914, when does interest begin thereon and when will the interest payment be due? How much interest will be due?

2. When does interest begin on a deposit opened Jan. 15, 1914? Feb. 28, 1914? March 4, 1914?

3. If you deposit, on March 31, 1914, $85 in a postal savings bank, receiving certificates for $5, $10, $20, and $50, how much interest will be due you on April 1, 1915?

4. If a boy deposits $15 in a postal savings bank April 30, 1914, and $50 May 30, 1914, how much interest will be due him May 1, 1915 on the $15 deposit and June 1, 1915 on the $50 deposit?

5. If $500 worth of postal certificates are exchanged on Jan. 1, 1914 for United States bonds, how much interest will be due on them July 1, 1914 at $2\frac{1}{2}\%$ per annum?

6. If $350 worth of postal certificates are exchanged on July 1, 1914 for United States bonds, how much interest will be due on them Jan. 1, 1915 at $2\frac{1}{2}\%$ per annum?

EXCHANGE

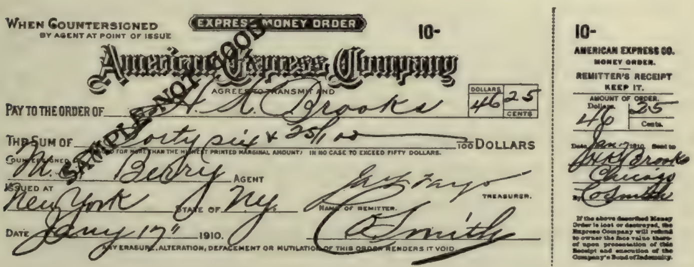
Exchange is the method of paying debts or collecting credits in distant places without transmission of money.

Exchange between two places in the same country is called **domestic exchange.**

Payments in the same country may be made by *postal money order*, by *express money order*, by *telegraphic money order*, by *bank check*, by *bank draft*, or by a *commercial draft* of a creditor on a debtor.

A **postal money order** is a written order by the postmaster in one place, to the postmaster in another place, to pay a specified sum of money to the person named in the order.

An **express money order** issued by an agent of an express company to another agent is similar to a postal money order. Both are negotiable.


WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE — EXPRESS MONEY ORDER — 10-

American Express Company AGREES TO TRANSMIT AND

PAY TO THE ORDER OF H. M. Brooks — DOLLARS 46 CENTS 25

THE SUM OF Forty six & 25/100 DOLLARS

COUNTERSIGNED M. E. Berry AGENT — E. Fargo TREASURER

ISSUED AT New York STATE OF N.Y.

DATE Jany 17th 1910. — NAME OF REMITTER C. Smith

ANY ERASURE, ALTERATION, DEFACEMENT OR MUTILATION OF THIS ORDER RENDERS IT VOID

10- AMERICAN EXPRESS CO. MONEY ORDER. REMITTER'S RECEIPT KEEP IT. AMOUNT OF ORDER. Dollars 46 Cents 25

If the above described Money Order is lost or destroyed, the Express Company will refund to owner the face value thereof upon presentation of this Receipt and execution of the Company's Bond of Indemnity.

Postal and express money orders, in addition to the sum specified on the face, cost as follows :

Orders for sums not exceeding	$2.50	3¢
Over $2.50 and not exceeding	5.00	5¢
Over 5.00 and not exceeding	10.00	8¢
Over 10.00 and not exceeding	20.00	10¢
Over 20.00 and not exceeding	30.00	12¢
Over 30.00 and not exceeding	40.00	15¢
Over 40.00 and not exceeding	50.00	18¢
Over 50.00 and not exceeding	60.00	20¢
Over 60.00 and not exceeding	75.00	25¢
Over 75.00 and not exceeding	100.00	30¢

NOTE. Postal orders are not issued for sums larger than $100.

A **telegraphic money order** is similar to the two orders mentioned. In addition to the regular charge for a 15-word message between the two places, the rates are:

For orders for not more than $25	25 cents
For more than $25 but not more than $50	35 cents
For more than $50 but not more than $75	60 cents
For more than $75 but not more than $100	85 cents

After the first $100 up to and including $3000, add 25 cents for each $100 or part thereof.

A **bank check** is an order on a bank, given by one of its depositors, to pay a specified sum to the order of a certain person. (See p. 357.)

A **certified check** is a notice to the payee that the amount of the check has been taken from the maker's deposit and placed with the bank's funds for the payment of the check when presented. Certified checks are used when the payee does not wish to accept a personal check.

As a rule, a small fee, called **exchange**, is charged for collecting a check in any town outside the location of the bank on which the check is drawn.

A **bank draft**, or **bill of exchange**, is a check drawn by one bank upon another.

By whom is the draft on p. 371 drawn? For what sum is it drawn? on what bank?

BANK DRAFT

No. 769

Third National Bank

Fort Wayne, Ind., June 8, 1914.

Pay to the order of Robert Saunders $700.00

Seven hundred $\frac{no}{100}$ .... Dollars.

To Corn Exchange Bank

NEW YORK, N. Y.

Frank Thompson,

Cashier.

Banks usually keep some money on deposit in other banks, called **correspondence banks**, located in large cities. Against this money they draw drafts, which they sell to customers who desire to remit payments to such cities. A small charge, called **exchange**, is usually made by banks for such service. This charge ranges from 10¢ to 50¢ on small drafts, and from $\frac{1}{10}$% to $\frac{1}{4}$% on drafts for larger amounts.

SIGHT COMMERCIAL DRAFT

3000\frac{no}{100}$ Milwaukee, Wis., April 10, 1914.

At sight pay to the order of Traders' Bank of Milwaukee

Three thousand $\frac{no}{100}$ .... Dollars.

Value received, and charge to account of

To Henry Smith, William Brown.

533 Wabash Ave., Chicago, Ill.

Collections may also be made by **commercial draft.** If Henry Smith of Chicago owes William Brown of Milwaukee \$3000, which Mr. Brown wishes to collect at once, he may draw on Henry Smith a sight draft like the one on p. 371.

The Milwaukee bank sends this draft to some Chicago bank, which collects the amount of Henry Smith and remits it to the Milwaukee bank, which in turn pays the amount, less a small fee for collection, to William Brown.

The following draft, due "Ninety days after sight," is called a **time draft.**

TIME COMMERCIAL DRAFT

\$3000 00/000 *Milwaukee, Wis., April 10, 1914.*

Ninety days after sight .... Pay to

the order of .... Traders' Bank of Milwaukee ....

Three thousand no/100 .... *Dollars.*

Value received, and charge to account of

To Henry Smith, *William Brown.*

533 Wabash Ave., Chicago, Ill.

If Mr. Smith intends to pay the draft when presented to him, he writes across the face in red ink "*Accepted*," and then adds the date and his signature. This draft has now the force of a promissory note and may be discounted, for the time it has to run, after date of acceptance.

Formerly, if trade between two trade centers like Chicago and New Orleans was equal, exchange was at par. If Chicago owed New Orleans, the demand in Chicago for drafts on New Orleans was greater than the demand in New Orleans for Chicago drafts. Hence, the drafts were at a premium in Chicago. Under reversed conditions, they were at a discount. For a long time, however, there has been little variation of this kind.

PROBLEMS

Written

1. Find the cost of a postal or an express money order for \$2.35; \$2.55; \$8; \$55.75; \$98.60.

2. Find the cost of sending a telegraphic money order for \$2345 at 1% plus \$.75 for the message.

3. What must be paid in Indianapolis for a draft on Chicago for \$800, at $\frac{1}{10}$% exchange?

SOLUTION. $\frac{1}{10}$% of \$800 = \$.80, cost of exchange.
\$800 + \$.80 = \$800.80, cost of draft for \$800.

4. A merchant in Chicago owes a merchant in Fort Wayne \$4750. How much must be paid in Chicago for a draft to settle the account, at $\frac{1}{4}$% exchange?

5. How much must be paid for a draft of \$354 at $\frac{1}{5}$% exchange?

6. Find the proceeds of a draft for \$5000, payable 30 days after sight, if discounted at 6% for the full term.

SOLUTION. The bank discount on \$1 for 30 da. at 6% = \$.005
5000 × \$.005 = \$25, bank discount on \$5000.
\$5000 − \$25 = \$4975, proceeds.

7. Find the proceeds of a draft for \$21,000, for ninety days after date, if discounted at 5% for the full term.

8. T. C. Arnold & Co. of New York drew a sight draft for \$2000 on R. J. Steele of Evansville, Ind. This draft was sold at a bank at $\frac{1}{4}$% discount. Find the proceeds.

9. Find the proceeds of a sight draft for \$3000, exchange and collection together being $\frac{1}{8}$%.

10. Find the proceeds of a draft for \$12,000, for sixty days after date, if discounted at 6% for the full term.

11. Mr. Bruce sends Mr. Jameson a check for \$480 in payment of a debt. Mr. Jameson cashes the check at a bank which charges him $\frac{1}{10}\%$, for collection. How much does the bank pay Mr. Jameson?

New York, Chicago, and San Francisco are the principal exchange or money centers in the United States.

12. Find the cost in Chicago of a draft on New York for \$2500 if \$1.00 per \$1000 is charged for collection.

13. Find the cost in South Bend, Indiana, of a draft on New York for \$5000 if $\frac{1}{10}\%$ is charged for collection.

14. Find the cost in Chicago of a draft for \$3500 on Vincennes, Indiana, if \$1.80 per \$1000 is charged for collection.

People intending to travel abroad often secure a letter of credit or a foreign express check.

A **letter of credit** is a letter from a bank in one country to one or more of their correspondent banks in another country, directing them to pay to the holder any sum not exceeding a certain amount specified in the letter.

Travelers' checks are guaranteed checks issued by express companies, banks, steamship companies, etc., in denominations of \$10, \$20, \$50, \$100, and \$200, at a fixed rate of $\frac{1}{2}\%$ of the face value.

The following values were printed on a \$20 check:

£ *s.* *d.*	FRANCS	MARKS	LIRE	CROWNS	FLORINS
4 1 2	102.50	82.50	102.50	73.39	49.02

15. Find how much English, French, and German money would be received in return for a \$50 check; for a \$10 check; for a \$100 check.

NOTE. There are 12 **pence** (*d.*) in one **shilling** (*s.*) and 20 shillings (*s.*) in 1 **pound** (£).

STOCKS AND BONDS

A **company** is an association of individuals for the transaction of business.

A **corporation** is a company governed in its operations by a general law or a special charter.

A **charter** is the legal act of incorporation. It states the powers and duties of the corporation.

The **stock** of a company is the money invested by it to carry on its business. The owners of stocks are called **stockholders.**

A **share** is one of the equal parts into which the stock has been divided. A share is usually of the value of $100 ; but may be more or less. In this book shares are $100 in value.

The **face**, or **par value**, is the sum for which the share is issued.

The **market value** is the sum for which a share can be sold.

A **dividend** is a sum paid to the stockholders out of the gains of the company. It is computed as a per cent of the face or par value of the stock.

An **assessment** is a sum required of the stockholders, according to the amount of stock held by each, to meet losses or expenses of the company.

Corporations often issue two kinds of stock, called **preferred stock** and **common stock.** The holders of preferred stock generally receive a specified rate of dividend that is paid before the holders of common stock receive any share of the profits.

Bonds are promissory notes issued by corporations and secured by mortgages on some property owned by the corporation. The usual face value of a bond is $1000.

Coupon bonds have interest certificates attached to them, which are redeemed as the interest comes due. They are payable to bearer. When interest is due the coupon is cut off and may be cashed at a bank or at the place where issued.

Registered bonds are recorded in the name of the owner in the books of the company issuing them. They cannot be transferred without indicating the transfer in the company's records.

Par and market value in bonds have the same meaning as in stocks.

NOTE. **Stockholders** are owners of the property, while **bondholders** are creditors.

Stocks and bonds are **at par** when they sell for the face value; **above par**, or at a premium, when they sell for more than the face value; **below par**, or at a discount, when they sell for less than the face value.

STOCK CERTIFICATE

Bonds bear interest at the rate specified on the face. It is always computed on the par value. A 4% bond is one that bears \$40 interest per \$1000 bond in one year. U. S. 5-20's are United States bonds bearing 5% interest, and running for 20 years.

A **broker** is a person who buys and sells stocks and bonds for another.

Brokerage is the compensation of the broker. It is computed on the par value. "Brokerage $\frac{1}{8}$" means that the broker receives $\frac{1}{8}$ of a dollar for each \$100 share bought or sold; that is, $\frac{1}{8}$% of the par value.

1. What is the name of the corporation that issued the stock certificate represented on p. 376? Where is this corporation located? What is the amount of the capital stock? What is the par value of each share? Who is the owner of this certificate? How many shares of the stock are represented by it? What is its face value?

2. If the company declares a 5% dividend, what is Mr. Fluker's income from the stock?

3. If he sells it at a premium of 10%, how much will he receive for it?

QUOTATIONS

STOCKS		BONDS	
Baltimore and Ohio . . .	$92\frac{3}{4}$	U. S. 4s coup., 1925	112
Baltimore and Ohio pfd. . .	98	U. S. 3s reg., 1918	105
Pennsylvania	107	Can. So. 5s	104

The above quotations from a newspaper mean: Baltimore and Ohio common stock, at $92\frac{3}{4}$% of its par value; Baltimore and Ohio preferred stock, at 98% of its par value; Pennsylvania railroad common stock at 107%; United States coupon bonds, paying 4% interest, and due in 1925, at 112%; United States registered bonds, paying 3% interest and due in 1918, at 105%; and Canada Southern railroad bonds, paying 5% interest, at 104%.

Study Recitation

1–24. Find the cost of the following stocks and bonds, including $\frac{1}{8}\%$ for brokerage.

Note. The par value of 1 share of each stock is \$100 and the par value of each bond is \$1000.

Stocks					Bonds		
	No. of Shares	Name of Stock	Market Price		No. of Shares	Name of Bond	Market Price
1.	10	Amalgamated Copper	$70\frac{7}{8}$	*13.*	4	U. S. 3s coup.	$102\frac{3}{8}$
2.	5	Atchison, Topeka & Santa Fé	92	*14.*	5	Adams Ex. 4s	$73\frac{7}{8}$
				15.	10	Jap. 1st $4\frac{1}{2}$s	$88\frac{1}{4}$
3.	10	Baltimore and Ohio	$92\frac{3}{4}$	*16.*	3	N. Y. Canal 4s	$98\frac{1}{2}$
4.	5	Canadian Pacific	223	*17.*	2	N. Y. City 4s	$96\frac{3}{8}$
5.	10	Central Leather	24	*18.*	6	Balt. & O. 4s	$91\frac{1}{8}$
6.	100	Consolidated Gas	130	*19.*	10	Can. So. 5s	104
7.	20	Erie	27	*20.*	8	N. Y. Cen. $3\frac{1}{2}$s	$80\frac{1}{8}$
8.	30	Illinois Central	106	*21.*	5	Southern Railway 5s	$102\frac{3}{4}$
9.	50	Lehigh Valley	$148\frac{1}{4}$				
10.	100	Pennsylvania	107	*22.*	4	Union Pacific 4s	$85\frac{7}{8}$
11.	10	Southern Pacific	$85\frac{7}{8}$	*23.*	10	Wabash 1st 5s	$101\frac{1}{2}$
12.	5	United States Steel	56	*24.*	3	West Shore 4s	$90\frac{1}{2}$

Solution to Example 1

Cost of 1 share at $70\frac{7}{8} + \frac{1}{8} = .71$ of \$100, or \$71.
Cost of 10 shares $= 10 \times$ \$71, or \$710.

Solution to Example 13

Cost of 1 bond at $102\frac{3}{8} + \frac{1}{8} = 102\frac{1}{2}\%$ of \$1000, or \$1025.
Cost of 4 bonds $= 4 \times$ \$1025, or \$4100.

25–48. Find the proceeds from the sale of the above stocks and bonds, deducting $\frac{1}{8}\%$ for brokerage.

Solution to Stock in 1

Proceeds from sale of 1 share at $70\frac{7}{8} - \frac{1}{8} = .70\frac{3}{4}$ of \$100, or \$70.75.
Proceeds from sale of 10 shares $= 10 \times$ \$70.75, or \$707.50.

Solution to Bond in 13

Proceeds from sale of 1 bond at $102\frac{3}{8} - \frac{1}{8} = 1.02\frac{1}{4}$ of \$1000, or \$1022.50.
Proceeds from sale of 4 bonds = $4 \times$ \$1022.50, or \$4090.

49–60. Find the yearly interest on bonds numbered 13 to 24.

Solution to Bond in 13

3% of \$1000 = \$30; $4 \times$ \$30 = \$120.

61. If I buy a \$1000 5% bond at 102, which includes brokerage, what rate of income, to the nearest tenth of 1 per cent, do I realize on my investment?

Solution

The interest on the bond is \$50 a year. The investment is \$1020. \$50 is $\frac{50}{1020}$, or 4.9%, of \$1000. *Ans.* 4.9%.

Written

62. Add $\frac{1}{8}$% brokerage to each of the quotations on bonds in numbers 13 to 24, and find the rate of income on each to the nearest tenth of 1 per cent.

63. How much must be paid, including $\frac{1}{8}$% brokerage, for 15 shares of National Lead Co. stock at $106\frac{7}{8}$?

64. If I buy 10 shares of Northern Pacific Stock at $116\frac{3}{8}$, with $\frac{1}{8}$% brokerage, and sell it at $120\frac{5}{8}$, with $\frac{1}{8}$% brokerage, how much do I gain?

65. How much yearly interest will be received on five U. S. 3's and what rate of income will they yield when bought at 102?

66. How much must I pay for a 4% bond to realize 5% on my investment?

Solution. \$40, the yearly interest on the bond, is 5% of the cost of the bond, \$40 ÷ .05 = \$800; hence the bond would be quoted at 80.

67. Make and solve problems like the above, using quotations from a daily newspaper.

GENERAL REVIEW

PROBLEMS

Oral

1. John had 36 pencils and used $33\frac{1}{3}\%$ of them. How many had he left?

2. A knife costing \$.80 is sold at a profit of 25%. What is the selling price?

3. What per cent of 30¢ is 30¢?

4. I buy a watch for \$16 and sell it for \$24. Find the per cent of gain.

5. A merchant, after deducting 10% from the list price of an article, receives \$2.70 for it. Find the list price.

6. James sells an article for \$.96, thereby gaining $33\frac{1}{3}\%$ of the cost. What is the cost?

7. By selling apples at 4¢ a dozen more than cost, a boy gained 10%. How much did he pay per dozen?

8. How much is paid for a carriage listed at \$240, with discounts of 25% and 10%?

9. By selling 3 qt. of milk for the price of 1 gal., I gain what per cent?

10. If Mr. Shaw buys lemons at 3 for 2¢ and sells them at 3 for 5¢, what per cent does he gain?

11. Mr. Baker has a 5% mortgage of \$10,000 on Mr. Beard's house. What amount of interest must Mr. Beard pay him each 6 months?

12. If a gain of \$500 on some goods is a gain of 20%, what is the cost of the goods?

13. A farmer set out 30,000 celery plants, but only $83\frac{1}{3}\%$ of them matured. If these were sold at \$3 per hundred, how much did the farmer receive for them?

14. Mr. Bruce buys 10 shares of stock at 80 and sells them at 100. Find the cost and the selling price, including $\frac{1}{8}\%$ for brokerage in each case.

15. Find the commission at 5% on a sale of $850.

16. $40 is 200% of what sum?

17. An agent buys a lot for $1500 and charges 2% commission. Find the entire cost of the lot.

18. A building worth $4000 is insured for $\frac{7}{8}$ of its value at 80¢ a hundred. Find the premium.

19. A hardware merchant sold two knives for 90¢ each. On one he lost 25%, and on the other he gained 25%. Did he lose or gain on the transaction, and how much?

20. An insurance company charged $240 premium for insuring a store for $\frac{3}{4}$ of its value at $1 per hundred. Find the value of the store.

21. A school district having an assessed valuation of $50,000 wishes to raise a tax of $250. What is the tax rate?

22. What is the tax of A, living in the school district mentioned, if his property is assessed at $5000?

23. What is the duty on 1000 yards of Brussels carpet, invoiced at $2.40 a yard, at 25% ad valorem?

24. What is the interest on $500, at 5%, for three years?

25. Find the amount due on a $600 note, bearing 6% interest, for 2 years and 6 months.

26. Find the bank discount at 8% on a $1000 note, due in 3 months, without interest.

27. Find the cost of a $1000 bond quoted at $98\frac{7}{8}$ with $\frac{1}{8}\%$ brokerage.

28. Find the yearly interest on a $1000 5% bond.

29. Find the cost of a draft for $1000 at $\frac{1}{10}\%$ exchange.

Written

30. Mr. Brown sold his farm for $6600, thereby gaining 20%. What was his gain?

31. A horse costing $125 is sold for $140. Find the gain per cent.

32. 25% of the cost is lost in selling an article for $4.50. Find the loss.

33. Find the commission at 2% for selling 6 carloads of hogs, weighing in all 90,000 pounds, at 8 ¢ a pound.

34. An agent receives $160 as a 2% commission for the purchase of a lot. Find the price paid for it.

35. A commission merchant purchases $15,500 worth of cotton, and charges $2\frac{1}{2}$% commission. What is the amount of the bill he sends to his employer?

36. Find the number of days between Jan. 4, 1914 and August 10 of the same year.

37. Find the exact interest on $450 at $4\frac{1}{2}$% from Jan. 4, 1914, to Aug. 10, 1914.

38. Find the bank discount on a note amounting to $8020, discounted for 65 days, at 8%.

39. Mr. Smith sold his farm for $6360, thereby gaining 20%. Would he have gained or lost, and what per cent, if he had sold it at $5500?

40. If 5% is lost by selling an article for $2.47, what per cent is gained or lost by selling it for $2.99?

41. A stock of goods worth $16,400 is insured for $\frac{3}{4}$ of its value at $8\frac{1}{2}$%. What is the premium? If the entire stock should burn, what would be the owner's loss by fire?

42. Find the cost of 10 shares of stock quoted at $95\frac{1}{2}$, brokerage $\frac{1}{8}$%.

43. Find the semiannual interest on 5 U. S. 3's, par value $1000.

44. A merchant buys through an agent 7300 yd. of carpeting at $1.25 a yard, paying $\frac{3}{4}$% commission. The freight amounts to $7.37. At what price per yard must the carpet be sold to realize a profit of 20%?

45. An agent sold 13,125 bu. of wheat at $.80 a bushel, on 5% commission. What was his commission?

46. If 24 bu. of corn are raised from 1 pk. of corn, what per cent is the increase? What is the increase?

47. A farmer buys 24 head of cattle at $80 a head, and after losing 6 sells the remainder at $105 a head. What per cent does he gain or lose?

48. A grocer selling eggs at 20 cents a dozen cleared 25% of the cost. How much did the eggs cost him? Another grocer, selling at the same price, cleared $33\frac{1}{3}$% of the cost. What was the cost?

49. A merchant bought silk at $1.50 a yard. He marked it so as to give him a profit of 20%. As it did not prove a good seller, he offered it at a discount of $33\frac{1}{3}$% and 10%. What was his asking price? his selling price? his loss per cent?

50. A farmer bought 40 acres of land at $40 an acre and spent $600 for improvements. For how much must he sell it to gain 20%?

51. I receive $4000 as commission at 4% on a sale of goods. Find the amount of sale.

52. A merchant sold a house for $3680, losing 8%. For how much should he have sold it to gain $12\frac{1}{2}$%?

53. If a lamp bought for $\frac{1}{8}$ less than its value is sold for $\frac{1}{4}$ more than its value, what is the per cent of gain?

54. A hotel is insured for $\frac{2}{3}$ of its value, at $1\frac{1}{2}$ % premium, and the premium amounts to \$200. What is the value of the hotel?

55. I sold two machines for \$300 each. On one I gained 20 % and on the other I lost 20 %. Did I gain or lose on the transactions, and how much?

56. A merchant bought a bankrupt's stock at 45 cents on the dollar, and sold it at 10 % below the original price. What per cent did he gain?

57. An article lost 10 % by wastage, and was sold for 30 % above cost. What was the gain per cent?

58. What per cent of 100 bushels are 50 pecks?

59. A dealer bought an article at 10 % below the price asked, and sold it for \$2400, gaining $33\frac{1}{3}$ %. What was the price asked?

60. A commission merchant receives 17 firkins of butter, each containing 43 lb., 630 lb. of chickens, and 68 cases of eggs, of 16 doz. each. He sells the butter at 30 cents a pound, the chickens at 24 cents a pound, and the eggs at 30 cents a dozen. Find the amount of his commission at 4 %.

61. If 3 % more is gained by selling a horse for \$83.25 than by selling it for \$81, what is the cost of the horse?

62. If a boy gains 12 % by selling 5 oranges for 14 cents, what per cent would he gain by selling them at 6 for 18 cents?

63. Mr. Jones insured his house, worth \$4800, for 3 years for $\frac{5}{6}$ of its value, at 75 cents per hundred. Find the rate per cent charged, the premium, and the company's loss, if the house should burn.

64. Find the duty, at 40 % ad valorem, on an invoice of 800 doz. linen handkerchiefs, valued in Berlin at \$3.75 a dozen.

65. A tax of \$25,000 is assessed in a city upon a property valuation of \$125,000,000. What is the rate of taxation, and what does Mr. T. Gray pay, whose store is assessed at \$16,500, house at \$2250, and personal property at \$950?

66. What price must be asked for goods that cost \$240 in order that there may be a clear profit of $16\frac{2}{3}$ % after allowing a discount of $12\frac{1}{2}$ % from the list price?

67. A 4 % note of \$400 was given March 8, and paid December 15 of the same year. Find the amount of the note.

68. Write a negotiable note for \$300, bearing interest at the rate of 7 % for 2 years, drawn by G. J. Jones in favor of R. S. Brown, July 5, 1913. Underscore the words that make it negotiable. What must R. S. Brown do if he sells the note to M. A. Starr?

69. Find the proceeds of the note in problem 68 if it is discounted at a bank Jan. 3, 1914 at 8 %.

70. What should the Mechanics' Bank pay for a note of \$1500, dated July 12, due in 4 months, bearing 8 % interest, discounted Sept. 1 at 6 %?

71. A merchant is given the choice of paying \$500 in 6 months, or receiving a 2 % discount for cash. Which is the better offer, and by how much, if money is worth 6 %?

72. How much interest will be paid annually on a \$1000, 5 % bond? a $4\frac{1}{2}$ % bond? a 4 % bond?

73. What rate of income do I realize on a \$1000 4 % bond bought at 80?

74. Find the cost of 5 \$100 shares of stock bought at $68\frac{1}{8}$ with brokerage at $\frac{1}{8}$ %.

Problems without Numbers

75. If you know the principal, the rate, and the time, how can you find the interest and the amount?

76. If you know the principal, the rate, and the interest, how can you find the time?

77. If you know the principal, the interest, and the time, how can you find the rate?

78. If you know the interest, the rate, and the time, how can you find the principal?

79. Write out and indorse a check made payable to Robert Hall by Frank Clark.

80. Write a negotiable promissory note.

81. If you know the amount of a mortgage and the rate of interest, how can you find the semiannual interest due on the mortgage?

82. If you know the face of a note, the term of discount, and the rate, how can you find the bank discount? the proceeds?

83. How can you find how much interest is due each half year on a savings bank account?

84. When does interest begin on a deposit made in a postal savings bank?

85. How can you pay money at a distance?

86. If you know the face of a draft and the rate of exchange, how can you find the cost of the draft?

87. If you know the market price of a stock and the rate of brokerage, how can you find the cost of a given number of shares? the proceeds from the sale of a given number of shares?

88. If you know the denomination of a bond and the rate of interest, how can you find the yearly interest?

89. If you know the market price of a bond and the yearly interest, how can you find the rate of income it yields?

USE OF SYMBOLS

In the analysis of problems it is frequently necessary to perform long operations in addition, subtraction, multiplication, or division in order to find the several results that must be obtained to secure the final result.

The analysis of the following problems illustrates the necessity of such operations in order to find the answer.

1. If 125 acres of land are bought for \$62.50 an acre and sold for \$9375, what is the gain per acre?

ANALYSIS. If land costs \$62.50 an acre, 125 acres cost 125 × \$62.50, or \$7812.50. If land costing \$7812.50 is sold for \$9375, the gain equals \$9375 − \$7812.50, or \$1562.50. If \$1562.50 is gained by the sale of 125 acres of land, the gain on 1 acre is \$1562.50 ÷ 125, or \$12.50.

In this analysis three operations are required and three results are found. \$62.50 must be multiplied by 125; \$7812.50 must be subtracted from \$9375; \$1562.50 must be divided by 125.

When pupils have been trained to add, subtract, multiply, and divide accurately and rapidly, it is entirely unnecessary to have *all* the problems in the arithmetic worked out completely, as the analysis demands all the reasoning involved in the solution. Much time usually given to figuring in arithmetic can thus be saved for work in other subjects.

In the solutions of problems, any letter may be substituted for, or used instead of, any number which might be the result of any operation, provided the same letter is not used instead of any other number in the solution of the problem. Numbers resulting from other operations in the same solution may be represented by other letters.

The use of letters for numerical results of indicated operations is illustrated in the following analysis of problem 1.

Analysis. If land costs $62.50 an acre, 125 acres cost 125 times $62.50, or x dollars. If land costing x dollars is sold, at a gain, for $9375 the gain equals $9375 $-\ x$ dollars, or y dollars. If y dollars are gained by the sale of 125 acres of land, the gain on one acre is $\frac{1}{125}$ of y dollars, or y dollars $\div$ 125, or $\frac{y}{125}$ dollars. The expression $\frac{y}{125}$ dollars represents the answer. If desired, z dollars may be substituted for $\frac{y}{125}$ dollars.

In an analysis of this kind, wherever we indicate an operation, we at once assume it to have been performed and substitute a letter for the result. As we proceed with the analysis, whenever it is necessary to use this result, we use instead the substituted letter, and substitute another letter for the result of the next operation, and so on until the expression or letter representing the final result is found. We can thus proceed with the analysis without the delay or interruption of our course of reasoning, resulting from the performance of each operation.

This analysis might also be indicated as follows:

$$\frac{\$9375-(125\times\$62.50)}{125}=x \text{ dollars gain. } \textit{Ans.}$$

A number of illustrations of the first form of analysis are here given. The same method may be used at the teacher's discretion with other problems throughout the book.

2. What change should be received from $50, in payment for 16 doz. buttons at $.36 a dozen, 6 yards of velvet at $4.75 a yard, and 15 yards of lining at $.18 a yard?

Analysis. 16 doz. buttons at $.36 a dozen cost 16 times $.36, or x dollars. 6 yards of velvet at $4.75 a yard cost 6 times $4.75, or y dollars. 15 yards of lining at $.18 a yard cost 15 times $.18, or z dollars. The cost of the buttons, velvet, and lining equals $x+y+z$, or p, dollars. The amount of change to be received is $50 $-\ p$ dollars or r dollars.

Or, $\$50-(16\times\$.36+6\times\$4.75+15\times\$.18)=x$ dollars. *Ans.*

3. At $1.50 a day, how much does a man earn in 3 weeks, if he works $5\frac{3}{4}$ days the first week, $4\frac{3}{5}$ days the second week, and $5\frac{2}{3}$ days the third week?

Analysis. During the 3 weeks he works $5\frac{3}{4}$ days + $4\frac{3}{5}$ days + $5\frac{2}{3}$ days, or x days. If he earns \$1.50 a day, in x days he earns x times \$1.50, or y dollars.

Indicate this analysis in another way.

4. A drover with \$2144 bought as many horses as possible for \$165 each, and spent the remainder for sheep at \$4 a head. How many of each did he buy?

Analysis. If horses cost \$165 each, for \$2144 a drover could buy $\frac{\$2144}{\$165}$, or x horses, and have y dollars left. At \$4 each, for y dollars he could buy $\frac{y}{4}$, or z sheep. He bought x horses and z sheep.

5. Corn weighs 56 lb., and wheat 60 lb., a bushel. If a farmer sells 6640 lb. corn at \$.35 a bushel, and 4290 lb. wheat at \$.60 a bushel, how much should he receive?

Analysis. 6640 lb. corn $= \frac{6640}{56}$ bu., or x bu. x bu. corn at \$.35 a bushel are worth $x \times$ \$.35, or y dollars. 4290 lb. wheat $= \frac{4290}{60}$ bu., or z bu. z bu. wheat at \$.60 a bushel are worth $z \times$ \$.60, or r dollars. The farmer should receive y dollars + r dollars, or s dollars.

Or, $\frac{6640}{56} \times \$.35 + \frac{4290}{60} \times \$.60 = x$ dollars. *Ans.*

6. A bin 8 ft. square and 6 ft. deep is $\frac{2}{3}$ full of wheat. What is the value of the wheat at \$.70 a bushel?

Analysis. If the bin is 6 ft. in depth and is $\frac{2}{3}$ full, the depth of the wheat in the bin is $\frac{2}{3}$ of 6 ft., or 4 ft. If the mass of wheat in the bin is 8 ft. wide, 8 ft. long, and 4 ft. deep, its volume is $8 \times 8 \times 4$ cu. ft., or 256 cu. ft. 256 cu. ft. $= 256 \times 1728$ cu. in., or x cu. in. One bushel contains 2150.4 cu. in. x cu. in. $= \frac{x}{2150.4}$ bushels, or y bushels. The value of y bushels of wheat at \$.70 a bushel $= y \times$ \$.70, or z dollars.

As in the first part of the above analysis, whenever the results of the indicated operations can be found mentally without trouble, letters should not be substituted for such results.

7. The interest of any sum is equal to the product of the principal by the rate by the time expressed in years. Ex-

press this rule by a formula using p for principal, r for rate, and t for time.

8. A lot 120 ft. long and 80 ft. wide is surrounded by a sidewalk 8 ft. wide. What will be the cost of the sidewalk at \$1.50 per square yard?

ANALYSIS. Since the sidewalk is 8 ft. wide, the length of each end walk, including the corners, is 80 ft. + 8 ft. + 8 ft., or 96 ft. The length of both end walks is 2×96 ft., or 192 ft. The length of each side walk is 120 ft.: of both side walks is 2×120 ft., or 240 ft. The entire length of the walk is 192 ft. + 240 ft., or x ft. The area of the walk is $x \times 8$, or y sq. ft. As there are 9 sq. ft. in a square yard, the number of square yards in the walk is $\frac{y}{9}$. At \$1.50 per square yard, the cost of $\frac{y}{9}$ sq. yd. is $\frac{y}{9} \times$ \$1.50, or z dollars.

The above problem may be analyzed correctly in a number of ways, in any one of which letters may be used for the results that cannot be easily found mentally.

9. How much will a planter receive who sends to market 127 bales of cotton, averaging 500 pounds, if the cotton is sold at \$.09¾ a pound, the freight being \$.57 a hundredweight, drayage \$60, and commission $3\frac{1}{2}$%?

ANALYSIS. If the bales average 500 pounds, 127 bales weigh 127×500 pounds, or x pounds. x pounds, at \$.09¾ per pound, sell for $x \times$ \$.09¾, or y dollars. The commission for selling is $.03\frac{1}{2}$ of y dollars, or z dollars. As the freight rate is \$.57 a hundred pounds, the expense for freight on x pounds is $\frac{x}{100} \times$ \$.57, or r dollars. The entire expense of selling is, — commission, z dollars, + freight, r dollars, + drayage, \$60, or z dollars + r dollars + \$60, or s dollars. Since the cotton sells for y dollars and the expense for selling is s dollars, the planter will receive y dollars − s dollars, or t dollars.

10. What is the rule for finding the area of a rectangle? Express it in a formula, using A for area, b for base, and h for height or altitude.

EXPRESSING EQUATIONS

Study Recitation

1. If x represents the number of dollars A earns in 1 day, 3 times x, which may be written $3x$, will represent the number of dollars he can earn in 3 days at the same rate. $\frac{x}{2}$ will represent the number of dollars he can earn in half a day, and $\frac{5}{2}x$ or $\frac{5x}{2}$ will represent the number of dollars he can earn in $2\frac{1}{2}$ days, or $\frac{5}{2}$ days. What, then, will represent the number of dollars he can earn in 2 days? in 5 days? in 10 days? in $\frac{1}{3}$ of a day? in $2\frac{1}{3}$ days? in $4\frac{1}{4}$ days?

2. How many are $1¢ + 5¢ + 6¢$? $1x + 5x + 6x$? ($1x$ or once x is written simply x.)

3. If John has x dollars and Henry has 4 times as many dollars, how many dollars has Henry? How many dollars have John and Henry? (*Ans.* Once x plus 4 times $x = 5$ times x, or $x + 4x = 5x$.) How many more dollars has Henry than John? (*Ans.* $4x - x = 3x$.) The number of dollars John has is what part of the number Henry has? $\left(\textit{Ans.}\ x \text{ is } \frac{1}{4} \text{ of } 4x, \text{ or } \frac{x}{4x} = \frac{1}{4}.\right)$

4. John has 2 more apples than James. If we let x equal the number James has, what is the expression for the number John has? (*Ans.* $x + 2$.) If we let x equal the number John has, how many has James? (*Ans.* $x - 2$.)

5. If Henry has $\frac{1}{2}$ as many apples as Fred has, and Fred's are represented by x, what is the expression for the number Henry has? $\left(\textit{Ans.}\ \frac{x}{2}.\right)$ If Fred gives Henry 2 apples, what is the expression for the number Fred then has? (*Ans.* $x - 2$.) For the number Henry then has? $\left(\textit{Ans.}\ \frac{x}{2} + 2.\right)$

6. If A has x dollars and B has \$8 more than A, how many dollars has B? (*Ans.* $x+8$.) If C's money is \$8 less than A's, how many dollars has C? (*Ans.* $x-8$.) If D's money is 8 times A's, how many dollars has D? (*Ans.* $8x$.) If E's money is $\frac{1}{8}$ of A's, how many dollars has E? $\left(Ans.\ \frac{x}{8}.\right)$ If F's money is \$2 more than 8 times A's, how many dollars has F? (*Ans.* $8x+2$.) If G's money is \$2 less than 8 times A's, how many dollars has G? (*Ans.* $8x-2$.)

7. If James has 8 apples and William has 6 apples, they both have $(8+6)$ apples. If Henry has $\frac{1}{2}$ as many apples as both James and William, he has $\frac{8+6}{2}=\frac{8}{2}+\frac{6}{2}=\frac{14}{2}, =7$ apples. Substitute x for the number of apples James has, and y for the number William has, and find the expression for the number Henry has.

8. What number added to 8 will equal 10? This may be stated $?+8=10$. The **known numbers** here are 8 and 10, and the **unknown number** represented by ? is the answer 2.

If we represent the unknown number by x, the statement is $x+8=10$. This is an **equation**, since what precedes the equality sign equals what follows it. The part on the left, $x+8$, is called the **first member**, and the part on the right, 10, the **second member.**

Written

Make equations expressing the following, representing the unknown number by x:

9. If \$4 is added to Frank's money, he will have \$6.

10. James, who has \$3 less than John, has \$6.

11. Five times Ida's age equals Eva's age, 25 years.

12. One fourth of Anna's age equals 4 years.

13. What number increased by 7 equals 25?

14. The sum of two numbers is 30, and one of the numbers is 10.

15. The difference of two numbers is 5, and the larger number is 17.

16. Twice a number increased by 4 equals 50.

17. One fourth a number diminished by 3 equals 10.

18. One half of a number plus one fourth of it equals 8.

19. Two pieces of lead weigh 35 pounds, and one piece weighs 4 pounds more than the other.

20. If a piece of work can be done in 6 days, what part of it can be done in 1 day? in 3 days? in y days? $\left(Ans.\ \frac{1}{6};\ \frac{3}{6};\ \frac{y}{6}.\right)$ If a piece of work can be done in x days, what part of it can be done in 1 day? in 3 days? in y days? $\left(Ans.\ \frac{1}{x};\ \frac{3}{x};\ \frac{y}{x}.\right)$

21. If A can do a piece of work in 4 days, and B can do the same piece of work in 6 days, what part can each do in 1 day? What part can both do in 1 day? If x equals the number of days it takes A and B to do the work together, what is the expression for the part they can both do in 1 day? From these expressions form an equation.

22. If 6 bushels of potatoes cost \$3, the expression for the part of a dollar 1 bushel costs is $\frac{3}{6}$. The expression for the number of bushels \$1 will buy is $\frac{6}{3}$. If c bushels of potatoes cost d dollars, what is the cost of 1 bushel? How many bushels will \$1 buy? How many bushels will \$2 buy?

23. The sum of three numbers is 21. The second is twice the first and the third is twice the second. What is the ex-

pression for the second? for the third? for the sum of the first and the second? for the sum of the three? for the difference between the first and the second?

24. The sum of two numbers is 25. If the first number is 8, what is the expression for the second? If the first number is b, what is the second?

25. The sum of two numbers is a. The first number is 6. What is the second?

26. A father's age is three times that of his son. Make an expression for the age of each. What is the expression for the age of each 10 years later? The son is then one half as old as his father. Using the son's age, make an expression for the father's age. What two expressions are equal? Put them in the form of an equation. State the problem which the solution of the equation will answer.

SOLVING EQUATIONS

Study Recitation

To **solve an equation** is to find the value of the unknown number.

1. Solve the equation $x + 8 = 10$.

In solving equations it is best, by transposing, to get the unknown numbers represented by letters alone in the first member and the known numbers in the second member.

Solution. Since 10 is 8 more than x, x must be 8 less than 10; $x = 10 - 8$, or 2. Proof. $2 + 8 = 10$.

2. Solve the equation $x - 8 = 10$.

Solution. Since 8 less than x equals 10, x must equal 8 more than 10; $x = 10 + 8 = 18$. Proof. $18 - 8 = 10$.

Notice in each of the preceding solutions that the known number is transposed from the left to the right member of the equation by changing the sign preceding it. In example 1, $+8$ is changed to -8 on being transposed. In example 2, -8 is changed to $+8$ on being transposed.

Any term may be transposed from one member to another of an equation, if the sign + preceding the term is changed to − or the sign − to +.

Find the value of x in the following:

3. $x+3=5$	**5.** $3+x=5$	**7.** $x+7=15$
4. $x-8=12$	**6.** $20+x=24$	**8.** $x-10=20$

9. Add 2 to both members of the equation $x+3=5$. (*Ans.* $x+3+2=5+2$, or $x+5=7$.) Solve $x+5=7$ and compare the result with the solution of $x+3=5$.

10. Subtract 2 from both members of the equation $x+3=5$. (*Ans.* $x+3-2=5-2$, or $x+1=3$.) Solve $x+1=3$ and compare with the solution of $x+3=5$.

The same number may be added to, or subtracted from, both members of an equation without changing the value of the unknown number.

If $x=4$, $4\times x$, or $4x$, $=4\times 4=16$.
If $4x=16$, $\frac{1}{4}$ of $4x$, or x, $=\frac{1}{4}$ of 16, or 4.

Both members of an equation may be multiplied, or divided by, the same number without changing the value of the unknown number.

Written

Find the value of x in the following equations:

11. $5x=25$	**13.** $4x+x=45$	**15.** $2x+3x=35$
12. $6x=36$	**14.** $5x-x=24$	**16.** $9x-5x=12$

17. If $\frac{1}{3}$ of x, or $\frac{x}{3}$, equals 12, how much does x equal? (*Ans.* 3×12, or 36.) If $\frac{2}{3}$ of x, or $\frac{2x}{3}$, equals 12, how much does x equal? (*Ans.* $\frac{1}{2}$ of 3×12, or $\frac{3}{2}$ of 12, or 18.)

18. If $\frac{2x}{3} = 3$, find the value of x.

SOLUTION. Since 3 times $\frac{2x}{3}$ equals $2x$, multiply both members by 3 to make the first member an integer, $2x = 9$. Then divide both members by 2. *Ans.* $x = 4\frac{1}{2}$.

Solve the following equations:

19. $\frac{x}{2} = 10$

20. $\frac{2x}{3} = 8$

21. $\frac{3x}{5} = 15$

22. $\frac{4x}{9} = 16$

23. $\frac{3x}{4} = 9$

24. $\frac{5x}{7} = 25$

PROBLEMS

WRITTEN

In every problem certain statements, or suppositions, are given, and a question asks for a number whose value is unknown. The finding of this unknown number requires the making and solving of an equation in which the unknown number is represented by a letter.

The first step is to let x, or some other letter, represent the unknown number. This letter may then be used whenever it is necessary to employ the unknown number in the solution of the problem.

1. If a number is increased by 250, the result is 3296. What is the number?

SOLUTION

$$\text{Let } x = \text{the number}$$
$$\text{Then } x + 250 = 3296.$$
$$x = 3296 - 250.$$
$$x = 3046.$$

PROOF. $3046 + 250 = 3296.$

2. If 295 is subtracted from a number, the result is 488. What is the number?

3. Ella's age is 12 years less than Frank's, who is 25 years old. How old is Ella?

4. Eva and Ruth together have \$54. Eva has twice as much as Ruth. How much has each?

SOLUTION

Let $x =$ the number of dollars Ruth has.
Then $2x =$ the number of dollars Eva has,
and $x + 2x = 54.$
Hence, $3x = 54.$
$x = 18$, the number of dollars Ruth has.
$2x = 2 \times 18 = 36$, the number of dollars Eva has.

PROOF. $36 = 2$ times 18; $36 + 18 = 54$.

5. Separate the number 600 into two parts, one of which shall be 5 times the other.

6. A horse and carriage cost \$274. Find the cost of each if twice the cost of the carriage is \$34 less than the cost of the horse.

7. If $\frac{3}{5}$ of a number is 72, what is the number?

8. What number added to 3 times itself equals 240?

9. If 6 times a number plus 15 equals 315, what is the number?

10. If 5 times a number less 48 equals 302, what is the number?

11. If a number is increased by 3 times itself, the result will be 72. What is the number?

12. A school in which there are 500 pupils has 56 more girls than boys. How many of each are there?

13. Find the interest of \$1000 at 6% for 5 yr., using the formula $i = p \times r \times t$.

Solution. Substitute \$1000 for p, 6%, or $\frac{6}{100}$, for r, 5 for t

$$\$\overset{10}{\cancel{1000}} \times \frac{6}{\cancel{100}} \times 5 = \$300.$$

14. Form five problems in interest and solve them by the above formula.

15. Find the area of a rectangle having a base of 6 in. and a height, or altitude, of 8 in., using the formula, $A = b \times h$.

$A = b \times h$. $A = 4 \times 8 = 32$. *Ans.* 32 sq. in.

16. Find the area of a rectangle having a base of 6 in. and an altitude of 10 in., using the above formula.

17. Form five problems like 15 and 16, and solve them by the use of the formula given above.

18. Three men, A, B, and C, formed a partnership with \$30,000 capital. B's capital was 3 times A's, and C's as much as the difference between B's and A's. How much capital did each furnish?

19. A girl made three purchases which amounted to \$27. For shoes she paid \$2 more than for gloves, and for a dress \$17 more than for shoes. How much did each purchase amount to?

20. The sum of three numbers is 24. The second is 5 less than the first, and the third is 10 more than the second. What are the numbers?

21. Divide the number 33 into three parts such that the second shall be 3 less than the first, and the third 4 times the first.

22. What must be added to 15 to make 42? What must be added to x to make 42?

23. Divide 18 into two parts, the smaller of which is x. What is the other part? The smaller part equals $\frac{1}{2}$ the larger part. Find each part.

24. A pipe will fill a tank in 8 hours. What part of the tank will it fill in 1 hour? in 3 hours? A pipe will fill a tank in b hours. What part of the tank will it fill in 1 hour? in c hours?

25. 60 is 75% of what number?

26. Divide the number 57 into three parts such that the second shall be 4 more than the first and the third 8 less than the second.

27. If a number is increased by 7 times itself and 40, the result is 360. What is the number?

28. Separate the number 240 into three parts, the second of which is 3 times the first and the third 4 times the first.

29. What number diminished by 50 equals 712?

30. What number divided by 17 equals 20?

31. What number multiplied by 25 equals 625?

32. What number increased by 83 equals 214?

33. What number decreased by 20% of itself equals 40?

34. What number increased by 20% of itself equals 30?

35. Separate 100 into two parts, the smaller of which, x, equals $\frac{1}{3}$ of the larger part.

36. A mother is 4 times as old as her daughter, and the sum of their ages is 40 years. How old is each?

37. The sum of the three numbers is 30. If the second number is twice the first, and the third number is 3 times the first, what is each number?

38. Construct and solve six problems, one each like problems 29–34 above.

POWERS AND ROOTS

Study Recitation

A **power** is the product arising from using the same number two or more times as a factor.

$3 \times 3 = 3^2 = 9$.
9 is the product of 3 used twice as a factor.
In 3^2 the figure 2 shows how many times 3 is to be used as a factor.

If a number is used twice as a factor, the product is the **square**, or the **second power**, of the number.

9 is the square of 3, or 9 is the second power of 3.

If a number is used 3 times as a factor, the product is the **cube**, or the **third power**, of the number.

$4 \times 4 \times 4 = 4^3 = 64$. The 3 in 4^3 shows that 64 is the product of 4 used 3 times as a factor. 64 is called the cube of 4, or the third power of 4.

A number written at the right of, and a little above, another number is called an **exponent**, and shows how many times the other number is to be used as a factor.

In the expressions 5^2, 7^3, 8^4, 9^6, the exponents are 2, 3, 4, and 6. The expressions are read, 5 square, or the second power of 5, or 5 to the second power; 7 cube, or the third power of 7, or 7 to the third power; the fourth power of 8, or 8 to the fourth power; the sixth power of 9, or 9 to the sixth power.

What does each of these expressions mean? Write the equivalent of each expression in two ways.

TO THE TEACHER. A number is sometimes called the first power of itself; as, 2 is the first power of 2; but as there must be two factors in order to have a product, and as a power is the product of equal factors and 2 is not such a product, it cannot properly be called a power of itself. In the higher mathematics, however, the expression *first power* is used.

Read the following expressions. Give the value of each at sight:

1. 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 20^2, 50^2.

Learn this table:

$13^2 = 169$	$16^2 = 256$	$19^2 = 361$	$23^2 = 529$
$14^2 = 196$	$17^2 = 289$	$21^2 = 441$	$24^2 = 576$
$15^2 = 225$	$18^2 = 324$	$22^2 = 484$	$25^2 = 625$

To find a power of a fraction, reduce it to its lowest terms, and find the required power of each of its terms.

Find the powers indicated in the expressions below:

2. $26^2 = ?$ $28^2 = ?$ $48^2 = ?$ $55^2 = ?$ $75^2 = ?$ $100^2 = ?$

3. $8^3 = ?$ $7^3 = ?$ $9^3 = ?$ $10^3 = ?$ $100^3 = ?$ $110^3 = ?$

4. $(\frac{1}{2})^2 = ?$ $(\frac{2}{3})^2 = ?$ $(\frac{4}{5})^2 = ?$ $(\frac{3}{8})^2 = ?$ $(\frac{5}{6})^2 = ?$ $(\frac{9}{12})^2 = ?$

5. $.1^2 = ?$ $.5^2 = ?$ $.01^2 = ?$ $.15^2 = ?$ $.001^2 = ?$ $.99^2 = ?$

Involution is the operation of finding the product of equal factors, or of raising a number to a power.

A **root** of a number is one of the *equal* factors of the number. If it is one of **two** equal factors of a number, it is the **square root** of the number.

$6^2 = 36$. 36 is the square of 6, or the second power of 6. 6 is one of the two *equal* factors of 36, and is therefore the square root of 36.

$5^3 = 125$. 125 is the cube of 5, or the third power of 5. 5 is one of the three *equal* factors of 125, and is therefore the cube root of 125.

In the expression $5^3 = 125$, 125 is the power, 5 is the root of the power, and 3, the exponent, shows to what power 5 is to be raised.

Evolution is the operation of finding the equal factors, or the roots of a power.

A root is indicated by the **radical sign** $\sqrt{}$, placed over the number whose root is to be found. $\sqrt{25}$ indicates that the square root of 25 is to be found. The figure 2 is not written with the sign; but for cube root, fourth root, etc., a figure called the **index** is placed before the sign to tell what root is to be found. $\sqrt[3]{125}$ indicates the cube root of 125. $\sqrt[4]{81}$ indicates the fourth root of 81.

Study Recitation

1. Square 25. $25 = 20 + 5$, hence it may be squared in two ways:

$$\begin{array}{rrl}
25 = & 20+5 & \\
25 = & 20+5 & \\
\hline
125 = & 20\times5+5^2 & \text{(Product of } 20+5 \text{ by } 5) \\
500 = & 20^2+20\times5 & \text{(Product of } 20+5 \text{ by } 20) \\
\hline
625 = & 20^2+2(20\times5)+5^2 & \text{(Product of } 20+5 \text{ by } 20+5)
\end{array}$$

The square of any integer composed of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.

Written

Square the following:

2. 15	**4.** 35	**6.** 16	**8.** 27	**10.** 150
3. 25	**5.** 40	**7.** 19	**9.** 84	**11.** 250

The **squares** of the smallest and the largest integers composed of one, two, and three figures are as follows:

$1^2=1$	$10^2=100$	$100^2=10000$
$9^2=81$	$99^2=9801$	$999^2=998001$

12. Separate each of these squares into periods of two figures each, beginning at the right. Thus, 98′01, 1′00′00, 99′80′01.

13. How does the *number of periods* in each square compare with the *number of figures* in the corresponding roots?

The number of figures in the square root of a perfect square is the same as the number of periods of two figures each into which the number can be separated, beginning at units.

The left-hand period may contain only one figure.

ROOTS BY FACTORING

Study Recitation

Since $5 \times 5 = 25$, $\sqrt{25} = \sqrt{5 \times 5} = 5$.

Since $5 \times 5 \times 5 = 125$, $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$.

1. Find, by factoring, the square root of 900.

Solution. The prime factors of 900 are $3 \times 3 \times 2 \times 2 \times 5 \times 5$. Arranging these in two equal groups, you have $(2 \times 3 \times 5) \times (2 \times 3 \times 5)$. Hence $\sqrt{900} = 2 \times 3 \times 5$, or 30.

2. Find, by factoring, the cube root of 216.

Solution. The prime factors of 216 are $2 \times 2 \times 2 \times 3 \times 3 \times 3$. Arranging these in three equal groups, you have $(2 \times 3)(2 \times 3)(2 \times 3)$. Hence $\sqrt[3]{216} = 2 \times 3$, or 6.

Find, by factoring, the square root of:

	a.	*b.*	*c.*	*d.*	*e.*
3.	49	81	144	729	2500
4.	64	121	361	324	3600

Find, by factoring, the cube root of:

5.	27	125	343	729	4096
6.	64	216	512	1000	3375

SQUARE ROOT

Study Recitation

1. Extract the square root of 625.

$$\begin{array}{r|l} & 6'25(25 \\ & 4 \\ \hline 45 & 2\ 25 \\ & 2\ 25 \\ \hline \end{array}$$

Steps:

1. Separate the number (625) into periods of two figures each, beginning with units.

2. Find the greatest square (4) equal to or less than the left-hand period (6), write its square root (2) for the first figure in the root, and subtract the square (4) from the left-hand period (6).

3. To the remainder (2) annex the next period (25) for a dividend.

4. Double the root (2) already found, and write the product (4) at the left of the remainder for a trial divisor.

5. Divide the new dividend (225), exclusive of the right-hand figure (5) by the trial divisor (4); write the quotient (5) for the next figure of the root, and also at the right of the trial divisor, to make the complete divisor (45).

6. Multiply the complete divisor (45) by the last figure (5) of the root, and subtract the product (225) from the dividend.

7. If there are other periods remaining, proceed in the same way.

Note. The seven steps should be *memorized*, omitting the numbers, as they apply to this example only.

2. Extract the square root of 1296.

Follow the directions above, without regarding the numbers.

$$\begin{array}{r|l} & 12'96(36 \\ & 9 \\ \hline & 3 \\ & 3\ 96 \\ 6 & 3\ 9\cancel{6} \\ 66 & 3\ 96 \\ & 3\ 96 \end{array}$$

Step 1. State it. What is the first period at the left?

Step 2. State it. What is the greatest square less than 12?

Step 3. State it. What is the new dividend?

Step 4. State it. What is the trial divisor?

Step 5. State it. What is the new dividend, exclusive of the right-hand figure? 6 is contained in 39 how many times? Where is the quotient placed?

Step 6. State it.

Bringing the work together (as in example 1) to shorten the operation, you have the following:

```
    12′96(36
     9
66 | 3 96
   | 3 96
```

3. Extract the square root of 60,516.

```
    6′05′16(246
    4
 44 | 2 05
    | 1 76
486 |   29 16
    |   29 16
```

In the fifth step, on dividing 20 by the trial divisor 4, you get 5; but when 5 is annexed to the trial divisor and the complete divisor 45 is multiplied by 5, the product is found to be larger than the dividend. This shows that the quotient 5 is too large, and must be decreased by 1, making it 4.

Bring down the next period and proceed as before, doubling the number already found in the root, and dividing 291 by 48, etc. If there were still other periods, the process would be continued in the same way.

4. Extract the square root of 94,249.

```
    9′42′49(307
    9
607 |  4249
    |  4249
```

The trial divisor 6 is not contained in 4. In such cases, place a zero in the root and at the right of the trial divisor, and bring down the next period. Divide 424 by 60; place the quotient 7 in the root and at the right of the trial divisor.

If, after annexing a zero as above and bringing down the next period, the trial divisor is not contained in the dividend, exclusive of the right-hand figure, annex another zero, bring down another period, etc.

When both terms of a fraction whose square root is to be extracted are squares, extract the square root of each. Otherwise, reduce the fraction to a decimal, carrying on the division until the decimal has twice as many places as there are figures desired in the root. Then extract the square root of the decimal.

In extracting the square root of decimals, begin at the decimal point and point off to the right into periods of two places, annexing zeros if necessary.

5. Point off .47856.

.47′85′60 Begin at the decimal point and point off to the right, annexing one zero, so that each period shall have two places; thus, .47′85′60.

In pointing off mixed decimals, point from the decimal point to the left in the integral part and to the right in the decimal part, annexing a zero if necessary to make the last period in the decimal two places.

A **perfect square** is the product of two equal factors.

When the number whose root is to be extracted is not a perfect square, the root can be found approximately, but not exactly, by annexing periods of zeros after the decimal point, carrying the work as far as desired. It is usually sufficient to carry the work out two decimal places.

6. Extract the square root of 295.

7. Extract the square root of 6.

```
      2′95.′00′00(17.17+
      1
 27  |195
     |189
  341  |600
       |341
 3427  |25900
       |23989
```

```
      6.′00′00′00(2.449
      4
 44  |200
     |176
  484  |2400
       |1936
 4889  |46400
       |44001
```

WRITTEN

Find the square root of:

8.	2304	**15.**	1156	**22.**	492804	**29.**	39.8161
9.	2025	**16.**	5184	**23.**	256036	**30.**	50.8369
10.	256	**17.**	133225	**24.**	62500	**31.**	66.2596
11.	4225	**18.**	104976	**25.**	490000	**32.**	* 345
12.	7225	**19.**	522729	**26.**	45.1584	**33.**	* 5
13.	4096	**20.**	98596	**27.**	3003.04	**34.**	$\frac{16}{25}$
14.	729	**21.**	390625	**28.**	27.5625	**35.**	$\frac{9}{16}$.

NOTE. For further explanation of the process of square root, see Supplement, p. 485.

* To 2 decimal places.

PROBLEMS

Written

1. A square piece of land contains 1440 A. How many rods long and wide is it?

2. What is the cost of fencing a square farm of 160 A. at the rate of $4 per rod?

3. At 60¢ a rod, what is the cost of fencing a square farm whose area is 262 A. 105 sq. rd.? How much additional will it cost to divide it into 3 equal parts by building cross fences?

4. A farmer wishes to fence a field in the form of a rectangle twice as long as it is wide, and containing 20 A. How many rods of fence are required?

How many more rods of fence are necessary to divide the field into two square fields? Draw a figure to represent the rectangle divided into two equal squares.

5. Having the area of a rectangle twice as long as it is wide, how can you find the length of the sides?

6. A hallway having an area of 108 sq. ft. is three times as long as it is wide. What is its width and length?

Suggestion. 108 sq. ft. ÷ 3 = area of each of the three squares into which the rectangle may be divided.

7. A square 10-acre field is planted with corn. The hills are four feet apart each way. The outside hills are on the extreme edge of the field on each side. How many hills of corn are there in the field?

8. A strawberry bed has an area of 2592 sq. ft. It is twice as long as it is wide. Find its dimensions in rods and the fractions of a rod.

REVIEW

1. Define power; square; cube; exponent; involution.

2. What is meant by evolution in arithmetic? by square root?

3. What is the difference between a root and a factor of a number?

4. What does the radical sign indicate? What is the index of a root?

5. What is the relation of the number of figures in a root to the number of figures in its square?

6. What is a trial divisor? How is it found? On what principle?

7. Extract the square root of 5329 and explain each step in the operation.

Problems without Numbers

8. How can you find the square of a number consisting of tens and units?

9. How can you find the square root of a number by factoring?

10. How can you find the cube root of a number by factoring?

11. Give the rule for the extraction of square root.

12. How is the square root of a common fraction extracted? the square root of a decimal?

13. When a number is not a perfect square, how may its approximate square root be found?

14. Make a problem demanding for its solution the extraction of a square root. Solve it.

MENSURATION

Mensuration treats of the measurements of lines, angles, surfaces, and solids.

A **plane surface** or a **plane figure** is a surface such that a straight line joining any two parts of it lies wholly in the surface.

An **angle** is the difference in direction of two lines that meet.

RIGHT ANGLE

ACUTE ANGLE

OBTUSE ANGLE

Two straight lines that meet so as to form a square corner make a **right angle.** The lines are said to be **perpendicular** to each other. If one line is drawn perpendicular to another, not at its extremity, two equal angles are formed. Each of these angles is a right angle.

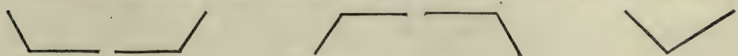
All right angles are equal.

RIGHT ANGLES

An **acute angle** is an angle less than a right angle.

ACUTE ANGLES

An **obtuse angle** is an angle greater than a right angle.



OBTUSE ANGLES

QUADRILATERALS

A **quadrilateral** is a plane figure having four sides.

Parallel lines are lines that are everywhere equally distant from each other.

A **parallelogram** is a quadrilateral whose opposite sides are parallel.

A **rectangle** is a parallelogram whose angles are all right angles.

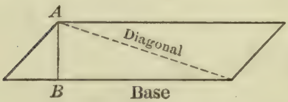
A **square** is a rectangle whose sides are equal.

A **trapezoid** is a quadrilateral having only two sides parallel.

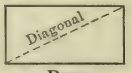
Any line (*AB* below) perpendicular to and connecting the parallel sides of a parallelogram or of a trapezoid is called the **altitude**.

The **diagonal** of a quadrilateral is a straight line joining the vertices of opposite angles.

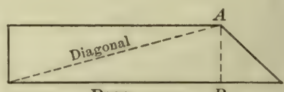
The line on which the parallelogram is supposed to stand is called the **base**. The opposite side may also be regarded as a base.



PARALLELOGRAM

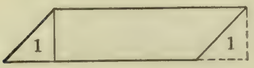


RECTANGLE



TRAPEZOID

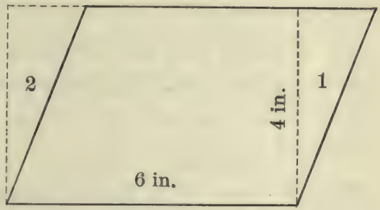
TO THE TEACHER. Teach the definitions by means of blackboard drawings, figures cut from pasteboard, etc. When the definitions have been taught, test the pupil's knowledge thereof by requesting him to draw the figures you designate by name.



Prove by means of cardboard figures similar to the accompanying one the rules for finding the areas of plane figures, and the reasons for the rules.

Study Recitation

Define square, quadrilateral, parallelogram, rectangle. What is the difference between a square and a parallelogram? between a quadrilateral and a parallelogram?



Cut from paper or pasteboard a rectangular parallelogram 6 inches long and 4 inches high. Cut another parallelogram of the same size, similar in shape to the accompanying figure. What is the area of the rectangular parallelogram? Cut off the projecting part marked "1," and put it at the other end of the figure, marking it "2." Compare the area of the parallelogram thus formed with the preceding one. What is its area?

Any parallelogram has the same area as a rectangle having the same base and altitude.

To find the area of a parallelogram, multiply the base by the altitude.

Note. In this and similar rules, by *base* and *altitude*, or other dimensions, is meant the numbers representing them, when expressed in *like units.* In these cases, the product represents the number of surface units in the area, each surface unit being the square of one of the linear units of the base or the altitude.

PROBLEMS

Oral

1. What is the area of a parallelogram 10 in. wide and 15 in. long?

2. How many acres are there in a piece of land 80 rd. long and 60 rd. wide?

3. A field is 80 rd. long and 40 rd. wide. How long a fence will be required to divide the field into two equal squares? How many acres are there in each square?

Written

4. What is the area of a parallelogram having a base of 15 in. and an altitude of $9\frac{1}{2}$ in.?

5. A strip of land in the shape of an oblique parallelogram is 150 rd. long and $60\frac{2}{3}$ rd. wide. How many acres does it contain?

6. What is the difference in area of two strips of land in the shape of parallelograms, one having a 40-rd. base and 165-ft. altitude, and the other a 150-yd. base and 181.5-ft. altitude?

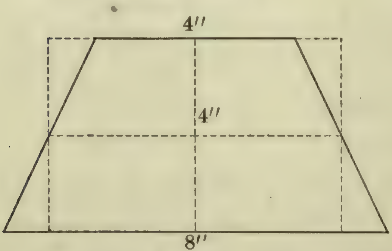
7. The area of a parallelogram is 650 sq. ft., and the base is 40 ft. Find the altitude.

8. The altitude of a parallelogram that has an area of 36 sq. yd. is 8 ft. What is the length of the base?

9. Given the area of a parallelogram and either the base or the altitude, how can you find the other dimension?

Study Recitation

What is a trapezoid?



Construct a trapezoid 4 in. high, with an 8-in. base and a 4-in. top. Halfway between the bases draw a line parallel to them. Cut off the corners of the trapezoid, and place them as shown in the illustration. What kind of figure have you? What are its dimensions? What is its area? Solve the example by the rule for finding the area of a trapezoid, and compare the result with the preceding answer.

To find the area of a trapezoid, multiply one half the sum of the parallel sides by the altitude.

PROBLEMS

Written

1. Find the area of a trapezoid whose sides are 12 in. and 18 in., the perpendicular distance being 10 in.

2. Find the area of a trapezoid, one side of which is 125 in. and the other 96 in., the altitude being 48 in.

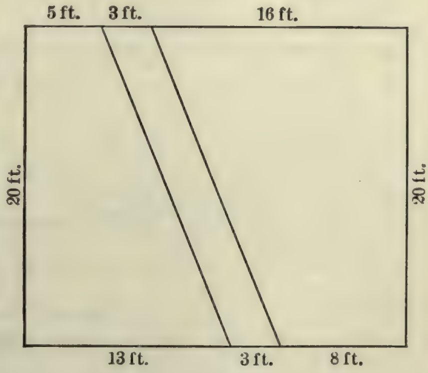
3. What is the area of a plank $16\frac{1}{2}$ in. on one end, $8\frac{3}{4}$ in. on the other, and 10 ft. long?

NOTE. First make a mental estimate, omitting the fractions.

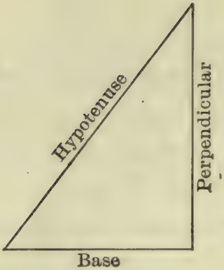
4. A has a field in the form of a trapezoid. The parallel sides are 75 rd. and 95 rd., and the perpendicular distance between them is 66 rd. How many acres are there in the field?

5. A board 18 ft. long is 12 in. wide at one end and 8 in. wide at the other. If the board is $1\frac{1}{2}$ in. thick, how many board feet does it contain?

6. The accompanying figure represents a garden with a 3 ft. walk running across it as shown. Using the dimensions given, find the area of the part of the garden on each side of the walk. Find the area of the entire plot. Find the area of the walk. See whether the sum of the areas of the walk and the parts of the garden on both sides of the walk equals the area of the entire plot.



TRIANGLES



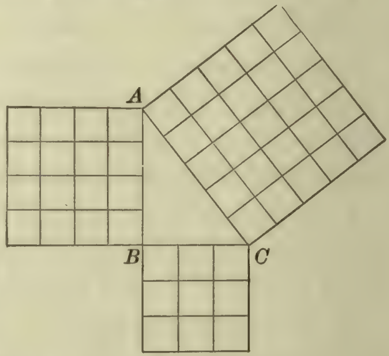
A triangle is a plane figure having three sides.

A triangle having a right angle is called a **right-angled triangle**.

The **hypotenuse** is the side opposite the right angle. The other two sides are the **base** and the **perpendicular**.

Right-angled triangles.

STUDY RECITATION



1. How many small squares are there in the large square on side AB of the triangle? on side BC? on side AC?

2. Compare the number of squares on the hypotenuse with the sum of the squares on the other two sides.

To find one side of a right-angled triangle when the other sides are given, square the two sides. If the hypotenuse is not given, extract the square root of the sum of the two squares to find the hypotenuse. If the hypotenuse is given, extract the square root of the difference of the squares to find the required side.

3. If the given sides are 4 in. and 3 in., find the hypotenuse.

SOLUTION

$$4^2 + 3^2 = 16 + 9 = 25; \ \sqrt{25} = 5. \quad \textit{Ans. } 5 \text{ in.}$$

4. If the hypotenuse is 5 in. and one side is 3 in., find the other side.

SOLUTION

$$5^2 - 3^2 = 25 - 9 = 16; \ \sqrt{16} = 4. \quad \textit{Ans. } 4 \text{ in.}$$

PROBLEMS

Written

1. The two sides of a right-angled triangle are 51 ft. and 68 ft. respectively. What is the length of the hypotenuse?

Solution. $\sqrt{51^2 + 68^2} = \sqrt{7225} = 85$, number of feet.

2. The hypotenuse of a right-angled triangle is 115 in., and the base is 92 in. What is the perpendicular?

3. A ladder 65 ft. long is placed against a house on level ground so that it reaches a window 60 ft. from the ground. How far from the house is the foot of the ladder?

4. One vessel sails due north from a port at the rate of 3 miles an hour. Another, sailing at the same time, goes east at the rate of 4 miles an hour. How far are they apart in 2 days?

5. A road runs on two sides of a square 40-acre field. In going from one corner to the diagonally opposite corner, how much will be saved by going in a straight line rather than by following the road?

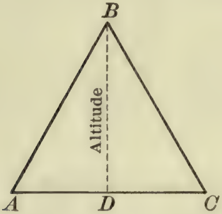
6. A smoke stack has guy wires anchored at points 80 ft. distant on a horizontal line from the base of the chimney. The wires are 120 ft. long. At what height above the base of the chimney are the wires attached to it? Give the answer in the nearest whole number of feet.

7. A triangular sail measures 9 ft. on the base and 12 ft. on the perpendicular. What is the perimeter of the sail?

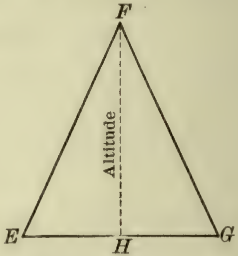
8. If a ladder 65 ft. long leans against a house and the foot of the ladder is 25 ft. from the base of the house, find the height of the top of the ladder from the ground.

Other triangles.

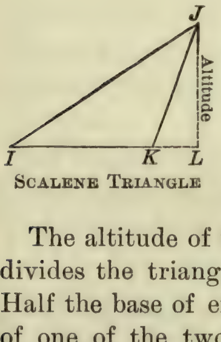
A triangle with three equal sides is an **equilateral triangle.** The angle opposite the base of a triangle is the **vertex**.



Equilateral Triangle



Isosceles Triangle



Scalene Triangle

The **altitude** of a triangle is the length of a perpendicular line from the vertex to the base, or to the base extended.

A triangle having two sides equal is an **isosceles triangle**.

A triangle having no two sides equal is a **scalene triangle.**

The altitude of the equilateral or of the isosceles triangle divides the triangle into two equal right-angled triangles. Half the base of either of the original triangles is the base of one of the two equal triangles formed from it. If the base and another side of an equilateral, or of an isosceles, triangle are known, its altitude and area may be found.

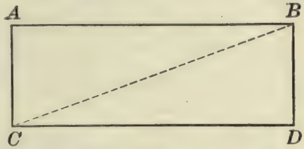
In a scalene triangle, a line may be drawn from one of its angles, regarded as the vertex of the triangle, to the opposite side and perpendicular to that side, dividing the triangle into two right-angled triangles. If the length of this perpendicular and the length of another side of each of the two triangles are known, their areas can be found. The sum of the areas equals the area of the original triangle.

PROBLEMS

Oral

1. What is the difference between a right-angled triangle and an equilateral triangle? between an equilateral triangle and an isosceles triangle? between a scalene triangle and an isosceles triangle?

2. Construct a parallelogram 6 in. long and 4 in. high. What is its area?



3. Cut the parallelogram in two on a line joining two opposite corners. Compare the two triangles produced. What is the area of each? What is the length of each? the height?

4. Draw rectangles of different dimensions.

5. Draw a diagonal in each rectangle and compute the area of the rectangle and of each triangle.

6. What is the area of a triangle with a 10-in. base and 12 in. high? of a triangle with a 6-in. base and 8 in. high?

7. How many acres are there in a triangular field having a base of 320 rd. and an altitude of 60 rd.?

To find the area of a triangle, multiply the base by one half of the altitude.

Written

8. What is the area of a triangle whose base is 75 rd. and altitude 81 rd.?

9. A triangle has a $17\frac{1}{2}$-in. base and a 20-in. altitude. What is its area?

10. What is the area of a scalene triangle with a 30-in. base and a $1\frac{1}{2}$-yd. altitude?

Study Recitation

To find the area of a triangle when its three sides are known, find half the sum of the three sides; subtract each side from this number; find the product of half the sum and the three differences; find the square root of this product.

1. The sides of a triangle are 6, 8, and 10.

Solution

$\frac{1}{2}(6+8+10)=12$; $12-6=6$; $12-8=4$; $12-10=2$.
$12 \times 6 \times 4 \times 2 = 576$; $\sqrt{576}=24$, area of the triangle.

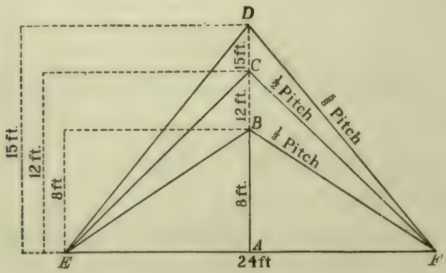
Written

2. Find the area of a triangular piece of land whose sides measure 30 rd., 20 rd., and 40 rd., respectively.

3. The sides of a triangular piece of linoleum measure 9 in., 12 in., and 15 in., respectively. What is its area? This area is what part of a square foot?

Find the area of the triangles, the lengths of whose sides are given below:

	a.	*b.*	*c.*		*a.*	*b.*	*c.*
4.	12 ft.	16 ft.	24 ft.	**6.**	14 yd.	12 yd.	18 yd.
5.	15 rd.	25 rd.	30 rd.	**7.**	7 in.	12 in.	16 in.



Pitch of roofs.

In the figure on p. 418 let the lines EBF, ECF, and EDF represent the front edges of three roofs, respectively, EF, a horizontal line, representing the distance between the upper outside edges of the plates on which the rafters rest.

The line EF represents the width of the building, 24 ft.

The line AB is the perpendicular distance from the peak to the middle of EF.

The roof EBF has a $\frac{1}{3}$ pitch.

The height of the peak from the line connecting the top of the plates is $\frac{1}{3}$ the distance from E to F. $\frac{1}{3}$ of 24 ft. = 8 ft., length of AB.

The roof ECF has a $\frac{1}{2}$ pitch.

The height of the peak from the line EF is $\frac{1}{2}$ EF. $\frac{1}{2}$ of 24 ft. = 12 ft., length of AC.

The roof EDF has a $\frac{5}{8}$ pitch.

The height of the peak from the line EF is $\frac{5}{8}$ EF. $\frac{5}{8}$ of 24 ft. = 15 ft., length of AD.

BAF, CAF, and DAF are right-angled triangles.

PROBLEMS

Written

1. Find the length of the rafters in each roof, from the peak to the outer edge of the plate. If the rafters extend 1 ft. beyond the plate, what is their length in each roof? What is the width of the roof on each side of the ridge?

2. If the building is 36 ft. long, and the ends of the roof project 1 ft. beyond the end wall, what is the length of the roof?

3. Using the dimensions found in the preceding problems, find the area of each roof.

4. How many bunches of shingles, 250 in each bunch, will be required to cover each roof, allowing 1000 to a square?

5. How can you find the pitch of the roof when the height of the peak above the top of the plates, and the width of the house, are known?

6. If the line EF (p. 418) is 30 ft. long and the line AB is 5 ft. long, what is the pitch of the roof?

7. If the line EF (p. 418) is 30 ft. long, and the roof ECF has a $\frac{1}{2}$ pitch, what is the length of the line AC?

8. If the line EF is 30 ft. long and the line AD is $9\frac{3}{8}$ ft. long, what pitch has the roof?

Allowing for the same projection of the roof beyond the plates and at the ends of the building as in problems 1 and 2, and for 1000 shingles to a square, find the number of shingles required to shingle:

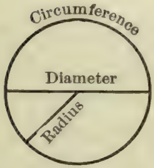
9. The roof indicated in problem 6.

10. The roof indicated in problem 7.

11. The roof indicated in problem 8.

CIRCLES

A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the center.



The **circumference** is the curved line that bounds the circle.

The **diameter** of a circle is a straight line passing through the center, with both ends terminating in the circumference.

The **radius** of a circle is a straight line from the center to the circumference. It is equal to half the diameter.

Define circle; diameter; circumference; radius.

By means of a piece of paper wrapped about a cylinder the diameter of which is known, find the circumference and determine how many times the diameter it is. It will be found to be a trifle more than $3\frac{1}{7}$ times the diameter.

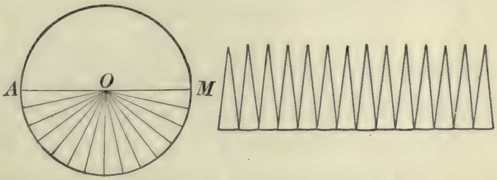
The circumference of a circle is more exactly 3.1416 times the length of its diameter. 3.1416 is often represented by the symbol π, called **pi.**

Study Recitation

To find the circumference of a circle when the diameter is known, multiply the diameter by 3.1416.

1. What is the circumference of a circle having a 12-inch diameter? a 5-inch radius?

2. Construct a circle on pasteboard and cut it out. Divide one half of the circle into smaller sections as shown in the figure.



3. Cut the circle into two equal parts along the line *AM*. From *O* cut along the lines indicated toward the circumference, but do not separate the parts entirely. Spread the resulting figures out and combine them as indicated in the figure at the right.

If the sections cut out are small enough, each may be regarded as a triangle whose base is a straight line, and a small part of the circumference.

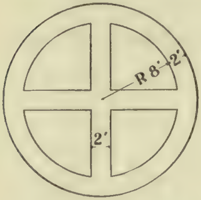
What is the length of the combined bases of the triangles as compared with the circumference of the circle? of the height of the triangles as compared with the radius? How is the area of the figure found?

To find the area of a circle, multiply the circumference by one half of the radius. Or, multiply the square of the radius by 3.1416.

Written

4. What is the area of a circle 10 feet in diameter? What is the area of a circle having a 20-foot radius?

5. Over how many square feet of surface can a horse graze if it is fastened to a post with a rope that allows it to graze 100 feet from the post?



6. A circular garden having a radius of 8′ is divided into quadrants by cross walks 2′ wide. It is also bounded by a circular walk 2′ wide. How many bricks are required to lay the walks when the bricks, $2\frac{1}{4}''$ by 4″ by $8\frac{1}{4}''$, are laid flat? Regard the cross walks as rectangles 16′ long and 2′ wide. (Allow 39 bricks to 1 sq. yd.)

Indicating Radius by R, Diameter by D, Circumference by C, and Area by A, the two rules given on p. 421 may be expressed by formulas 1, 2, and 3 below.

Formulas.

1. $D \times 3.1416 = C.$

2. $C \times \frac{R}{2} = A.$

3. $R^2 \times 3.1416 = A.$

Formula 3 is derived from formula 2 by substituting for C its value as given in formula 1, and substituting $2 \times R$ for its equal D, $2 \times R \times 3.1416 \times \frac{R}{2} = A$. Canceling the factor 2, and multiplying R by R, you get formula 3, $R^2 \times 3.1416 = A$.

Since the square of any number is $\frac{1}{4}$ the square of twice the number, the square of the radius is $\frac{1}{4}$ the square of the diameter, or $R^2 = \frac{D^2}{4}$. Substituting $\frac{D^2}{4}$ for R^2 in formula 3, and canceling the factor 4, you get,

4. $D^2 \times .7854 = A.$

From the foregoing formulas, other formulas may be derived by applying the principle: *If the product of two factors is divided by either factor, the quotient is the other factor.*

	Derived from Formula	
5. $\frac{C}{3.1416} = D,$	1.	Read: Circumference divided by 3.1416, equals the diameter.

	Derived from Formula	
6. $\frac{A}{C}=\frac{R}{2}$,	2.	Read: Area divided by circumference equals one half the radius.
7. $\frac{A}{\frac{R}{2}}=C$,	2.	Read: Area divided by one half the radius equals the circumference.
8. $\frac{A}{3.1416}=R^2$,	3.	Read: Area divided by 3.1416 equals the square of the radius.

Formulas 1 and 2 and 3 should be thoroughly memorized; the remaining formulas may be derived from these when needed.

9. $\frac{A}{.7854}=D^2$.	4.	Read: Area divided by .7854 equals the square of the diameter.

Find the required element in the following exercises and express the formula employed for finding it:

7.	$D = 100$	$C = ?$	**13.**	$A = 50.2656$	$R = ?$
8.	$C = 628.32$	$D = ?$	**14.**	$A = 50.2656$	$C = ?$
9.	$R = 7$	$C = ?$	**15.**	$A = 1$ acre	$D = ?$
10.	$R = 5$	$A = ?$	**16.**	$C = 157.08$	$A = ?$
11.	$D = 120$	$A = ?$	**17.**	$R = 31$ ft. 6 in.	$A = ?$
12.	$A = 218.25$	$D = ?$	**18.**	$D = 10$	$A = ?$

19. The area of a circular garden is 2578.23 sq. rd. Find the diameter and the circumference.

20. The circumference of a circle is 15.708 ft. What is its area? How many circles, each 15 in. in diameter, will equal the area of the above circle?

21. A hot air flue is 24 in. by 36 in. What is the diameter of a circular flue having the same area?

22. What is the difference in area of two flues, one 16 in. square, and the other a circular flue 16 in. in diameter?

23. Draw a figure to represent an 8-in. square and circumscribe a circle about it. What is the difference in the areas of the two figures?

24. The diameter of a water pipe is 3 in. What is the area of the cross section of the pipe?

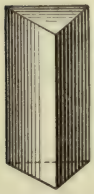
25. A circular silo has an inside diameter of 20 ft. How many square feet are there in the base of the silo?

SOLIDS

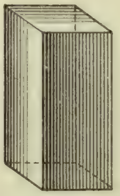
A **solid** or body has length, breadth, and thickness.

A **prism** is a solid whose sides are parallelograms, and whose two ends are equal polygons parallel to each other.

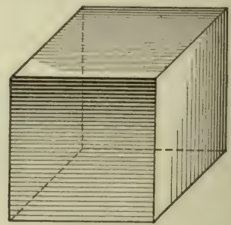
Prisms, according to the form of their bases, are triangular, quadrangular, pentagonal, etc.



TRIANGULAR PRISM



QUADRANGULAR PRISM



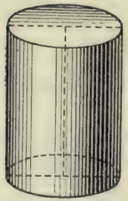
CUBE

A **triangular** prism is one that has triangles for its bases.

A **quadrangular** prism is one that has quadrilaterals for its bases. Quadrangular prisms are also called **parallelopipeds**.

A **cube** is a solid with six equal square faces.

A **cylinder** is a solid with a curved surface and two equal parallel circular bases.



CYLINDER

The **convex surface** of a cylinder is the area of its curved surface.

The **axis** of a cylinder is the line joining the center of the two bases.

The **altitude** of a prism or of a cylinder is the perpendicular distance between its bases.

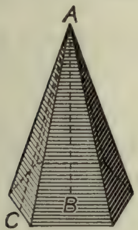
A **pyramid** is a solid whose base is a polygon, and whose sides are triangles meeting at the vertex.

The **altitude** of a pyramid is the distance from the vertex to the center of the base; as AB in the figure below.

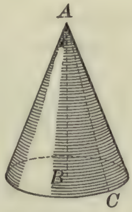
The **slant height** of a pyramid is the distance from the vertex to the middle of the base of any of its triangular sides; as AC in the figure.

A **cone** is a solid whose base is a circle and whose surface tapers uniformly to a point.

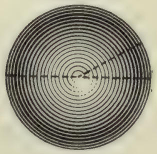
A **sphere** is a solid with a curved surface, every point of which is equally distant from a point within called the **center.**



PYRAMID



CONE



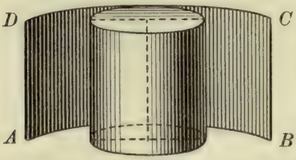
SPHERE

The **diameter** of a sphere is a straight line passing through the center of the sphere and terminating in its surface. One half the diameter is the **radius** of the sphere. The greatest distance around the sphere is its **circumference.**

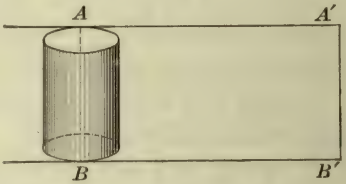
All the surface of a solid except its base or bases is called the **lateral surface.** The entire surface is the sum of the lateral surface and of the bases.

Study Recitation

Note. Derive the rules for finding the area of the surface of different solids in manner similar to the one that follows:



Cover exactly the lateral surface of any cylinder with a sheet of paper of the same height. Unroll the paper. Find its area. What is the circumference of the cylinder? Multiply the circumference of the cylinder by its height. How does the product compare with the area of the paper? State the rule for finding the lateral surface of a cylinder.



The cylinder in the illustration is represented as lying on a plane touching it along the line AB. Conceive it rolled to the right. When A reaches the plane at A', B is exactly at the point B', and the cylinder has made one rotation. The path described by it is a rectangle equal to the entire lateral surface of the cylinder.

The sides of any prism are rectangles. If these rectangles are placed side by side, they form one rectangle whose length is the perimeter of the base and whose width is the altitude of the prism.

The surface of a cylinder may be regarded as made up of an infinite number of rectangles, the sum of whose bases is the circumference of the cylinder. The length of each rectangle is the altitude of the cylinder.

The lateral surface of a pyramid is made up of triangles. The slant height of the pyramid is the altitude of each of the triangles. The perimeter of the base is the sum of the bases of the triangles.

A cone may be regarded as a pyramid with an infinite number of bases, therefore its lateral surface is made up of an infinite number of triangles whose altitude is the slant height of the cone and the sum of whose bases is the circumference of the base of the cone.

In finding the lateral surface of a prism, a cylinder, a pyramid or a cone, you simply find the area of three or more rectangles, or three or more triangles.

Surfaces of solids.

I. *The lateral surface of a prism or a cylinder equals the product of the perimeter or the circumference of the base by the altitude.*

II. *The lateral surface of a pyramid or a cone equals one half the product of the perimeter of the base by the slant height.*

III. *The surface of a sphere equals the product of the diameter by the circumference, or it equals the area of four times the circle having the diameter of the sphere.*

Written

1. What is the lateral surface of a cylinder 8 in. in circumference and 14 in. high? of a cylinder 10 in. in circumference and 10 in. high?

2. What is the lateral surface of a triangular prism whose sides are 3 in., 4 in., and 5 in., and whose height is 20 in.? What is the entire surface of this prism?

3. What is the lateral surface of a square prism whose base is 4 in. and altitude 9 in.? What is the entire surface?

4. What is the lateral surface of a cylinder with an altitude of 16 in. and a diameter of 12 in.? What is the entire surface?

5. What is the lateral surface of a cylinder having an altitude of 20 in. and a radius of 8 in.? the entire surface?

6. What is the lateral surface of a square pyramid with a 12-in. base and 20-in. slant height? the entire surface?

7. What is the lateral surface of a pyramid whose base is an equilateral triangle having 12-in. sides, and whose slant height is 18 in.?

8. What is the lateral surface of a cone 20 in. in diameter with a slant height of 40 in.? the total surface?

9. What is the surface of a 10-in. sphere?

10. What is the surface of a 20-in. sphere; of a 20-in. cube; of a circle 20 in. in diameter?

Volume of solids.

STUDY RECITATION

1. Define cube; parallelopiped; prism; pyramid; cone; sphere.

2. Draw or construct a cube 1 in. on each side. How many cubic inches does it contain?

3. Draw or construct a parallelopiped that has a 1-in. base and is 2 in. high. How many cubic inches does this solid contain? Construct it 3 in. high. How many cubic inches does it contain?

4. Draw or construct a prism or a parallelopiped 3 in. long, 2 in. wide, and 1 in. thick. How many cubic inches does it contain?

I. *The volume of a rectangular solid equals the product of its three dimensions, or it is equal to the product of the number representing the area of the base by the altitude.*

II. *The volume of a cylinder equals the product of the number representing the area of the base by the altitude.*

III. *The volume of a pyramid or a cone equals the product of the number representing the area of the base by one third of the altitude.*

IV. *The volume of a sphere equals the product of the number representing the area of the surface by one third of the radius.*

5. How many cubic inches are there in a solid 10 in. long, 8 in. wide, and 10 in. thick?

6. How many cubic feet of air are there in a room 20 ft. long, 15 ft. wide, and 10 ft. high?

7. How many cords of wood are there in a pile 32 ft. long, 4 ft. wide, and 4 ft. high?

8. How many cubic inches are there in a cylinder 10 in. in diameter and 10 in. high?

9. Construct a square prism of soft wood or of a turnip. Ascertain its weight. By cutting the edges with a knife, transform the prism into a pyramid having the same base and the same height. Ascertain its weight. Compare the weight of the pyramid with that of the prism.

10. A prism of a certain size weighs 10 oz. What will be the weight of the largest pyramid that can be constructed from it? State the rule for finding the volume of a pyramid or of a cone.

11. A square prism is 12 in. wide and 30 in. high. What is its volume?

12. A cone is 10 in. in diameter and 10 in. high. Find its volume.

13. How many $\frac{1}{4}$-in. cubes can be cut from a 1-in. cube?

14. How many 2-in. cubes can be cut from a 4-in. cube?

Written

15. How many cubic inches are there in an 8-in. cube? in a 16-in. cube?

16. How many cubic feet of water are there in a tank 16 ft. long, 4 ft. 6 in. wide, and 18 in. deep?

17. What is the cost of excavating a cellar 60 ft. long, 30 ft. wide, and 6 ft. deep, at 10¢ per cubic yard?

SOLUTION. $\frac{60 \times 30 \times 6}{27} = 400$, number of cubic yards.

$400 \times 10¢ = \$40$. *Ans.*

18. Find the cost of excavating for the basement of a store 150 ft. long, 80 ft. wide, and 9 ft. deep, at $8\frac{1}{3}$ cents per cubic yard.

19. How many cubic yards of soil are removed in digging a sewer on a level street 2 mi. long, the ditch being 2 ft. wide and 6 ft. deep?

20. How many cubic yards of soil are removed in digging a ditch for the foundation of a house 64 ft. $\times$ 30 ft., the foundation being 2 ft. 6 in. thick, and extending 2 ft. 6 in. beneath the surface of the ground?

Illustrate the surface of the ditch by a drawing.

21. How many barrels of water will a cistern 11 ft. long, 9 ft. wide, and 7 ft. deep hold?

SOLUTION. $\dfrac{11 \times 9 \times 7 \times 1728 \times \frac{2}{63}}{231} = 164\frac{4}{7}$, number of barrels.

State the reason for the employment of the different factors in the operation.

22. How many barrels of water are there in a reservoir 44 ft. long, 36 ft. wide, and 14 ft. deep?

23. How many barrels of water are there in a rectangular tank 21 ft. long, 3 ft. 6 in. wide, and 1 ft. 6 in. deep?

24. How many bushels of grain are there in a bin 12 ft. long, 10 ft. wide, and 8 ft. deep?

25. How many cubic inches are there in a cylindrical vessel 8 in. in diameter and 12 in. high?

26. What is the volume of the largest cone that can be constructed from a cylindrical piece of wood 12 in. in diameter and 21 in. high? the volume of the largest sphere?

27. What is the weight of water in a cylindrical bucket 14 in. in diameter and 18 in. high? (A cubic foot of water weighs $62\frac{1}{2}$ lb.)

28. What is the weight of an iron sphere 8 in. in diameter, the iron being $7\frac{1}{5}$ times as heavy as an equal bulk of water?

29. How many cubic inches of water will a cone-shaped funnel 8 in. in diameter and 12 in. high, hold?

30. A circular tank 5 ft. in diameter and $3\frac{1}{2}$ ft. high is filled with water. How many gallons does it contain?

31. What is the capacity in gallons of a tank 6 ft. × 5 ft. × 10 ft.?

32. Find the weight of the water when the tank in Ex. 31 is full.

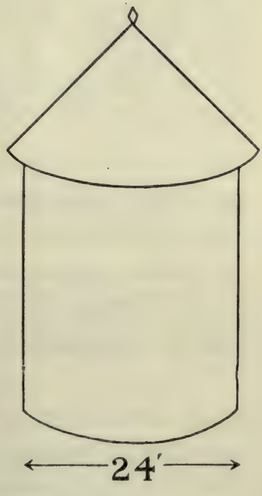
33. A tank to contain 1472.625 gal. is to be 10′ high. What should be its diameter?

34. A man wishes to build a circular cesspool 9′ in diameter and 24′ deep. How many loads of dirt will have to be excavated?

(1 load = 1 cu. yd.)

35. A silo 24′ in diameter is to be constructed so as to contain 250 tons of silage. If silage averages 40 lb. per cubic foot of volume, what must be the depth of the silo?

36. How many bricks laid edgewise would it require to lay the floor of this silo if 2% is allowed for waste in cutting bricks?



37. A silo is built 36′ high and contains 280 tons of silage. What is the diameter of the silo?

REVIEW

PROBLEMS

Oral

1. What is the unit used by contractors in estimating cost of plastering? of papering? of painting? of macadamizing roads? What is the area of the cover of this book?

2. What measure is used in stating the area of a lot? the area of a city block? the area of a farm? by the geographer in stating the area of the country? of the state?

3. What is the area of the floor of a room 12 ft. long and 10 ft. wide? What is the area of a blackboard 16 ft. long and 4 ft. wide? What is the unit of measurement?

4. What is the cost of painting a solid board fence 150 ft. long and 10 ft. high at a cost of 50¢ per square of one hundred square feet? Work in two ways. Which way is the better?

5. A room is 15 ft. long, 12 ft. wide, and 9 ft. high. How many square yards are there in the walls? in the ceiling?

Written

6. What is the cost of painting a floor 18 ft. long and 16 ft. wide at 10¢ per square yard?

7. How many square yards of plastering are there in a hall 90 ft. long, 30 ft. wide, and 20 feet high, no allowance being made for doors, windows, and baseboard?

Solution

90 ft. $\times$ 2 + 30 ft. $\times$ 2 = 240 ft., length of walls.

$\frac{240 \times 20}{9} = 533\frac{1}{3}$, number square yards in walls.

$\frac{90 \times 30}{9} = 300$, number square yards in ceiling.

$533\frac{1}{3} + 300 = 833\frac{1}{3}$, number square yards of plaster in hall.

8. What is the cost of lathing and plastering a store 120 ft. long, 50 ft. wide, and 22 ft. high, at $33\frac{1}{3}$ cents per square yard, an allowance of 10 sq. yd. being made for doors, windows, and baseboard?

9. What is the cost of plastering a room 18 ft. 6 in. long, 16 ft. 3 in. wide, and 10 ft. 6 in. high, at 15¢ per square yard, allowance being made for one half of the space occupied by three doors and two windows, each 6 ft. 9 in. in height and 3 ft. 6 in. in width? (No allowance for baseboard.)

10. A roll of paper is usually 8 yd. long and $\frac{1}{2}$ yd. wide. What will be the cost of paper for the walls of a room 18 ft. 6 in. long, 16 ft. 3 in. wide, and 10 ft. 6 in. high at 20¢ per roll? (See p. 216.)

11. A dining room in a hotel 50 ft. long, 28 ft. wide, and 19 ft. high is being renovated, — the floor oiled and polished at 5¢ per square foot, the ceiling papered with paper at 50¢ a roll including labor, the walls frescoed at $37\frac{1}{2}$¢ per square yard. Write a receipted bill for the contractor.

12. A room is 18 ft. long and 16 ft. wide. What is the cost of carpet for it at 75¢ a yard, the strips to run crosswise and the carpet being a yard wide? the strips to run lengthwise?

NOTE. In laying the carpet lengthwise, 5 strips of the carpeting 1 yd. wide would cover only 15 ft., leaving a strip of the floor 1 ft. wide and 18 ft. long still to be covered. Hence another strip must be bought. Therefore 6 strips 18 ft. long, or 108 ft., or 36 yd., are needed.

13. At $1.25 a yard, what is the cost of Brussels carpet 27 in. wide for a room 16 ft. 6 in. long and 15 ft. wide, strips to run lengthwise, making an allowance of 9 inches per strip after the first strip is laid for matching designs? What is the cost if the strips run crosswise?

14. The floor of a hall 120 ft. long and 50 ft. wide is being covered with 30-in. carpet, at \$1.60 per yard. If 9 in. are allowed on each strip but the first for matching designs, what is the cost of the carpet?

15. What is the value of a piece of land 120 rd. long and 40 rd. wide, at \$55 an acre?

16. A map is drawn to a scale of $\frac{1}{4}$ inch to the mile. How many acres are represented by 1 sq. ft. of the map?

17. A road is ordinarily 4 rd. wide. How many miles of it will make 20 A.?

18. What is the cost of bricks for paving a street $2\frac{1}{2}$ mi. long and 4 rd. wide, at \$15 per thousand, the bricks being $8\frac{1}{4}$ in. long, 4 in. wide, and $2\frac{1}{4}$ in. thick?

NOTE. The bricks are laid on edge. See p. 248, note.

19. Which is cheaper, and how much, — to macadamize a yard 125 ft. long and 80 ft. wide, at 50¢ a square yard, or to pave it with cut stones 10 in. long and 8 in. wide, at \$20 per M?

20. The length of the base of a parallelogram is 60 ft.; its altitude is 39 ft. Find its area.

21. How many acres are there in a farm in the form of a parallelogram 68 rd. long and 45 rd. wide?

22. What is the area of a trapezoid whose parallel sides are 68 rd. and 42 rd., the perpendicular distance between them being $37\frac{1}{2}$ rd.?

23. The area of a trapezoid is 300 sq. in. The perpendicular distance between the two bases is 12 in. If the length of one of the parallel sides is 30 in., what is the length of the other parallel side?

24. What is the area of a triangular piece of land with a base of 62 rd. and a perpendicular distance of 95 rd. from the vertex to the base?

25. What is the cost of a triangular piece of land with a base of 2112 ft. and an altitude of 1650 ft., at $50 per acre?

26. How much cloth is wasted in cutting the largest circle possible from a piece of cloth 16 in. square?

27. What is the difference in area between an 8-in. circle and a 16-in. circle?

28. Find the total atmospheric pressure, at 15 lb. per square inch, upon the top of a circular glass plate 32 in. in diameter.

29. Two circles, 5 in. and 12 in., respectively, in diameter, have the same center. What is the area of the ring between their circumferences?

30. The area of a circle is 1 sq. in. What is its diameter?

31. Find the entire surface of a 6-in. cube; of a 12-in. cube.

32. A crib is 9 ft. 6 in. by 9 ft. 4 in. by 8 ft. 7 in. Find its contents in cubic feet.

33. A wagon box is 10 ft. long, $3\frac{1}{2}$ ft. wide, and 44 in. deep. How many bushels ($1\frac{1}{4}$ cu. ft. each) of wheat will it hold?

34. If the "spaces or middles" between corn rows are $3\frac{1}{2}$ ft. wide, how many rows are there in a field 42 rd. wide? 56 rd. wide?

35. If each shock of corn fodder is 12 hills square ($3\frac{1}{2}$ ft. between hills), how many shocks can be cut from a field 56 rd. wide and 84 rd. long? How many hills are there in the field?

36. A field of oats containing 36.5 A. yields on an average 37.2 bu. per acre. The owner of the land gets .6 and the tenant .4 of the total crop, which is marketed at $.45 per bushel. How much money does each realize from the crop?

37. How many cubic inches of water can be poured into a cone 10 in. in diameter and 15 in. high?

38. A heap of wheat in the corner of a bin is 5 ft. high, and extends 5 ft. each side from the corner. Find the number of bushels in the heap.

39. A 12-inch sphere is reduced to 10 in. How many cubic inches are there in the shell removed?

40. How many square inches of cloth will cover the top and sides of a cabinet 14 in. long, 8 in. wide, and 7 in. high?

41. How many square yards of cement plastering are required for the sides and the bottom of a cistern 10 ft. 3 in. long, 7 ft. 6 in. wide, and 6 ft. deep?

42. What is the lateral surface of a prism 16 in. high, having a triangular base with sides of 4 in., 5 in., and 6 in.?

43. What is the lateral surface of a cylinder 10 in. in diameter and 15 in. high? its entire surface?

44. How many square yards of cement plastering are required for a cylindrical cistern 8 ft. in diameter and 8 ft. deep? (The wall and the bottom of the cistern must be considered.)

45. What is the lateral surface of a triangular prism having a height of 12 in., and having for its base an equilateral triangle the entire length of whose sides is 36 in.?

46. How many square inches of metal are required for tinning the entire surface of a cone having a base 12 in. in diameter and a slant height of 21 in.?

47. What is the area of the surface of a 1-in. sphere? of a 10-in. sphere?

48. At $1.50 per square yard, what is the cost of gilding a 3-ft. globe?

Problems without Numbers

In answering the following questions, make and solve an original problem to show the correctness of your answer. When possible, draw a figure to illustrate your problem.

1. Having given the side of a square, how can you find its area?

2. Having given the area of a square, how can you find its side?

3. Having given the dimensions of a rectangle, how can you find its area?

4. Having given the area of a rectangle and one side, how can you find the other side?

5. Having given the area of a parallelogram and its altitude, how can you find its length?

6. Having given the base and the altitude of a triangle, how can you find its area?

7. Having given the area of a triangle and its base, how can you find its altitude?

8. Having given the hypotenuse and one of the other sides of a right-angled triangle, how can you find the third side?

9. Having given the base and the perpendicular of a right-angled triangle, how can you find the hypotenuse?

10. Having given the sides of an equilateral triangle, how can you find its area?

11. Having given the base and the altitude of an isosceles triangle, how can you find its area?

12. Having given the three sides of any triangle, how can you find its area?

13. Having given the diameter of a circle, how can you find its circumference?

14. Having given the radius of a circle, how can you find its circumference?

15. Having given the circumference of a circle, how can you find its diameter? its radius?

16. Having given the radius of a circle, how can you find its area? Give formula.

17. Having given the area of a circle, how can you find its diameter?

18. Having given the dimensions of a prism, how can you find its lateral surface? its entire surface?

19. Having given the diameter and the altitude of a cylinder, how can you find its lateral surface? its entire surface?

20. What must be known to find the lateral surface of a pyramid or a cone? How can you find it?

21. Having given the diameter of a sphere, how can you find its surface?

22. How can you find the volume of a prism? of a cylinder? of a cone? of a pyramid? of a sphere?

GENERAL REVIEW

Oral

15 14 12 21 33 45 54 76 67 98 89 90 86 75

1. Rapidly add 5 to each of the above numbers; then 6; 4; 2; 3; 8; 7; 9; 10; 12; 11.

2. Rapidly subtract 3 from each of the above numbers; then 2; then 6; 7; 4; 5; 8; 9; 12; 10; 11.

3. Rapidly multiply each of the above numbers by 3; by 4; by 5; by 7; by 6; by 8; by 9; by 10; by 11; by 12.

4. Rapidly divide each of the above numbers by 2; by 4; by 5; by 3; by 6; by 8; by 7; by 9; by 10; by 11; by 12. Give quotients and remainders.

Rapidly make change from $1 for the following amounts:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
5.	56 ¢	75 ¢	19 ¢	38 ¢	97 ¢	40 ¢
6.	62 ¢	83 ¢	24 ¢	26 ¢	29 ¢	70 ¢

Reduce to the lower denomination mentioned:

	a.	*b.*	*c.*	*d.*
7.	2 qt. 1 pt.	1 lb. 2 oz.	2 min. 15 sec.	1 yd. 2 ft.
8.	2 gal. 1 qt.	2 T. 1 lb.	2 hr. 5 min.	1 bu. 1 qt.
9.	3 pk. 1 qt.	3 ft. 2 in.	2 wk. 3 da.	3 lb. 5 oz.
10.	2 bu. 1 pk.	2 yd. 1 in.	1 sq. yd. 1 sq. ft.	2 yd. 5 in.

Reduce to the next higher denomination :

	a.	b.	c.	d.
11.	16 qt. (liq.)	24 in.	18 sq. ft.	120 min.
12.	16 qt. (dry)	9 ft.	48 oz.	135 da.
13.	4 pt.	640 rd.	120 sec.	148 hr.
14.	16 pk.	72 in.	4000 lb.	200 yr.

15. Change $\frac{1}{2}$ to fourths ; to sixths ; to eighths ; to tenths ; to twelfths ; to sixteenths.

16. Change $\frac{1}{3}$ to sixths ; to ninths ; to fifteenths ; to eighteenths.

17. Change $\frac{3}{4}$ to eighths ; to twelfths ; to sixteenths.

18. Change $\frac{4}{5}$ to tenths ; to fifteenths ; to twentieths ; to twenty-fifths.

19. Change $\frac{1}{8}$ to sixteenths ; $\frac{5}{8}$ to sixteenths ; $\frac{7}{8}$ to sixteenths.

20. Change to twelfths : $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{2}{3}$; $\frac{4}{4}$; $\frac{5}{6}$.

21. Change to sixteenths : $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{3}{4}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$.

22. Change $\frac{4}{8}$ to halves ; $\frac{6}{9}$ to thirds ; $\frac{8}{12}$ to thirds ; $\frac{8}{12}$ to sixths.

23. Change to lower terms : $\frac{2}{4}$; $\frac{4}{6}$; $\frac{6}{8}$; $\frac{6}{10}$; $\frac{8}{12}$; $\frac{10}{16}$.

Change to lowest terms :

	a.	b.	c.	d.	e.	f.	g.	h.
24.	$\frac{2}{4}$	$\frac{3}{9}$	$\frac{6}{8}$	$\frac{10}{15}$	$\frac{15}{20}$	$\frac{12}{15}$	$\frac{10}{16}$	$\frac{10}{12}$
25.	$\frac{4}{6}$	$\frac{4}{10}$	$\frac{8}{12}$	$\frac{12}{16}$	$\frac{15}{25}$	$\frac{10}{20}$	$\frac{9}{12}$	$\frac{4}{20}$

Change to improper fractions :

	a.	b.	c.	d.	e.	f.
26.	$3\frac{1}{2}$	$6\frac{1}{4}$	$9\frac{2}{5}$	$6\frac{3}{8}$	$3\frac{3}{10}$	$2\frac{3}{16}$
27.	$4\frac{1}{3}$	$7\frac{3}{4}$	$7\frac{1}{6}$	$4\frac{5}{8}$	$4\frac{5}{12}$	$2\frac{1}{20}$
28.	$5\frac{2}{3}$	$8\frac{1}{5}$	$7\frac{5}{6}$	$5\frac{7}{8}$	$1\frac{5}{16}$	$3\frac{1}{25}$

Change to whole or mixed numbers:

	a.	b.	c.	d.	e.	f.	g.	h.
29.	$\frac{4}{2}$	$\frac{7}{3}$	$\frac{10}{5}$	$\frac{15}{8}$	$\frac{24}{8}$	$\frac{22}{11}$	$\frac{28}{14}$	$\frac{64}{16}$
30.	$\frac{5}{2}$	$\frac{12}{4}$	$\frac{14}{5}$	$\frac{18}{9}$	$\frac{35}{8}$	$\frac{36}{12}$	$\frac{30}{16}$	$\frac{50}{25}$
31.	$\frac{6}{3}$	$\frac{15}{4}$	$\frac{12}{6}$	$\frac{25}{10}$	$\frac{40}{10}$	$\frac{39}{12}$	$\frac{49}{16}$	$\frac{40}{20}$

Reduce to fractions having the l. c. d., and add:

	a.	b.	c.	d.
32.	$\frac{1}{2}, \frac{1}{4}$	$\frac{1}{2}, \frac{3}{4}$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$
33.	$\frac{1}{2}, \frac{1}{3}$	$\frac{1}{4}, \frac{3}{8}$	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$	$\frac{1}{4}, \frac{1}{5}, \frac{1}{10}$
34.	$\frac{1}{3}, \frac{1}{5}$	$\frac{1}{8}, \frac{3}{16}$	$\frac{1}{2}, \frac{1}{8}, \frac{1}{16}$	$\frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

Subtract:

	a.	b.	c.	d.	e.
35.	$\frac{1}{2}-\frac{1}{4}$	$\frac{3}{4}-\frac{3}{8}$	$\frac{1}{2}-\frac{5}{12}$	$\frac{3}{5}-\frac{3}{10}$	$\frac{3}{4}-\frac{2}{5}$
36.	$\frac{1}{2}-\frac{1}{8}$	$\frac{5}{6}-\frac{1}{2}$	$\frac{5}{8}-\frac{3}{16}$	$\frac{3}{8}-\frac{5}{16}$	$\frac{5}{8}-\frac{1}{3}$
37.	$\frac{1}{2}-\frac{1}{16}$	$\frac{7}{8}-\frac{3}{4}$	$\frac{7}{12}-\frac{1}{6}$	$\frac{2}{3}-\frac{1}{6}$	$\frac{2}{3}-\frac{5}{12}$

Find products:

	a.	b.	c.	d.	e.
38.	$\frac{2}{3}$ of 36	$4 \times \frac{3}{8}$	$\frac{2}{5}$ of $\frac{5}{8}$	$\frac{5}{8}$ of $\frac{3}{10}$	$\frac{5}{12}$ of $\frac{3}{10}$
39.	$\frac{3}{8}$ of 48	$5 \times \frac{7}{15}$	$\frac{3}{4}$ of $\frac{4}{5}$	$\frac{2}{3}$ of $\frac{3}{7}$	$\frac{3}{16}$ of $\frac{8}{9}$
40.	$\frac{5}{16}$ of 64	$4 \times 2\frac{1}{4}$	$\frac{5}{16}$ of $\frac{4}{5}$	$\frac{2}{3}$ of $\frac{3}{16}$	$\frac{3}{10}$ of $\frac{5}{6}$

Find quotients:

	a.	b.	c.	d.	e.
41.	$\frac{3}{4} \div 3$	$3 \div \frac{3}{4}$	$\frac{3}{4} \div \frac{3}{8}$	$\frac{3}{8} \div \frac{2}{3}$	$\frac{5}{6} \div \frac{5}{12}$
42.	$\frac{4}{5} \div 4$	$4 \div \frac{4}{5}$	$\frac{3}{8} \div \frac{3}{4}$	$\frac{3}{10} \div \frac{3}{5}$	$\frac{5}{12} \div \frac{5}{6}$
43.	$\frac{4}{5} \div 5$	$5 \div \frac{4}{5}$	$\frac{2}{3} \div \frac{3}{8}$	$\frac{3}{5} \div \frac{3}{10}$	$\frac{5}{16} \div \frac{5}{8}$

Find what part the first number is of the second:

	a.	b.	c.	d.	e.
44.	3, 4	10, 6	8, 6	9, 6	$\frac{1}{4}$, $\frac{1}{2}$
45.	4, 5	9, 36	6, 8	18, 4	$\frac{3}{8}$, $\frac{3}{4}$
46.	6, 10	36, 9	16, 3	32, 24	$\frac{3}{16}$, $\frac{3}{8}$

Find the ratio of:

	a.	b.	c.	d.	e.	f.
47.	3 to 4	4 to 20	6 to 8	10 to 15	$\frac{1}{4}$ to $\frac{1}{8}$	$\frac{3}{4}$ to $\frac{3}{16}$
48.	4 to 3	20 to 4	8 to 6	15 to 10	$\frac{1}{8}$ to $\frac{1}{4}$	$\frac{3}{16}$ to $\frac{3}{4}$

Find the number of which:

	a.	b.	c.	d.	e.
49.	4 is $\frac{1}{2}$	8 is $\frac{4}{5}$	12 is $\frac{4}{5}$	20 is $\frac{5}{6}$	18 is $\frac{3}{8}$
50.	6 is $\frac{1}{3}$	9 is $\frac{3}{5}$	14 is $\frac{7}{8}$	25 is $\frac{5}{8}$	16 is $\frac{4}{5}$
51.	7 is $\frac{1}{5}$	10 is $\frac{5}{8}$	16 is $\frac{2}{3}$	30 is $\frac{3}{10}$	24 is $\frac{3}{8}$

Reduce to decimals and to per cents:

	a.	b.	c.	d.	e.	f.	g.	h.	i.	j.
52.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{16}$	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{12}$	$\frac{1}{25}$
53.	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{12}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{1}{20}$

Reduce to common fractions in their lowest terms:

	a.	b.	c.	d.	e.	f.	g.
54.	.50	.40	$.12\frac{1}{2}$	$.62\frac{1}{2}$	$.16\frac{2}{3}$	$.66\frac{2}{3}$	$.06\frac{1}{4}$
55.	.25	.75	$.37\frac{1}{2}$	$.87\frac{1}{2}$	$.33\frac{1}{3}$	$.83\frac{1}{3}$	.04

Find the cost of 48 articles at the following prices:

	a.	b.	c.	d.	e.
56.	10¢	50¢	$12\frac{1}{2}$¢	$33\frac{1}{3}$¢	\$1.25
57.	20¢	60¢	$37\frac{1}{2}$¢	$66\frac{2}{3}$¢	\$1.75
58.	25¢	75¢	$62\frac{1}{2}$¢	$16\frac{2}{3}$¢	\$1.37$\frac{1}{2}$
59.	50¢	80¢	$87\frac{1}{2}$¢	$83\frac{1}{3}$¢	\$1.33$\frac{1}{3}$

How many articles can be purchased for \$10 at each of the following prices:

	a.	b.	c.	d.	e.
60.	10¢	25¢	$16\frac{2}{3}$¢	$62\frac{1}{2}$¢	$87\frac{1}{2}$¢
61.	20¢	$12\frac{1}{2}$¢	$33\frac{1}{3}$¢	$83\frac{1}{3}$¢	$6\frac{1}{4}$¢

Find:

	10% of:	50% of:	75% of:	$12\frac{1}{2}$% of:	$33\frac{1}{3}$% of:
62.	10	12	4	8	10
63.	20	16	8	16	20
64.	50	18	12	24	30

Find what per cent the first number is of the second:

	a.	b.	c.	d.	e.
65.	5 of 10	3 of 9	4 of 16	5 of 8	2 of 3
66.	2 of 20	4 of 12	3 of 4	8 of 5	3 of 2

Of what number is

	a.	b.	c.	d.
67.	2, 50%	6, 25%	8, 10%	15, $37\frac{1}{2}$%
68.	3, $33\frac{1}{3}$%	7, 20%	9, $12\frac{1}{2}$%	16, 25%

Find the profit or loss:

	a.		b.		c.	
	Cost	Rate of gain	Cost	Rate of Loss	Cost	Rate of Gain
69.	\$10	10%	\$4	25%	\$ 9	$33\frac{1}{3}$%
70.	5	20%	8	$12\frac{1}{2}$%	10	20%

Find the per cent of profit or loss:

	a.		b.		c.	
	Cost	Selling Price	Cost	Selling Price	Cost	Selling Price
71.	\$.10	\$.15	\$10	\$18	\$10	\$8
72.	\$.10	\$.12	\$ 9	\$12	\$10	\$9

Find the discount and the net proceeds :

	a. List Price	Rate of Disc.	*b.* List Price	Rate of Disc.	*c.* List Price	Rates of Disc.
73.	\$10	10 %	\$36	$33\frac{1}{3}$ %	\$10	20 %, 10 %
74.	\$40	25 %	\$20	20 %	\$40	25 %, 20 %

Find the commission :

	Sales	Rate of Com.	Sales	Rate of Com.	Sales	Rate of Com.
75.	\$24	25 %	\$18	$33\frac{1}{3}$ %	\$10	10 %
76.	\$40	20 %	\$16	$12\frac{1}{2}$ %	\$20	25 %

Find the premium on the following insurance policies :

	Face of Policy	Rate	Face of Policy	Rate	Face of Policy	Rate
77.	\$1000	25 ¢ per 100	\$1000	$\frac{1}{5}$ %	\$8000	$\frac{1}{8}$ %
78.	\$1500	20 ¢ per 100	\$2000	$\frac{3}{5}$ %	\$4000	$\frac{3}{8}$ %

Find the taxes on the following pieces of property :

	Assessment	Tax Rate	Assessment	Tax Rate	Assessment	Tax Rate
79.	\$1000	.02	\$1600	.05	\$3000	$.01\frac{1}{2}$
80.	\$1200	.04	\$2000	$.01\frac{1}{4}$	\$3200	$.02\frac{1}{4}$

Find the duty:

	Value	Ad Val. Duty	Value	Ad Val. Duty	Quantity	Specific Duty
81.	\$100	10 %	\$3000	$33\frac{1}{3}$ %	200 lb.	\$1 a lb.
82.	\$200	20 %	\$4000	25 %	100 bu.	5 ¢ a bu.

Find the interest and the amount of :

83. \$100, 5 yr., 5 %

84. \$200, 6 yr., 3 %

85. \$1000, 2 yr., $2\frac{1}{2}$ %

86. \$300, $2\frac{1}{2}$ yr., 4 %

87. \$600, $3\frac{1}{3}$ yr., 6 %

88. \$1000, 4 yr., $3\frac{1}{2}$ %

Written

Time yourself with these exercises. Then work some of them again, trying to beat your record.

Write from dictation, add, and test results:

	a.	b.	c.	d.
89.	653127	805672	$9324.73	$7894.67
	894238	790745	6849.89	5639.86
	757899	983286	7787.75	8427.95
	562328	648354	3538.79	6458.43
	429899	573729	4305.84	7894.09
	701572	784648	5664.68	8345.05
	834999	896349	8976.88	7263.97
90.	.0356	18.09	256.0407	785.05
	12.7043	5.0425	14.953	25
	13.02	184.7653	2157.647	185.0003
	15.6351	3157.634	3082.0057	.0007
	17.7401	463.05	9743.0562	7.6534

	a.	b.	c.	d.	e.
91.	$12\frac{3}{4}$	$13\frac{3}{5}$	$125\frac{2}{3}$	$62\frac{1}{2}$	$75\frac{3}{8}$
	$156\frac{7}{8}$	$14\frac{3}{10}$	$356\frac{5}{9}$	$37\frac{3}{4}$	$87\frac{3}{10}$
	$54\frac{1}{2}$	$17\frac{2}{15}$	$714\frac{5}{18}$	$16\frac{2}{3}$	$15\frac{3}{16}$

	a.	b.	c.
92.	13 ft. 4 in.	14 pk. 4 qt.	16 lb. 5 oz.
	15 ft. 10 in.	15 pk. 5 qt.	23 lb. 14 oz.
93.	4 gal. 3 qt.	3 yd. 2 ft.	10 mi. 210 rd.
	3 gal. 2 qt.	4 yd. 1 ft.	50 mi. 240 rd.

Subtract and test:

	a.	b.	c.	d.
94.	563412	721000	\$8205.72	\$4000.00
	486789	356879	4056.98	3678.29
95.	213.05	356.0057	562.	1825.095
	12.9746	64.92	.0576	746.9
96.	$112\frac{3}{4}$	$113\frac{1}{6}$	$169\frac{5}{16}$	$562\frac{5}{12}$
	$15\frac{7}{8}$	$17\frac{2}{3}$	$143\frac{3}{8}$	$314\frac{5}{16}$

	a.	b.	c.
97.	15 ft. 8 in.	25 lb. 8 oz.	25 wk. 4 da.
	13 ft. 10 in.	16 lb. 10 oz.	16 wk. 6 da.

Find the difference in time between:

98. Aug. 15, 1913, and Jan 4, 1915. April 20, 1913, and Feb. 25, 1915.

99. Sept. 22, 1912, and Feb. 16, 1914. Sept. 29, 1912, and Mar. 12, 1914.

Make change from \$1 and \$2 for each of the purchases in column *a*; from \$5 and \$10 for each of the purchases in column *b*; from \$15 and \$20 for each of the purchases in columns *c*, *d*, and *e*.

	a.	b.	c.	d.	e.
100.	\$.78	\$1.54	\$12.36	\$14.98	\$10.75
101.	\$.65	\$2.75	\$10.16	\$13.52	\$11.98
102.	\$.39	\$3.65	\$10.54	\$12.39	\$12.16
103.	\$.97	\$4.94	\$11.25	\$14.83	\$14.45

Find the products, canceling when possible. Test answers:

	a.	*b.*	*c.*
104.	3214×3256	$\frac{5}{6} \times \frac{3}{4}$	6.75×1.02375
105.	1056×2097	$\frac{7}{8} \times \frac{4}{5}$	$.072 \times .072$
106.	9457×9457	$13\frac{3}{16} \times 14\frac{3}{8}$	1.052×6.04
107.	$2360 \times \$28.97$	$15\frac{3}{5} \times 16\frac{5}{8}$	70.3×60.005
108.	$2745 \times \$102.965$	$205\frac{5}{16} \times 13\frac{3}{4}$	$.054 \times \$17.65$
109.	6234×1.002	$30\frac{2}{15} \times 14\frac{3}{10}$	$.065 \times \$118.54$
110.	75.15×91.005	$54\frac{3}{20} \times 12\frac{3}{10}$	$.037\frac{1}{2} \times 200$
111.	813.237×16.095	$15\frac{5}{16} \times 19\frac{3}{8}$	$.062\frac{1}{2} \times 800$

Find the quotients and remainders and test the answers:

112.	$59492 \div 573$	$\frac{5}{6} \div \frac{2}{3}$	3 ft. 6 in. $\div$ 6
113.	$120650 \div 457$	$\frac{7}{8} \div \frac{3}{4}$	4 gal. 2 qt. $\div$ 9
114.	$\$5942.97 \div 621$	$15\frac{3}{4} \div 5\frac{1}{4}$	5 lb. 1 oz. $\div$ 9
115.	$78105 \div 635$	$5\frac{3}{10} \div 4\frac{3}{5}$	7 wk. 3 da. $\div$ 12
116.	$5252.88 \div 1.527$	$60\frac{1}{8} \div 15\frac{3}{4}$	5 doz. 4 $\div$ 8
117.	$37366.692 \div 12.046$	$12\frac{4}{5} \div 1\frac{3}{8}$	5 yr. 4 mo. $\div$ 16
118.	$5372506 \div 15.007$	$10\frac{1}{8} \div 2\frac{1}{4}$	4 pk. 4 qt. $\div$ 12
119.	$318.0848 \div 1.5004$	$7\frac{1}{5} \div 1\frac{1}{6}$	4 yd. 3 ft. $\div$ 5

Find the ratio of:

120.	15 to 60	$\frac{3}{4}$ to $\frac{4}{5}$	2 in. to 3 ft.
121.	60 to 15	$\frac{4}{5}$ to $\frac{3}{4}$	2 gal. to 3 qt.
122.	$12\frac{1}{2}$ to $37\frac{1}{2}$	$\frac{3}{4}$ to $\frac{7}{8}$	2 oz. to 1 lb.
123.	$37\frac{1}{2}$ to $12\frac{1}{2}$	$\frac{7}{8}$ to $\frac{3}{4}$	1 lb. to 2 oz.

Reduce to improper fractions:

	a.	b.	c.	d.
124.	$3\frac{1}{2}$	$14\frac{3}{4}$	$15\frac{3}{8}$	$83\frac{1}{3}$
125.	$4\frac{2}{3}$	$15\frac{5}{8}$	$33\frac{1}{3}$	$20\frac{5}{12}$
126.	$5\frac{3}{4}$	$16\frac{2}{3}$	$62\frac{1}{2}$	$19\frac{3}{5}$
127.	$7\frac{3}{8}$	$25\frac{5}{12}$	$87\frac{1}{2}$	$43\frac{4}{5}$

Reduce to whole or to mixed numbers:

	a.	b.	c.	d.	e.
128.	$\frac{50}{3}$	$\frac{49}{4}$	$\frac{56}{2}$	$\frac{43}{5}$	$\frac{31}{6}$
129.	$\frac{49}{7}$	$\frac{23}{8}$	$\frac{81}{9}$	$\frac{45}{10}$	$\frac{65}{12}$
130.	$\frac{73}{15}$	$\frac{64}{16}$	$\frac{100}{20}$	$\frac{75}{25}$	$\frac{35}{11}$

Reduce to lowest terms:

	a.	b.	c.	d.	e.	f.
131.	$\frac{14}{16}$	$\frac{10}{15}$	$\frac{16}{24}$	$\frac{18}{36}$	$\frac{16}{20}$	$\frac{14}{35}$
132.	$\frac{36}{60}$	$\frac{21}{49}$	$\frac{24}{64}$	$\frac{25}{100}$	$\frac{15}{75}$	$\frac{16}{40}$
133.	$\frac{18}{81}$	$\frac{16}{56}$	$\frac{27}{63}$	$\frac{36}{144}$	$\frac{27}{72}$	$\frac{20}{48}$

Reduce to fractions having the l. c. d.:

	a.	b.	c.
134.	$\frac{1}{2}, \frac{2}{3}, \frac{1}{4}$	$\frac{2}{3}, \frac{3}{5}, \frac{3}{10}$	$\frac{1}{6}, \frac{2}{5}, \frac{3}{10}$
135.	$\frac{3}{4}, \frac{1}{6}, \frac{2}{5}$	$\frac{3}{8}, \frac{5}{12}, \frac{3}{16}$	$\frac{3}{4}, \frac{5}{12}, \frac{5}{6}$
136.	$\frac{1}{2}, \frac{3}{4}, \frac{3}{8}$	$\frac{1}{2}, \frac{3}{8}, \frac{3}{16}$	$\frac{3}{5}, \frac{3}{4}, \frac{3}{10}$

Reduce to decimals and to per cents:

	a.	b.	c.	d.	e.	f.	g.
137.	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{7}{12}$	$\frac{3}{25}$
138.	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{3}{5}$	$\frac{1}{7}$	$\frac{7}{8}$	$\frac{3}{16}$	$\frac{7}{50}$
139.	$\frac{3}{4}$	$\frac{1}{16}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{5}{12}$	$\frac{11}{12}$	$\frac{17}{20}$
140.	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{7}{16}$	$\frac{11}{25}$

Reduce to per cents and to common fractions in their lowest terms:

	a.	b.	c.	d.	e.
141.	.20	$.12\frac{1}{2}$	$.16\frac{2}{3}$	1.75	$1.12\frac{1}{2}$
142.	.40	$.37\frac{1}{2}$	$.33\frac{1}{3}$	1.25	$1.33\frac{1}{3}$
143.	.50	$.62\frac{1}{2}$	$.66\frac{2}{3}$	1.50	$1.37\frac{1}{2}$
144.	.60	$.87\frac{1}{2}$	$.83\frac{1}{3}$	1.40	$1.87\frac{1}{2}$
145.	.25	$.06\frac{1}{4}$	$.08\frac{1}{3}$	1.60	$1.66\frac{2}{3}$

Reduce the numbers in column (1) to the denominations indicated in column (2):

	a.		b.		c.	
	(1) Number	(2) Denomination	(1) Number	(2) Denomination	(1) Number	(2) Denomination
146.	16 pt.	quarts	5 lb.	ounces	5 sq. yd.	sq. feet
147.	5 qt.	pints	80 oz.	pounds	27 sq. ft.	sq. yards
148.	4 gal.	quarts	4000 lb.	tons	3 A.	sq. rods
149.	24 qt.	gal.	3 T.	pounds	54 cu. ft.	cu. yards
150.	40 qt.	pecks	36 in.	feet	5 cu. yd.	cu. feet
151.	5 pk.	quarts	4 ft.	inches	120 sec.	minutes
152.	100 pk.	bushels	72 in.	yards	5 hr.	minutes
153.	16 bu.	pecks	18 ft.	yards	180 min.	hours

Find the cost of:

	a.	b.	c.
154.	20 lb. @ 25¢	48 yd. @ $83\frac{1}{3}$¢	50 yd. @ \$1.50
155.	26 lb. @ 50¢	48 yd. @ $37\frac{1}{2}$¢	25 lb. @ \$1.25
156.	28 lb. @ 75¢	36 yd. @ $16\frac{2}{3}$¢	28 yd. @ \$1.40
157.	50 lb. @ 20¢	72 yd. @ $87\frac{1}{2}$¢	50 bu. @ \$1.75
158.	60 lb. @ 40¢	40 yd. @ $62\frac{1}{2}$¢	56 gal. @ $\$1.62\frac{1}{2}$
159.	24 lb. @ $12\frac{1}{2}$¢	64 yd. @ $6\frac{1}{4}$¢	64 yd. @ $\$1.87\frac{1}{2}$
160.	33 lb. @ $33\frac{1}{3}$¢	80 yd. @ $18\frac{3}{4}$¢	96 lb. @ $\$1.37\frac{1}{2}$
161.	42 lb. @ $66\frac{2}{3}$¢	84 yd. @ $8\frac{1}{3}$¢	66 lb. @ $\$1.33\frac{1}{3}$

Find in the shortest way the number of yards that can be bought for the given sums at the given prices:

	a. Amount	a. Price per Yard	b. Amount	b. Price per Yard
162.	$10	$.50	$36	$.33$\frac{1}{3}$
163.	$40	$.25	$72	$.66$\frac{2}{3}$
164.	$36	$.75	$85	$.83$\frac{1}{3}$
165.	$88	$.40	$96	$.06$\frac{1}{4}$
166.	$17	$.12$\frac{1}{2}$	$84	$1.20
167.	$15	$.37$\frac{1}{2}$	$1.21	1.37\frac{1}{2}$
168.	$25	$.62$\frac{1}{2}$	$1.30	1.62\frac{1}{2}$
169.	$49	$.87$\frac{1}{2}$	$1.50	1.87\frac{1}{2}$

Find:

	a.	b.	c.	d.	e.
	20 % of:	12$\frac{1}{2}$ % of:	133$\frac{1}{3}$ % of:	87$\frac{1}{2}$ % of:	16$\frac{2}{3}$ % of:
170.	50	88	$660	104	$600
171.	75	160	$720	128	$780
172.	100	248	$540	360	$834
	8$\frac{1}{3}$ % of:	6$\frac{1}{4}$ % of:	18$\frac{3}{4}$ % of:	83$\frac{1}{3}$ % of:	66$\frac{2}{3}$ % of:
173.	120	320	$800	180	$330
174.	624	480	$960	240	$450
175.	840	640	$1440	846	$792

176. Find 6% of each of the numbers in examples 170 to 175; then 5% of them; 4$\frac{1}{2}$%; 3$\frac{1}{2}$%; 2$\frac{1}{2}$%; 7%; 7$\frac{1}{2}$%; 18%; 23%; 45%; 67%; 125%; 115%; 150%.

Find what per cent the first number is of the second:

	a.	b.	c.
177.	15 of 45.	16 of 48.	1 in. of 1 ft.
178.	45 of 15.	48 of 16.	2 oz. of 2 lb.
179.	12$\frac{1}{2}$ of 62$\frac{1}{2}$.	$\frac{4}{5}$ of $\frac{3}{4}$.	4 qt. of 1 pk.
180.	62$\frac{1}{2}$ of 12$\frac{1}{2}$.	$\frac{3}{4}$ of $\frac{4}{5}$.	2 qt. of 2 gal.

Of what number is:

	a.	b.	c.
181.	15, 3 %	\$ 220, $137\frac{1}{2}$ %	\$3500, $62\frac{1}{2}$ %
182.	16, 4 %	\$2500, 40 %	\$7210, $87\frac{1}{2}$ %
183.	72, $12\frac{1}{2}$ %	\$8100, $112\frac{1}{2}$ %	\$8100, 75 %
184.	66, $66\frac{2}{3}$ %	\$7200, 120 %	\$7500, 125 %

Find the profit or loss:

	Cost	Rate of Gain	Cost	Rate of Loss	Cost	Rate of Gain
185.	\$50	20 %	\$75	45 %	\$64.30	10 %
186.	\$33	$33\frac{1}{3}$ %	\$240	24 %	\$56.80	$12\frac{1}{2}$ %
187.	\$44	25 %	\$690	18 %	\$20.50	20 %
188.	\$64.08	$12\frac{1}{2}$ %	\$544	26 %	\$14.40	28 %

Find the per cent of profit or loss:

	Cost	Selling Price	Cost	Selling Price	Cost	Selling Price
189.	\$20	\$24	\$90	\$150	\$25.75	\$20.60
190.	\$24	\$20	\$120	\$90	\$40.20	\$50.25
191.	\$56	\$70	\$150	\$165	\$28.32	\$31.86
192.	\$70	\$56	\$165	\$150	\$36.16	\$27.12

Find the discount and the net price of each of the following:

	List Price	Rate of Discount	List Price	Rates of Discount	List Price	Rates of Discount
193.	\$56	25 %	\$120	$33\frac{1}{3}$ %, 25 %	\$900	$33\frac{1}{3}$ %, 10 %
194.	\$80	$12\frac{1}{2}$ %	\$600	20 %, 10 %	\$1000	20 %, 10 %
195.	\$3.30	$33\frac{1}{3}$ %	\$480	$16\frac{2}{3}$ %, 10 %	\$5000	50 %, 5 %
196.	\$4.50	20 %	\$800	$12\frac{1}{2}$ %, 5 %	\$9600	$37\frac{1}{2}$ %, 10 %

Find the commission:

	a. Sales	a. Rate of Com.	b. Sales	b. Rate of Com.	c. Purchase	c. Rate of Com.
197.	$3600	25%	$1040	18%	$8800	$2\frac{1}{2}$%
198.	$4800	$12\frac{1}{2}$%	$2020	15%	$7500	5%
199.	$5000	20%	$6000	$33\frac{1}{3}$%	$6300	12%
200.	$6000	$16\frac{2}{3}$%	$4040	25%	$7500	18%

Find the premium on each of the following policies:

	a. Face of Policy	a. Rate	b. Face of Policy	b. Rate
201.	$5000	24¢ per $100	$4000	$\frac{1}{4}$%
202.	$6350	64¢ per $100	$16,000	$\frac{3}{4}$%
203.	$10,000	$1.50 per $100	$15,000	$\frac{3}{5}$%
204.	$16,000	$12\frac{1}{2}$¢ per $100	$20,000	$\frac{4}{5}$%

Find the amount of taxes on the following property:

	a. Assessment	a. Tax Rate	b. Assessment	b. Tax Rate	c. Assessment	c. Tax Rate
205.	$15,000	$.024	$10,000	$2\frac{3}{4}$%	$9500	$5\frac{1}{4}$ mills
206.	$18,000	$.0015	$50,600	$5\frac{1}{2}$%	$18,000	$4\frac{1}{2}$ mills
207.	$24,256	$.004	$75,250	$1\frac{1}{4}$%	$24,000	$2\frac{1}{8}$ mills

Find the duty on each of the following imports:

	Value	Ad Valorem Duty	Value	Ad Valorem Duty	Quantity	Specific Duty
208.	$500	30%	$12,000	25%	2000 lb.	$1.85 a pound
209.	$650	35%	$15,040	20%	200 bu.	$ .06 a bushel
210.	$800	60%	$10,500	15%	500 doz.	$1.20 a dozen
211.	$900	45%	$55,600	40%	1600 tons	$ .50 a ton

Find the interest and amount of:

	Principal	Time	Rate
212.	\$2000	3 yr. 2 mo.	6%
213.	\$430.50	9 mo.	6%
214.	\$5650	1 yr. 6 mo.	5%
215.	\$670.70	9 mo. 6 da.	$4\frac{1}{2}$%
216.	\$70.95	6 mo. 12 da.	6%
217.	\$63.42	2 yr. 3 mo. 6 da.	4%
218.	\$840.90	3 yr. 6 mo. 6 da.	$3\frac{1}{2}$%
219.	\$5320	1 yr. 4 mo. 10 da.	5%
220.	\$6250	1 yr. 4 mo. 10 da.	$3\frac{1}{2}$%
221.	\$575.80	5 yr. 4 mo. 1 da.	$2\frac{1}{2}$%
222.	\$8900	6 mo.	$5\frac{1}{2}$%
223.	\$650.40	4 yr. 3 mo. 10 da.	$3\frac{3}{4}$%

Find the date of maturity, the term of discount, the bank discount, and the proceeds of the following notes:

	Face	Time	Date of Note	Date of Discount	Rate of Discount
224.	\$2400	6 mo.	Jan. 2	Feb. 2	5%
225.	\$3200	3 mo.	Mar. 1	Mar. 15	6%
226.	\$5600	4 mo.	Apr. 3	May 3	5%
227.	\$7500	2 mo.	July 6	July 15	4%
228.	\$5000	90 da.	May 1	May 15	6%
229.	\$6000	60 da.	June 1	June 15	5%
230.	\$7000	30 da.	Aug. 15	Sept. 1	$5\frac{1}{2}$%
231.	\$8500	6 mo.	Sept. 7	Sept. 15	6%
232.	\$9500	5 mo.	Oct. 1	Oct. 15	4%
233.	\$10,000	4 mo.	Dec. 1	Dec. 15	5%

Find the cost of the following $100 shares of stocks and $1000 bonds, adding $\frac{1}{8}$ % for brokerage:

	a.		b.		c.		
	No. of Shares	Market Price	No. of Shares	Market Price	No. of Bonds	Market Price	Rate of Int.
234.	100	$88\frac{3}{8}$	5	119	5	$88\frac{3}{8}$	5 %
235.	500	$96\frac{7}{8}$	10	$102\frac{7}{8}$	10	$103\frac{1}{2}$	4 %
236.	50	$26\frac{1}{8}$	20	$124\frac{1}{2}$	20	$96\frac{7}{8}$	$3\frac{1}{2}$ %
237.	10	$100\frac{1}{2}$	100	$35\frac{3}{4}$	100	$102\frac{1}{4}$	4 %

238. Find the amount of interest on the bonds in examples 234 *c* to 237 *c* and the per cent of income.

Square the following numbers:

	a.	b.	c.	d.	e.
239.	13	17	22	125	436
240.	14	18	23	100	569
241.	15	19	24	500	718
242.	10	21	25	600	650
243.	16	36	49	106	720

Find the square root of:

	a.	b.	c.	d.
244.	576	324	5184	11,025
245.	529	1024	3025	11,881
246.	441	2025	4489	48,841
247.	361	9801	5329	112,896
248.	484	10,000	7225	60,025
249.	289	8649	7744	132,496

FARM PROBLEMS

Dairying

To the Teacher. Problems on pages 455–469 are intended especially for rural schools and may be omitted, if desired.

Note. In all these problems, compute per cents to hundredths of a per cent, amounts of butter fat and skimmed milk to hundredths of a pound, prices of a pound to mills, and final costs to the nearest cent.

Many creameries are owned by the patrons who furnish the milk. Officials elected by the patrons supervise the management of the factory, the sale of the butter, etc. At the close of each month a statement is formulated, showing the amount received from the sale of the butter, the expenses, and the balance that is to be divided among the patrons, in proportion to the amount of cream furnished by each.

The totals of a creamery of 10 patrons were as follows for the month of March:

Name	Pounds Milk	Average Test	Pounds Butter Fat
		Per Cent	
1. John Smith	6450	4.02	259.29
2. Captain Doe	4326	3.35	
3. James Dean	7640	3.39	
4. Henry King	3064	4.05	
5. Peter Rula	5675	4.25	
6. P. G. Riley	6940	4.20	
7. Roy Gimlin	5045	3.85	
8. Fred Johns	2480	3.96	
9. John Lund	4965	4.15	
10. W. Bell	2983	3.83	

1. The butter for the month was sold for \$634.85, and the expenses were \$75.97. Find the amount due each patron.

Suggestion. First find the amount of butter fat furnished by each patron (4.02% = 4.02 lb. per hundred pounds of milk). Divide the net proceeds by the total number of pounds of butter fat. This gives the price of a pound of butter fat. Multiply the number of pounds furnished by the different patrons by the price of one pound.

2. About 80 lb. of skimmed milk is returned for every 100 lb. of milk furnished. This is worth about 20¢ a hundred. What is the total gross value of the milk handled in this creamery?

3. L. M. Carpenter's milk sheet for the month of February showed the following entries:

1. 245 lb.	*8.* 240 lb.	*15.* 209 lb.	*22.* 250 lb.
2. 256 lb.	*9.* 237 lb.	*16.* 242 lb.	*23.* 245 lb.
3. 199 lb.	*10.* 225 lb.	*17.* 219 lb.	*24.* 256 lb.
4. 244 lb.	*11.* 210 lb.	*18.* 229 lb.	*25.* 260 lb.
5. 263 lb.	*12.* 209 lb.	*19.* 224 lb.	*26.* 262 lb.
6. 213 lb.	*13.* 230 lb.	*20.* 218 lb.	*27.* 260 lb.
7. 245 lb.	*14.* 217 lb.	*21.* 232 lb.	*28.* 275 lb.

Three tests were taken during the month, giving these results: 3.95%, 3.99%, and 4.30%. What was the average per cent of butter fat?

4. If the net price of butter fat was 25¢ a pound, find the amount due Mr. Carpenter.

5. What is the total value of his milk industry for the month? (For data for skimmed milk, see Example 2.)

The following figures are based on the records of a part of the herd of an Iowa farmer:

No. of Cow	Pounds of Milk	Av. Test Per Cent	Av. Price of Butter Fat
1	4832	3.35	33.6¢
2	5488	4.06	34.4¢
3	5896	3.98	33.6¢
4	6808	3.91	32.8¢
5	6972	3.67	34.8¢
6	5093	3.63	32.7¢
7	6771	3.39	32.7¢
8	5557	2.93	32.9¢
9	7489	3.89	33.2¢
10	6215	4.44	34.0¢

Find the pounds of butter fat and its value for :

6. Cow No. 1.

7. Cow No. 2.

8. Cow No. 3.

9. Cow No. 4.

10. Cow No. 5.

11. Cow No. 6.

12. Cow No. 7.

13. Cow No. 8.

14. Cow No. 9.

15. Cow No. 10.

16. Find the total number of pounds of milk produced.

17. How many pounds of butter fat did the ten cows produce ?

18. What was the total value of butter fat produced ?

19. What was the average number of pounds of milk produced ?

20. Which cows were above the average in milk production?

21. What was the average per cent of butter fat ?

22. Which cows were above the average in per cent of butter fat ?

23. Which cows were above the average in both milk production and in per cent of butter fat ?

24. What was the average number of pounds of butter fat produced ?

The cost of feeding this herd was as follows :

No. of Cow	Cost of Roughage	Cost of Grain
1	$17.43	$15.77
2	$22.43	$17.41
3	$22.43	$17.41
4	$22.43	$17.04
5	$22.43	$17.41
6	$22.43	$13.97
7	$22.43	$14.77
8	$22.43	$13.97
9	$22.43	$14.34
10	$22.43	$17.41

25. Find the total cost of feeding for each cow.

26. What was the profit on each cow?

27. What was the total profit?

28. Find the average profit per cow.

29. Which cows were above the average in profit?

Find the cost per pound of producing butter fat, and the value of butter fat produced for $1 for:

30. Cow No. 1.

31. Cow No. 2.

32. Cow No. 3.

33. Cow No. 4.

34. Cow No. 5.

35. Cow No. 6.

36. Cow No. 7.

37. Cow No. 8.

38. Cow No. 9.

39. Cow No. 10.

40. What was the average cost per pound of producing the butter fat?

41. Which cows were below the average in cost of production?

A farmer who was a member of a Cow Testing Association had the following record on his herd:

No. of Cow	Pounds of Milk	Av. Test Per Cent	Av. Price of Butter Fat
1	3552	3.89	34.6 ¢
2	2109	4.09	29.0 ¢
3	2682	3.84	30.7 ¢
4	5189	3.93	33.4 ¢
5	4414	4.41	32.0 ¢
6	6959	3.96	33.7 ¢
7	2848	3.88	30.8 ¢
8	1846	3.81	31.2 ¢
9	4780	3.81	34.9 ¢
10	5622	4.01	34.1 ¢

During the period of testing, the cost of feed was as follows:

No. of Cow	Cost of Roughage	Cost of Grain
1	$13.67	$16.43
2	$12.50	$10.50
3	$13.67	$10.71
4	$13.67	$10.71
5	$13.67	$10.71
6	$13.67	$10.71
7	$13.67	$10.71
8	$13.67	$10.71
9	$13.67	$19.63
10	$13.67	$19.63

42. Find the number of pounds of butter fat produced by each cow.

43. What was the total number of pounds of butter fat produced by the herd?

44. What was the value per cow of the butter fat produced?

45. What was the total cost of feed per cow?

46. What was the profit per cow?

47. What was the cost per pound of producing the butter fat for each cow?

48. What was the value of butter fat produced for each dollar expended for feed in the case of each cow?

49. Which cows produced more than the average amount of milk? Which produced more than the average amount of butter fat?

The following is a sample daily ration that was used during the testing period:

10 pounds timothy hay
6 pounds corn
5 pounds bran
2 pounds linseed meal

50. At $12 a ton for timothy hay, $.01 a pound for corn, $.012 a pound for bran, and $.017 a pound for linseed meal, find the cost of this ration.

The following figures are based on a study made by the United States Department of Agriculture of 719 cows owned in Southern states. The data cover a period of 12 months.

	Average Cow	Best Cow	Poorest Cow	Av. of Best Ten Cows	Av. of Poorest Ten Cows
Milk, pounds	4299.4	8325.5	1125	8181.9	1577.6
Butter fat	216.8	538.8	64.1	459.0	77.2
Value of skim milk	$8.17	$15.57	$2.12	$16.45	$3.00
Cost of feed per cow	$36.27	$72.03	$23.80	$65.73	$24.63

Find the value of butter fat @ $.34 produced by:

51. Average cow.

52. Best cow.

53. Poorest cow.

54. Average of best ten cows.

55. Average of poorest ten cows.

Find the difference in value between butter fat and skim milk produced by:

56. Average cow.

57. Best cow.

58. Poorest cow.

59. Average of best ten cows.

60. Average of poorest ten cows.

Deducting the cost of feed, find the total profit in each case on the butter fat and skim milk produced by:

61. Average cow.

62. Best cow.

63. Poorest cow.

64. Average of best ten cows.

65. Average of poorest ten cows.

Calculate the per cent of profit:

66. Average cow.

67. Best cow.

68. Poorest cow.

69. Average of best ten cows.

70. Average of poorest ten cows.

Find the per cent of butter fat:

71. Average cow.

72. Best cow.

73. Poorest cow.

74. Average of best ten cows.

75. Average of poorest ten cows.

Weight of Animals

76. A stock farmer of the Middle West gives the following figures on the cost of feeding a steer for a year. Find the total cost of feeding the steer a year.

Silage, 5 tons	@	$2.50	
Pasturage			$5.00
Corn, 20 bushels	@	$.60	
Labor			$2.00

77. A steer weighing 700 lb. is bought at $.07 per pound and fed for a year for $24.50. What is the amount of the investment at the end of the year?

78. If a steer after being fed for a year for $24.50 gains in weight from 700 lb. to 1400 lb., find the profit if the steer is sold at $.09½ a pound.

NOTE. Hogs and poultry lose about 20% of the live weight in dressing, while sheep and cattle lose from 33⅓% to 50%.

79. What is the dressed weight of a crate of chickens weighing 240 lb.?

80. A 380-lb. live hog, purchased at $6.75 a hundred, is retailed at an average of 15¢ a pound. What is the profit?

81. Experiments at agricultural stations have shown that 500 lb. of corn fed to hogs increases their weight about

100 lb. If hogs are worth \$10 per hundred dressed, does a farmer gain or lose by feeding them 360 bu. of corn worth 60 ¢ a bushel? (1 bu. corn weighs 56 lb.) How much?

82. When pork sells at \$7 per hundred live weight, how much ought corn to bring in order that neither loss nor gain may result in feeding it to hogs, on the basis of problem 81?

83. A drove of hogs average 200 lb. in December, and are fed corn on the basis given in problem 81 until they weigh 300 lb. If pork sells at \$6 per hundred, live weight, in December, and the corn is worth 50¢ per bushel, at what price per hundred should the 300-lb. hogs be sold to bring the same amount of money as in December?

84. If these hogs, which sold at \$6 per 100 lb. in December, were sold at \$6.25 per 100 lb., live weight, after being fed the corn, how much did the owner receive per hog for his labor in feeding and caring for the hogs after paying for the corn?

85. I sell 5 steers weighing 4950 lb., and some cows weighing 3460 lb. What is the total of the dressed beef produced by them, allowing a loss of 40 % in dressing?

86. A butcher bought a steer weighing 1250 lb. When killed and dressed, the marketable meat weighed 58 % of the live weight. If he paid \$6 per hundred for the steer and sold 375 lb. of the meat at an average of 10 ¢ per pound, at what average price must he sell the remainder of the carcass, to make 25% on his investment? What items of expense in conducting the business, besides the price paid for the steer, have to be considered in determining the net profit?

87. Hens average 150 eggs a year. Allowing 1080 eggs for hatching, what is the income from a yard of 120 hens, when eggs are 20¢ a dozen, and 1850 lb., live weight, of young chickens are sold at 15¢ a pound, dressed weight?

Potatoes

Reports from forty growers of potatoes in the state of Iowa showed the following items of expense in growing 1 acre of potatoes:

Rent, or interest, on land (valued at $100 per acre)	$6.00
Seed, 12 bu. @ $.60	
Treatment of seed	.50
Cutting (by hand)	.50
Plowing	2.00
Disking (twice) @ $.25	
Harrowing (5 times) @ $.13	
Planting	1.50
Cultivating (5 times) @ $.25	
Spraying (3 times)	5.00
Digging	5.00
Hauling to market (4 miles)	3.50
Extra hand labor (hoeing, etc.)	2.00

88. Find the total cost per acre.

89. The average yield per acre was 140 bushels. What was the cost of production per bushel?

90. What was the average cost per bushel for hauling the crop to market?

91. How much did the rent amount to for each bushel?

92. What was the cost per bushel for digging?

93. How much did the plowing, disking, harrowing, planting, cultivating, and spraying add to the cost per bushel?

94. The average yield per acre for the state of Iowa is 82 bushels. If these growers had raised only an average crop, what would have been the cost per bushel?

95. The growers received an average of $.71 per bushel for their potatoes. What was the profit per acre? per bushel?

The following table contains data received from twenty potato growers in a Western state, in 1911:

Grower	Price rec'd per Bushel	Av. Yield per Acre in Bushels	Cost of Production per Acre
1	$1.48	150	$31.10
2	.90	250	37.90
3	1.00	200	39.00
4	1.00	140	32.30
5	.50	275	42.00
6	.50	250	30.50
7	.55	215	40.60
8	.50	225	36.10
9	1.00	80	35.25
10	.60	140	28.50
11	.76	100	27.80
12	.45	176	33.70
13	1.60	125	39.30
14	.53	247	30.60
15	1.00	100	30.25
16	.80	100	31.00
17	.50	250	34.00
18	.40	250	48.45
19	1.00	125	22.95
20	.50	320	30.00

96. What was the gross return per acre for each grower?

97. Determine the net profit per acre in each case.

98. What was the profit per bushel for each grower?

99. What was the cost of production per bushel for each grower?

100. What was the average cost of production per acre?

101. What was the average yield per acre?

Hay in Mows and in Stacks

The only accurate method of determining the number of tons of hay in a stack or mow is by weighing. The amount, however, may be determined, approximately, by measurement.

In high stacks and deep mows:

Wild hay requires about 400 cu. ft. per ton.
Timothy hay requires about 500 cu. ft. per ton.
Clover hay requires about 600 cu. ft. per ton.

For stacks or mows less than 10 ft. high, add about 100 cu. ft. to each of the foregoing.

102. How many tons of clover are there in a mow 36 ft. by 40 ft. and 24 ft. deep?

103. How many tons of wild hay are there in a mow 6 ft. narrower and 2 ft. deeper than the foregoing?

104. A mow of clover is 11 ft. wide, 16 ft. long, and 9 ft. high. What is its value at $8.50 per ton?

105. How many tons of wild hay are there in a stack 40 ft. long, 20 ft. wide, and 44 ft. high?

106. A mow is 24 ft. by 30 ft., and contains 35 tons of clover. How high is it?

107. A haymow designed to hold 150 tons of clover is 60 ft. long and 24 ft. high. How wide must it be?

Fencing

Barbed wire averages about one pound per rod. It is sold in spools containing from 95 pounds to 115 pounds.

108. What is the cost of fencing a square 40-acre tract with a 4 barbed wire fence at $2.85 per hundred?

109. A field 80 rd. long and 40 rd. wide is fenced with woven wire 32 in. wide at 30¢ a rod, and 2 barbed wires at $3 per hundred. The posts are 1 rd. apart and cost $12\frac{1}{2}$¢ apiece. What is the price of the material?

110. An 80-acre farm of two adjoining square forties is to be surrounded by a fence, and to be separated into 20-acre fields by cross fences. What is the cost of the material if the same kind is used as given in problem 109?

111. What is the cost of material required for surrounding a 40-acre square farm with a 4-wire fence, at $2.75 per hundred, posts one rod apart at 15 ¢ apiece?

112. A man can drive 20 posts per hour and string 4 wires across 10 posts per hour. At $2 per day of 8 hours, what is the cost of labor on the fence mentioned in problem 111?

Silos

A **silo** is a structure used for preserving in its green state finely cut corn stalks, sugar cane, alfalfa, etc. This finely cut vegetable material is called **silage**, or **ensilage**. Silos are built of wood, stone, brick, or cement. The circular silo is the most economical form.

The average weight of silage in silos from 25 ft. to 36 ft. high is about 40 pounds a cubic foot. Forty pounds is also the amount of silage ordinarily fed to a cow per day.

113. 10 A. of good corn should provide silage for 20 cows for 180 da. If 40 lb. a day are allowed for each cow, how many tons of silage should be furnished by 10 A.?

114. On this basis, how many acres of good corn are required to fill a silo 16 ft. in diameter and 36 ft. high?

115. How many head of stock will this supply for 200 days?

116. A circular silo 20 ft. in diameter contains 200 tons of ensilage. If this averages 40 lb. per cubic foot, find the depth of the silo.

117. If a silo is filled at the rate of 8 tons per day, how long will it take to fill a silo 16 ft. in diameter and 40 ft. high?

118. How many cows can be supplied from it for 190 days?

Water in Plant Growth

Experiments conducted by the Department of Agriculture have proved that about 300 lb. of water are required to mature 1 lb. of corn. Oats need 500 lb., while clover, barley, and wheat need about 400 lb. each to mature 1 lb. of the product.

119. How much water is required to mature 40 acres of oats averaging 45 bu. per acre? (1 bu. of oats weighs 32 lb.)

120. To what depth would it cover the field if applied at one time? (1 cu. ft. of water weighs 62.5 lb.)

121. A 10-acre field of corn averaging 100 bu. per acre receives 6 in. of rain. If the rest is supplied by irrigation, how many cubic feet of water are required, no allowance being made for evaporation? (70 lb. = 1 bu. of corn on cob.)

122. From a given strip of land with a firm surface, 5.34 inches of water evaporated during a given time. From an adjacent strip of like size and soil, whose surface was kept well tilled to a depth of 3 inches, only .88 inch evaporated in the same time. How many tons of water were saved the soil by tilling 80 acres?

123. How much less rainfall is needed on a 40-acre piece of corn kept well tilled than on a field of the same size that is allowed to remain hard and firm? What will be the weight in tons?

124. Find the difference between the amount of water required to mature a 10-acre field of corn yielding 90 bu. per acre and a field of wheat of the same size yielding 32 bu. per acre. Make no allowance for evaporation. (1 bu. wheat weighs 60 lb.)

125. What is the average rainfall in your state per year? How much would the total fall for one year on an acre weigh?

Miscellaneous

126. The value of properly cured and stored corn stover averages about $10 per ton. The yield per acre averages about 2 tons. A 20-acre field of corn yields 40 bu. of corn per acre. If the corn is worth 56¢ per bushel, what is the entire value of the crop?

127. Of a quantity of seed corn used for planting, 80 kernels out of 100 germinate. The stand of corn is what proportion of what it should be if every kernel germinated?

128. If 85 kernels out of 100 germinate, the stand of corn is what proportion of a perfect stand?

129. If 99% of seed corn planted in a certain field germinates and the yield is 36 bu. per acre, what would have been the yield had all the seed corn germinated?

130. If corn shrinks 8% in weight between October and June, and sells at 60¢ per bushel in June, how much should it sell for in October, to bring the same amount of money from one acre yielding 50 bu., October weight?

131. If the same corn sells for 55¢ per bushel in October, how much should it sell for per bushel in June, in order to yield 6% more than in October?

132. Potatoes shrink an average of 1% a month for 5 months, after being taken from the ground. In September, a farmer harvests a crop of 500 bu. of potatoes. The market price at that time is $1.00 per bushel. At what price per bushel must he sell them 5 months later, to repay him for the interest lost at 6%?

133. A farmer sells 100 bu. of potatoes on October 1 at $1.00 per bushel. If he had kept them until March 1 and sold them for $1.20 per bushel, how much would he have gained, reckoning interest at 6%?

134. If 10% of the potatoes in problem 133 were lost because of decay, would the farmer have gained or lost, and how much, by holding the potatoes?

135. The number of bushels of new corn in a crib equals $\frac{1}{5}$ the number of cubic feet of corn in the crib. The number of bushels of old corn in a crib equals $\frac{1}{4}$ the number of cubic feet of corn in the crib. A corn crib is 20 ft. long, 6 ft. wide at the bottom, 8 ft. wide at the top, and 6 ft. high. How many bushels of new corn will it hold? of old corn?

136. A corn crib 24 ft. long, 8 ft. wide at the bottom, 10 ft. wide at the top, and 7 ft. high, holds how many bushels of new corn? of old corn?

137. A corn crib has an average width of 7 ft. It is 8 ft. high. What must be its length to hold 250 bu. of new corn? How many bushels of old corn will it hold?

138. A farmer grows 800 bu. of corn which he sells for 65¢ per bushel. By giving one extra day to selecting and caring for his seed corn, he increases his yield the next year 4%. If corn sells at the same price as the year before, how much does he receive for his extra day's work?

139. Investigations made in a certain district showed that there was a loss of 10% in the oat crop because of the presence of smut. What was the loss to a farmer who harvested 450 bu. of oats that year, if oats sold for 48¢ per bushel?

140. The farmer mentioned in the preceding problem could have treated his seed oats with a formaldehyde solution at a cost of $2 for material and labor, thus preventing the loss by smut. How much would he have gained by so doing?

MANUFACTURING AND FREIGHT PROBLEMS

1. In 1850 there were 123,025 manufacturing establishments in the United States, and in 1900, 512,254. Find the per cent of increase for 50 years.

2. In 1850 the value of manufactured products in the United States was $1,019,106,616, and in 1900, $13,004,400,143. What per cent of increase was there from 1850 to 1900?

3. If raw silk loses 24% in being boiled, how much will 5000 lb. weigh after being boiled?

4. The value of manufactured products in the city of New York in 1904 was $206,825,000, and in 1909, $266,034,000. What per cent did the value increase in 5 years?

5. In 1913 $381,032,495 of the foreign carrying trade was in American vessels, and $3,391,998,429 in foreign vessels. What per cent of the total trade was in American vessels?

6. In 1913 the freight products of agriculture in the United States amounted to 85,566,053 tons; of animals, to 23,763,262 tons; of mines, to 539,255,980 tons; of forests, to 108,506,272 tons; of manufactures, to 135,175,536 tons; of merchandise, to 36,519,321 tons; of other commodities, to 38,447,567 tons. Find the total freight product. Find the per cent each product was of the total.

Values of Principal Manufactures Exported from the United States in 1913

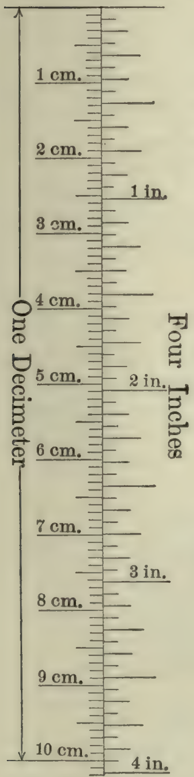
Iron and steel	$304,605,797	Chemicals, etc.	$26,574,519
Copper	$140,164,913	Leather	$63,893,351
Agricult. Implements	$ 40,572,352	Cotton	$53,743,977
Wood Manufactures	$115,704,777	Books, etc.	$10,092,719
Mineral oils	$129,666,995	Paper	$11,686,584

7. Find the total value of these manufactures. Find what per cent each value is of the total.

SUPPLEMENT

To the Teacher. The following subjects were omitted from the body of the book, because they have not much practical value. They are included in this supplement in case it is desired to teach them in some schools.

METRIC SYSTEM OF MEASURES



The system of weights and measures in our country is irregular and somewhat difficult to learn, as well as inconvenient to apply. The same held true with the old systems of other nations. In order to overcome these defects, France in 1799 adopted the metric system of weights and measures, based upon the decimal system of notation. This system is now in use in all civilized nations, England and the United States being the only exceptions. Congress in 1866 legalized its use in our country, but it is used here at the present time in scientific work chiefly.

Multiples of the primary units are represented by the Greek prefixes, **deka** (10), **hekto** (100), **kilo** (1000), **myria** (10,000); and decimal divisions, by the Latin prefixes, **deci** ($\frac{1}{10}$), **centi** ($\frac{1}{100}$), **milli** ($\frac{1}{1000}$).

LINEAR MEASURE

10 **millimeters** (mm.)	= 1 **centimeter** (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 **kilometer** (Km.)
10 kilometers	= 1 myriameter (Mm.)

The units most commonly used are those printed in black type; namely, the millimeter, centimeter, meter, and kilometer.

A decimeter, or 10 centimeters, as shown in the cut, equals a little less than 4 in. The meter is equal to 39.37 in., or 1.09 yd.

SQUARE MEASURE

100 sq. millimeters (sq. mm.)	= 1 sq. centimeter (sq. cm.)
100 sq. centimeters	= 1 sq. decimeter (sq. dm.)
100 sq. decimeters	= 1 **sq. meter** (sq. m.)
100 sq. meters	= 1 sq. dekameter (sq. Dm.)
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.)
100 sq. hektometers	= 1 **sq. kilometer** (sq. Km.)

LAND MEASURE

100 centares (ca.) or square meters	= 1 **are** (a.), or 1 sq. Dm.
100 ares	= 1 **hektare** (Ha.)

MEASURES OF VOLUME

1000 cubic millimeters (cu. mm.)	= 1 **cubic centimeter** (cu. cm.)
1000 cubic centimeters	= 1 **cubic decimeter** (cu. dm.)
1000 cubic decimeters	= 1 **cubic meter** (cu. m.), or **stere**

MEASURES OF WEIGHT

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 **gram** (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 **kilogram** (Kg.)

MEASURES OF CAPACITY

10 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 **liter** (l.)

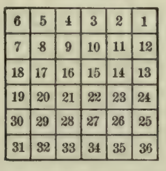
APPROXIMATE VALUES

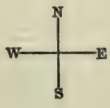
Metric Units	Approximate Equivalents
1 kilometer	$\frac{5}{8}$ mi.
1 meter	1.1 yards
1 centimeter	0.4 inch
1 hektare	2.5 acres
1 square meter	1.2 square yards
1 square centimeter	0.16 square inch
1 cubic meter	1.3 cubic yards
1 cubic centimeter	0.06 cubic inch
1 hektoliter	3 bushels
1 liter	0.9 quart (dry)
1 liter	1.06 quarts (liquid)
1 metric ton	1 long ton, nearly
1 kilogram	2.2 pounds
1 gram	15.4 grains

LAND SURVEY

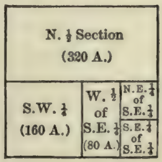
In most states the land is divided by survey into townships 6 miles square, and these are divided into 36 equal sections of a square mile, or 640 acres each. The sections are subdivided into half-sections and quarter-sections, and are numbered and designated as in the following diagrams.

A TOWNSHIP





A SECTION



The townships are numbered and designated east or west from a definite north and south line, called the **principal meridian**, and north or south from a definite east and west line called the **base line.**

PROBLEMS

Oral

1. How many acres are there in the N.E. $\frac{1}{4}$ of S.E. $\frac{1}{4}$ of section 18?

2. How many acres are there in the N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of section 34? in the W. $\frac{1}{2}$ of S.E. $\frac{1}{4}$ of section 30?

3. Find the cost of the S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of section 9, at $40 an acre.

4. How many rods of fence are required to inclose the land mentioned in problem 3?

5. Find the cost of the N. $\frac{1}{2}$ of S.E. $\frac{1}{4}$ of section 36, at $50 an acre.

6. At $100 an acre, find the cost of the S. $\frac{1}{2}$ of section 3 and the N.E. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of the same section.

LONGITUDE AND TIME

Circular measure is used in measuring arcs of circles and angles, and in estimating latitude and longitude.

Table

60 seconds ($''$)	=	1 minute ($'$)
60 minutes	=	1 degree (deg. or °)
360 degrees	=	1 circumference

The **longitude** of a place is its distance east or west from a given meridian. The meridian most commonly used for this purpose is the one passing through the observatory at Greenwich, England. It is called the prime meridian. All places on it have 0° longitude. Longitude is reckoned in degrees, minutes, and seconds, and ranges from 0° to 180° east or west. The earth revolves on its axis from

west to east once in every 24 hours, and this causes the sun to appear to revolve around the earth from east to west in the same time. Since the sun travels through the entire circumference, or 360°, in twenty-four hours, in one hour it travels $\frac{1}{24}$ of 360°, or 15°; in one minute it travels $\frac{1}{60}$ of 15°, or 15′; and in one second it travels $\frac{1}{60}$ of 15′, or 15″.

Places to the *east* of any meridian have *later* time; those to the *west*, have *earlier* time, since the sun appears first to those in the east. When the sun is directly overhead, it is noon on that meridian.

Table of Longitude and Time

A difference of 15° of longitude makes a difference of 1 hour of time.
A difference of 15′ of longitude makes a difference of 1 minute of time.
A difference of 15″ of longitude makes a difference of 1 second of time.

PROBLEMS

Oral and Written

1. How many degrees of longitude make a difference of 1 hour in time? 12 hours? 30 minutes? 20 seconds?

2. When it is noon at Chicago, what is the time 15° east? 15° west? 30° east? 30° west? 60° east? 60° west?

3. What is the difference in longitude between two cities, if the difference in time is 2 hours? 3 hours 30 minutes?

4. If I start at Denver and travel until my watch is 1 hr. 30 min. too fast, in what direction and how far do I travel?

5. The first shock of the San Francisco earthquake, April 18, 1906, was recorded at 5.12 A.M. in observatories in 120° W. and at 19 min. 20 sec. past 8 A.M. in 75° W. What length of time was consumed by the shock in passing between the two observatories?

6. The longitude of Philadelphia is 75° 10′ 0″ west, and of Vienna 16° 20′ 22″ east. What is the time in Vienna when it is 9 A.M. in Philadelphia?

$$\begin{array}{rrrl} & 16^\circ & 20' & 22''\text{ E.} \\ & 75^\circ & 10' & 0''\text{ W.} \\ \hline 15) & 91^\circ & 30' & 22'' \end{array}$$

6 hr. 6 min. $1\frac{7}{15}$ sec.

9 A.M. + 6 hr. 6 min. $1\frac{7}{15}$ sec. =

3 hr. 6 min. $1\frac{7}{15}$ sec. P.M. *Ans.*

Since one place is in east, and the other in west, longitude, the difference in longitude is found by *adding*, which gives 91° 30′ 22″. From the table on p. 475 it follows that the difference in time is as many hours, minutes, and seconds, respectively, as there are degrees, minutes, and seconds in $\frac{1}{15}$ of the difference in longitude, or 6 hr. 6 min. $1\frac{7}{15}$ sec. Since Vienna is in east longitude, and Philadelphia in west longitude, the sun rises earlier in Vienna; hence we must add the difference in time to 9 A.M., which gives 6 min. $1\frac{7}{15}$ sec. past 3 P.M.

7. The difference in longitude of two places is 40°. What is the difference in time between the places?

8. The longitude of New Orleans is 90° west. Find the difference in time between this place and Philadelphia, which is in longitude 75° 10′ 0″ west.

9. London is on the prime meridian, and Washington is 77° 3′ 6″ west. What change must a traveler make in his timepiece in going from Washington to London?

10. Paris is 2° 20′ east longitude, and San Francisco 122° 26′ 15″ west. What time is it in San Francisco when it is 7 P.M. in Paris?

11. Rome is 12° 27′ east, and Manila 120° 58′ 6″ east. Dewey began firing on the defenses of Manila at 5.41 A.M., May 1, 1898. At what time, by timepieces in Rome, was the fight begun?

12. The difference in time between Philadelphia and Cincinnati is 37 min. 20 sec. What is the difference in longitude?

$$\begin{array}{r} 37 \text{ min. } 20 \text{ sec.} \\ 15 \\ \hline 9^\circ\ 20'\ 0''.\ \textit{Ans.} \end{array}$$

Since a difference of 1 minute of time corresponds to a difference of 15′ of longitude, etc., 15 times the number of seconds of time will represent seconds of longitude (300″), and 15 times the number of minutes of time, the number of minutes of longitude (555′), 555′ + 300″ = 9° 20′ 0″.

13. The time at St. Louis is 53 min. earlier than the time at Washington. What is the difference in longitude?

14. In traveling from New York to Cincinnati I find that my watch is 41 min. 32 sec. too fast. Find the difference in longitude.

15. In traveling on a parallel of latitude from 89° 32′ 30″ west longitude, I notice that my watch has lost 1 hr. 25 min. 30 sec. In what direction am I traveling, and what is the longitude of the second place?

16. The longitude of New York is 74° 3′ west, and of Jerusalem 35° 13′ east. When it is 4.30 A.M. in New York, what is the time in Jerusalem?

17. Formulate a problem in longitude and time, getting the data from your geography. Solve the problem.

STANDARD TIME

In the work on longitude and time, it was shown that all places differing in longitude differ in time as reckoned by the sun. In order to avoid the confusing differences in time when each place has its own local time, the railroads of Canada and the United States in 1883 formulated a system

of **standard time.** In this system, meridians that are multiples of 15° are selected as principal meridians, and the belts $7\frac{1}{2}°$ on either side have the local time of the principal meridian. In practice, however, the time belts are not of uniform width, since the division terminals of railroads often extend beyond, or fall short of, the halfway points between the principal meridians. The time belts used in standard time are the Atlantic, Eastern, Central, Mountain, and Pacific, with principal meridians of 60°, 75°, 90°, 105°, and 120°, respectively.

A traveler leaves New York at 8 A.M. As he leaves Pittsburgh, he notices that his watch, an accurate timekeeper, is an hour fast. Pulling into Ellis, Kansas, his watch is still an hour fast, but as the train leaves the station he notices that his watch is two hours fast. When leaving Ogden, Utah, he notices that his watch has gained another hour. His timepiece now is three hours fast, and remains so during the rest of the trip to San Francisco. Explain.

STANDARD TIME BELTS

SIMPLE INTEREST

TABLE METHOD

People who have much interest to compute generally use **interest tables.** The following partial table is based on a year of 360 days and a rate of 6 %.

Years	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8000	$9000
1	60	120	180	240	300	360	420	480	540
2	120	240	360	480	600	720	840	960	1080
3	180	360	540	720	900	1080	1260	1440	1620
4	240	480	720	960	1200	1440	1680	1920	2160
5	300	600	900	1200	1500	1800	2100	2400	2700
6	360	720	1080	1440	1800	2160	2520	2880	3240
7	420	840	1260	1680	2100	2520	2940	3360	3780

Months	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8000	$9000
1	5	10	15	20	25	30	35	40	45
2	10	20	30	40	50	60	70	80	90
3	15	30	45	60	75	90	105	120	135
4	20	40	60	80	100	120	140	160	180
5	25	50	75	100	125	150	175	200	225
6	30	60	90	120	150	180	210	240	270
7	35	70	105	140	175	210	245	280	315
8	40	80	120	160	200	240	280	320	360

Days	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8000	$9000
1	.167	.333	.50	.667	.833	1.00	1.167	1.133	1.50
2	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	3.00
3	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	4.50
4	.667	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00
5	.833	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50
6	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00
7	1.167	2.333	3.50	4.667	5.833	7.00	8.167	9.333	10.50
8	1.333	2.667	4.00	5.333	6.667	8.00	9.333	10.667	12.00
9	1.500	3.000	4.50	6.000	7.500	9.00	10.500	12.000	13.50
10	1.667	3.333	5.00	6.667	8.333	10.00	11.667	13.333	15.00
11	1.833	3.667	5.50	7.333	9.167	11.00	12.833	14.667	16.50
12	2.000	4.000	6.00	8.000	10.000	12.00	14.000	16.000	18.00
13	2.167	4.333	6.50	8.667	10.833	13.00	15.167	17.333	19.50
14	2.333	4.667	7.00	9.333	11.667	14.00	16.333	18.667	21.00
15	2.500	5.000	7.50	10.000	12.500	15.00	17.500	20.000	22.50

Written

1. Find by the table on page 479 the interest on \$3560 for 5 yr. 6 mo. 6 da. at 6 %.

NOTE. The interest on \$500 is $\frac{1}{10}$ of that on \$5000; the interest on \$60 is $\frac{1}{100}$ that on \$6000.

SOLUTION.

$$\begin{array}{lrcrcrcr}
\text{Int. on} & \$3000 = & \$900 & + & \$90 & + & \$3.00 & \\
\text{Int. on} & 500 = & 150 & + & 15 & + & .50 & \\
\text{Int. on} & 60 = & 18 & + & 1.80 & + & .06 & \\
\hline
\text{Int. of} & \$3560 & \$1068 & + & 106.80 & + & 3.56 & = \$1178.36
\end{array}$$

Find by this method the interest, at 6 %, on:

2. \$2000, 1 yr. 1 mo. 5 da.

3. \$1520, 2 yr. 3 mo. 6 da.

4. \$3460, 3 yr. 2 mo. 7 da.

5. \$5220, 4 yr. 4 mo. 10 da.

6. \$8140, 5 yr. 5 mo. 12 da.

7. \$7260, 6 yr. 8 mo. 15 da.

8. \$6418, 3 yr. 5 mo. 8 da.

9. \$5105, 2 yr. 4 mo. 6 da.

CANCELLATION METHOD

NOTE. At 6% the interest on any principal for 1 day is $\frac{1}{1000}$ of the principal divided by 6. This result should be multiplied by the number of days.

1. Find the interest on \$4268 for 63 days (*a*) at 6 %; (*b*) at 4 %.

SOLUTION. (*a*) $\overset{\$2.134}{\$\cancel{4.268}} \times \dfrac{\overset{21}{\cancel{63}}}{\underset{3}{\cancel{6}}} = \$44.814$;

(*b*) $\overset{\$2.134}{\$\cancel{4.268}} \times \dfrac{\overset{7}{\cancel{\overset{21}{\cancel{63}}}}}{\underset{3}{\cancel{6}}} \times \dfrac{\overset{2}{\cancel{4}}}{\underset{\cancel{2}}{\cancel{6}}} = \$29.876$

Find by the cancellation method the interest of:

2. \$5000, 15 da., 6 %.

3. \$6000, 10 da., 4 %.

4. \$4500, 60 da., 5 %.

5. \$7200, 90 da., 3 %.

COMPOUND INTEREST

Amount of $1 *at various rates, compound interest,* 1 *to* 17 *years*

YEARS	1 PER CENT	1¼ PER CENT	1½ PER CENT	2 PER CENT	2½ PER CENT	3 PER CENT
1	1.010000	1.012500	1.015000	1.020000	1.025000	1.030000
2	1.020100	1.025156	1.030225	1.040400	1.050625	1.060900
3	1.030301	1.037971	1.045678	1.061208	1.076891	1.092727
4	1.040604	1.050945	1.061364	1.082432	1.103813	1.125509
5	1.051010	1.064082	1.077284	1.104081	1.131408	1.159274
6	1.061520	1.077383	1.093443	1.126162	1.159693	1.194052
7	1.072135	1.090850	1.109845	1.148686	1.188686	1.229874
8	1.082857	1.104486	1.126493	1.171659	1.218403	1.266770
9	1.093685	1.118292	1.143390	1.195093	1.248863	1.304773
10	1.104622	1.132271	1.160541	1.218994	1.280085	1.343916
11	1.115668	1.146424	1.177949	1.243374	1.312087	1.384234
12	1.126825	1.160755	1.195618	1.268242	1.344889	1.425761
13	1.138093	1.175264	1.213552	1.293607	1.378511	1.468534
14	1.149474	1.189955	1.231756	1.319479	1.412974	1.512590
15	1.160969	1.204829	1.250232	1.345868	1.448298	1.557967
16	1.172579	1.219889	1.268986	1.372786	1.484506	1.604706
17	1.184304	1.235138	1.288020	1.400241	1.521618	1.652848

YEARS	3½ PER CENT	4 PER CENT	4½ PER CENT	5 PER CENT	6 PER CENT	7 PER CENT
1	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.272279	1.315932	1.360862	1.407100	1.503630	1.605782
8	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.511069	1.601032	1.695881	1.795856	2.012197	2.252192
13	1.563956	1.665074	1.772196	1.885649	2.132928	2.409845
14	1.618695	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.675349	1.800944	1.935282	2.078928	2.396558	2.759032
16	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.794676	1.947901	2.113377	2.292018	2.692773	3.158815

Compound interest tables are used in computing compound interest. The table on page 481 shows the amount of \$1 at compound interest for any number of years up to 17, and for any rate per cent up to 7.

Written

1. Find, by using the table, the amount of \$500 for 10 years at 7 %, compound interest.

Solution

The amount of \$1 for 10 years at 7 %, as shown by the table (p. 481), equals \$1.967151.

Hence the amount of \$500 = 500 × \$1.967151, or \$983.58. *Ans.*

2. Find the amount of \$800 for 12 years at 6 %, compound interest.

3. What is the compound interest on \$2000 for 15 years at 7 %?

4. Find the amount of \$1200 for 8 years at 4 %, compounded semiannually.

Note. When interest is compounded semiannually, one half the given rate and twice the number of periods for which interest is compounded are used. Thus, in problem 4, find the interest at 2 % for 16 years.

5. What is the amount and the compound interest of \$400 for 2 years, 3 months at 6 %, interest compounded semiannually?

Note. Compute the interest for 4 years at 3 %, and then for the remaining time at 6 %.

6. Find the compound interest on \$5000 for 6 years at 7 %, compounded semiannually.

7. Find the compound interest on \$10,000 for 3 years at 6 %, compounded semiannually.

PARTIAL PAYMENTS

Sometimes the maker of a note is not able to pay it in full at maturity, or is allowed, if he chooses, to make part payment before the note is due. Such payment is then indorsed on the back of the note with the date of payment. Several such payments may be made before the entire amount is paid. Such payments are called **partial payments.**

The following problem is solved by the rule adopted by the United States Supreme Court, called the *United States Rule*. It is based on the following principles:

(1) *Payments are applied first to discharge or pay the interest due on the principal at the time the payment is made, and the balance, if any, is then applied to reduce the principal.*

(2) *Only the principal draws interest; hence, if the payment is less than the interest due when it is made, the unpaid interest must not be added to the principal and thus draw interest. The interest is then calculated to the next period before the payment is deducted.*

1. What amount is due on the following note:

$2000	*South Bend, Ind., Feb. 1, 1914.*
On demand, for value received, I promise to pay	
Arthur Carter ------------------------------ *or order,*	
Two thousand and $\frac{no}{100}$ ------------------------------ *Dollars.*	
With interest at 5%.	*Frank Morse.*

The following payments are indorsed on this note:

Aug. 1, 1914	$100	Nov. 1, 1915	$100
Aug. 1, 1915	50	Find balance due Feb. 1, 1916.	

SOLUTION

Principal	$2000
Int. on $2000, Feb. 1, 1914, to Aug. 1, 1914 (6 mo.)	50
Amt. of $2000, Aug. 1, 1914	2050
Payment, Aug. 1, 1914	100
New principal, Aug. 1, 1914	1950
Int. on $1950, Aug. 1, 1914, to Aug. 1, 1915 (1 yr.) $97.50. As the interest exceeds the payment of $50 made Aug. 1, 1915, no new principal is formed	
Int. of $1950, Aug. 1, 1914, to Nov. 1, 1915 (1 yr. 3 mo.)	121.88
Amt. of $1950, Nov. 1, 1915	2071.88
Less payments, $50 (Aug. 1, 1915) and $100 (Nov. 1, 1915)	150
New principal, Nov. 1, 1915	1921.88
Int. of $1921.88, Nov. 1, 1915, to Feb. 1, 1916 (3 mo.)	24.02
Amt. due, Feb. 1, 1916	$1945.90

2. A note of $1000, dated March 1, 1914, with interest at 4 %, is indorsed as follows: June 1, 1914, $90; March 1, 1915, $500. How much is due on the note March 1, 1916?

3. On a note for $5000, dated July 1, 1913, with interest at 6 %, the following indorsements are made: July 1, 1914, $40; July 1, 1915, $500. How much is due July 1, 1916?

When notes are fully paid within a year from date, the following rule, called the *Merchants' Rule*, is ofted used.

Find the amount of the principal from the time it begins to draw interest to the time of final settlement.

Find the amount of each payment from the time it is made to the date of settlement.

From the amount of the principal subtract the sum of the amounts of the payments. The difference is the balance due.

4. John Smith gives Henry Jones his note for $250, due in one year from Dec. 15, 1913, interest at 7 %.

On March 15, 1914, Smith makes a payment of $75, and on Sept. 15, 1914, he makes another payment of $50. How much is due at maturity?

SQUARE ROOT

Explanation of the Process

The square of any number composed of tens and units equals the square of the tens plus twice the product of the tens by the units plus the square of the units.

Or, substituting t for tens and u for units, $(t+u)^2 = t^2 + 2\ tu + u^2$.

1. Extract the square root of 625.

$$\begin{array}{r|r}
 & \underline{2\ 5} \\
 & 6'25 \\
 & 4\ 00 \\
40 & 2\ 25 \\
 & \underline{2\ 00} \\
 & 25 \\
 & 25
\end{array}$$

Beginning at units to point off into periods of two figures, you find that the first period consists of one figure, 6.

It has been shown that the square of the first figure of the root is found in the first period. The greatest square equal to or less than 6 is 4. Its square root is 2, the first figure of the root. The two periods show that there are two figures in the root, hence the value of its first figure in units is 20. The square of 2 tens, or 20, is 4 hundreds, or 400. To show clearly what is done, the two zeros are annexed to 4, the square of the first figure of the root, and the subtraction is then made. The result, 225, is the same as though the zeros had been omitted (as is ordinarily done), the 4 subtracted from the first period and the next period annexed to the remainder.

You have now taken from the square one of the three parts of which it is composed; namely, the square of the tens. The remainder is made up of the other two parts; namely, 2 times the product of the tens by the units + the square of the units ($2\ tu + u^2$).

You have found the tens' figure of the root, but have yet to find the units' figure. Of the remaining two parts, the second part, the square of the units is much the smaller, and in finding the trial units' figure may be ignored. Leaving it out of account, you may regard the 225 as made up of 2 times the product of the tens, or 20, by the units. Here you have the product of three factors, and two of them known, to find the other factor. The two known factors in 2 times the product of 20 by the units are 2 and 20. To find the trial units' figure in the root, divide 225, the supposed product of the three factors, by 40, the product of the known factors, 2 and 20, and you have 5 for the quotient, or the other factor, the units' figure of the root.

Multiplying 40 by 5, you have 200, or 2 times the product of the tens by the units, or the second part of the square. Subtracting, you have left 25, or the square of the units, the third part of the square. If the trial units' figure is correct, its square equals 25, which is true; hence 5 is the correct units' figure.

NOTE. The square subtracted from the first period, and the product of the second figure of the root by the complete divisor, together equal the entire number. If there were three figures in the root, these two numbers together would equal the square of the first two figures of the root considered as tens, and the units' figure would remain to be found in the same way as the second figure of the root was found, and so on, if the root contained more than three figures.

$$\begin{array}{r|l}
 & 2\;5 \\
 & \overline{625} \\
 & 4 \\ \cline{2-2}
45 & 225 \\
 & 225
\end{array}$$

In actual work all the zeros are omitted. The trial divisor, 4, is used to divide the dividend, 225, exclusive of the right-hand figure, or to divide 22, as this gives the same result as though 40 and 225 were used. When the trial units' figure is found, it is annexed to 4, the trial divisor, making 45. 45 is equal to 2 times the tens + 5. This being multiplied by 5, the units' figure, gives 2 times the product of the tens by the units + the square of the units in one operation.

2. Extract the square root of 3249, explaining the process.

CUBE ROOT

$1^3 = 1$	$4^3 = 64$	$7^3 = 343$
$2^3 = 8$	$5^3 = 125$	$8^3 = 512$
$3^3 = 27$	$6^3 = 216$	$9^3 = 729$

1. Extract the cube root of 12,167.

$$\begin{array}{lcr|l}
 & & 2\;3 & \\
 & & \overline{12'167} & \\
 & & 8 & \\ \cline{4-4}
3 \times 20^2 & = & 1200 & 4167 \\
3 \times 20 \times 3 & = & 180 & \\
3^2 & = & 9 & 4167 \\ \cline{3-4}
 & & 1389 &
\end{array}$$

Steps:

1. Separate the number (12,167) into periods of three figures each, beginning with units.

2. Find the greatest cube (8) equal to or less than the left-hand period (12); write its cube root (2) for the first figure of the root and subtract the cube (8) from the left-hand period (12).

3. To the remainder (4) annex the next period (167) for the dividend.

4. Write three times the square of the first figure (2) of the root, considered as tens (or 20), as trial divisor (1200), at the left of the dividend (4167).

5. Divide the dividend (4167) by the trial divisor (1200) and write the quotient for the next figure of the root.

6. Add to the trial divisor three times the first part of the root (2) considered as tens (or 20), multiplied by the last figure (3), (180); and also the square of the last figure (3) of the root (9), to make the complete divisor (1389).

7. Multiply the complete divisor (1389) by the last figure (3) of the root, and subtract the product from the dividend (4167).

8. Repeat steps 3 to 7 if there are more than two figures in the root, substituting the proper numbers for those given.

NOTES. 1. If at any time the trial divisor is not contained in the dividend, place a zero in the root, annex two zeros to the trial divisor, and annex the next period to the dividend and then divide.

2. If at any time the product of the complete divisor by the last figure of the root found is greater than the dividend, the root figure is too large. In that case reduce the root figure by 1, make the new additions to the trial divisor, and proceed as before.

3. In extracting the cube root of decimals, point off from the decimal point to the right. If necessary, annex zeros, so that each period may have three places.

4. In finding the cube root of a fraction whose terms are not perfect cubes, reduce the fraction to a decimal with three times as many places as the number of figures wanted in the root, and extract the root.

TO THE TEACHER. The statement of the steps 1 to 8, omitting the numbers, is the rule for extracting the cube root. Each step should be thoroughly learned. Pupils should be able to state the entire rule and to apply each step in order. After going through the rule with pupils to see that they understand and can apply its directions, give drills in applying each step to a number of examples before taking up the next step. Before this drill is begun on any step, have the exact wording of the step learned and then apply it without referring to the book.

Point off the following numbers for the extraction of their cube roots, keeping the work on each number on a separate sheet of paper:

2.	3.	4.	5.	6.
32768;	79507;	614125;	373.248;	.004913

State the first step.

Give the cubes of the numbers from 1 to 10.

State the second step, and apply it in each case.

State the third step, and apply it to each remainder.

In the same way, state and apply the succeeding steps with each of the results thus far obtained.

Explanation of the Process

Relation of the number and places of figures in a root to the number and places of figures in its cube.

$1^3 = 1$ The cube of the smallest number of one figure consists of one figure.

$9^3 = 729$ The cube of the largest number of one figure consists of three figures.

The cube of any number between 1 and 9 contains one, two, or three figures.

$10^3 = 1000$

$90^3 = 729{,}000$

Annex a zero to 1, making 10, and to 9, making 90, and cube these numbers. Notice that annexing a zero to the smallest and to the largest number of one figure increases the number of figures in the cube by three.

$19^3 = 6859$

$99^3 = 970{,}299$

Annex 9, the largest number of one figure, to 1, making 19, and to 9, making 99. It will be seen that annexing the figure 9 to the smallest and to the largest number of one figure increases the number of figures in each cube by three. Hence, annexing a figure of any value to any one-figure number increases the number of figures in its cube by three.

$$\begin{aligned} 10^3 &= 1\,000 \\ 100^3 &= 1\,000\,000 \\ 900^3 &= 729\,000\,000 \\ 990^3 &= 970\,299\,000 \\ 999^3 &= 996\,002\,999 \end{aligned}$$

In like manner, it may be shown that annexing a figure to any number increases the number of figures in its cube by three.

The cube of the first figure of the root is found in the first period of the cube; of the first two figures of the root, in the first two periods of the cube; of the first three figures of the root, in the first three periods of the cube, etc.

From the preceding, it appears that the cube of any number of one figure contains one, two, or at most three figures, and that if any figure is annexed to any one-figure number, the number of figures in its cube is increased by three; that for any figure after the first in the root there are three figures in its cube, and that for the first figure in the root there may be one, two, or three figures in the cube; that pointing off the number into periods of three places each, beginning with units, will show the first period, whether of one, two, or three figures, and the number of figures in the root.

Composition of the Cube of a Number

A number is cubed by multiplying its square by the number.

$$25^2 = 20^2 + 2(20 \times 5) + 5^2$$

Multiply $(20 + 5)^2$ by $20 + 5$, indicating the operations. Remember that multiplying any one of several factors of a product multiples the product.

$$\begin{array}{r} 20^2 + 2(20 \times 5) + 5^2 \\ 20 + 5 \\ \hline 20^2 \times 5 \quad + 2(20 \times 5^2) + 5^3 \\ 20^3 + 2(20^2 \times 5) \ + \ (20 \times 5^2) \qquad\quad \\ \hline 20^3 + 3(20^2 \times 5) \ + 3(20 \times 5^2) + 5^3 \end{array}$$

In the partial products we have 2 times 20×5^2 and once 20×5^2; the sum is 3 times 20×5^2.

We also have once $20^2 \times 5$ and 2 times $20^2 \times 5$; the sum is 3 times $20^2 \times 5$.

$$25^3 = (20 + 5)^3 = 20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3.$$

Substituting t for the tens, or 20, and u for the units, or 5, we have: $(t + u)^3 = t^3 + 3(t^2 \times u) + 3(t \times u^2) + u^3$.

$$\begin{array}{rcrl} t^3 = 20^3 & = & 8000 & = \text{Cube of the tens.} \\ 3(t^2 \times u) = 3(20^2 \times 5) & = & 6000 & = \text{Three times the square of the tens by the units.} \\ 3(t \times u^2) = 3(20 \times 5^2) & = & 1500 & = \text{Three times the tens by the square of the units.} \\ u^3 = 5^3 & = & 125 & = \text{Cube of the units.} \end{array}$$

$$t^3 + 3(t^2 \times u) + 3(t \times u^2) + u^3 =$$
$$20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3 = 15{,}625.$$

As any number of two or more figures may be regarded as made up of a certain number of *tens* + a certain number of units, it follows that the cube of any number of two or more figures equals:

The cube of the tens plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

$$75^3 = (70+5)^3 = 70^3 + 3(70^2 \times 5) + 3(70 \times 5^2) + 5^3 = ?$$
$$215^3 = (210+5)^3 = 210^3 + 3(210^2 \times 5) + 3(210 \times 5^2) + 5^3 = ?$$

1. Extract the cube root of 15,625.

We have found that the cube root of a number is made up of four parts:

(1) The cube of the tens. (2) Three times the square of the tens multiplied by the units. (3) Three times the tens multiplied by the square of the units. (4) The cube of the units.

We have found also that the cube of the first figure of the root is found in the first period of the cube.

$$\begin{array}{r|l} & \quad 2\;5 \\ \hline & 15'625 \\ t^3 = & \quad 8 \\ \hline & \;\; 7\,625 \end{array}$$

$$\left.\begin{array}{r} 3(t^2 \times u) \\ 3(t \times u^2) \\ u^3 \end{array}\right\} \text{A}$$

The first period is 15. The largest cube equal to or less than 15 is 8; its cube root is 2, the first figure of the required root.

As there are two periods, and therefore two figures in the root, the first figure, 2, is 2 tens, or 20 units. As the cube 8 is in thousands' place, you need not write the three zeros.

In subtracting 8 thousand from 15,625, you take away the cube of the tens, and have left the other three parts of the cube, as shown at A.

The first of these parts, $3(t^2 \times u)$, is the largest of the three, and as it contains all the different kinds of factors found in the other two parts, that is, 3, t, and u, you may disregard the other two parts and use only the first of these parts in finding the trial second figure of the root.

This is called a trial figure because as the other two parts of the cube have been disregarded, and 7625 has been considered as made up of only one of the three parts, the trial divisor may be found to be too large when

the two parts disregarded for the purpose of finding the units' figures are considered.

Regarding 7625, then, as made up of 3 times the square of the tens multiplied by the units, you see that two of these factors are known, namely: 3, and the tens, which you have found to be 2, or 20 units. The square of the tens is 20 × 20, or 400. 3 times the square of the tens is 1200. The units' figure is not known, but may be found approximately by dividing 7625 by 1200 as a trial divisor, or, what is the same thing, ignoring the zeros and dividing 76 by 12. The quotient is 6, the trial units' figure.

$$\left.\begin{array}{r} 3(20^2 \times 6) = 7200 \\ 3(20 \times 6^2) = 2160 \\ (6^3) = \underline{216} \\ 9576 \end{array}\right\} A \qquad \left.\begin{array}{r} 3(20^2 \times 5) = 6000 \\ 3(20 \times 5^2) = 1500 \\ 5^3 = \underline{125} \\ 7625 \end{array}\right\} B$$

Before writing the 6 in the root, test to see whether it is the proper figure. Writing out the value of each of the three parts as shown at A, you find their sum is 9576, or more than 7625. Therefore the trial units' figure, 6, is too large. Substituting for it the next smaller number, 5, and writing out the value of the three parts as shown above at B, you find that their sum is 7625, showing that 5 is the correct units' figure, which you place in the root over the second period.

In the process employed in extracting the cube root of numbers, you have shortened the above operation as follows:

After dividing 7625 by 1200 as a trial divisor, and finding the trial units' figure, 6, you may complete the divisor by adding to it 3(20 × 6), or 360, and 6^2, or 36, making 1200 + 360 + 36, or 1596, which equals $3 \times t^2 + 3(t \times u) + u^2$. Each of these three parts lacks the same factor, the units. On multiplying their sum, 1596, called the complete divisor, by 6, the product, 9576, is the same as you found the sum of the three full parts to be, in testing to determine whether 6 was the correct units' figure.

Trial divisor, or		7625
$3 \times t^2$	1200	
$3 \times t \times u$	300	
u^2	25	7625
Complete divisor	1525	

In testing 5, proceed in the same way: Adding to 1200, 3(20 × 5), or 300, and 5^2, or 25, you have 1525, the complete divisor, which, when multiplied by the units' figure, 5, gives 7625, the sum of the three parts of the cube remaining after the cube of the tens, 8000, has been subtracted.

The cube subtracted from the first period, and the product of the complete divisor by the second figure of the root, together equal the entire number or cube. If there were three figures in the root, these two numbers together would equal the cube of the first two figures of the root considered as tens, and the units' figure would be found in same way as the tens' figure was found, and so on, if the root had more than three figures.

2. Extract the cube root of 97,336, explaining the process. Work the example, answering each question in order.

The information necessary for answering these questions may be obtained by a careful study of the examination of the process of extracting the cube root, pp. 488–492.

1. Why do you point off the number into periods of three figures each, beginning at units' place?

2. How is the first figure of the root found? Why? What is it?

3. The number subtracted from the first period is what as related to the first figure of the root?

4. The number formed by annexing the next period to the remainder contains what three parts of the entire cube?

5. What part of the entire cube is it regarded as being for the purpose of finding the second figure of the root?

6. What factors does this part contain?

7. Which of these factors are known?

8. When the product of three factors and two of the factors are known, how can you find the other factor?

9. What is done in this case to find the trial second figure of the root? What is this figure?

10. What is the trial divisor? Of what factors is it the product? What two additions are made to it to form the complete divisor? Give the numbers added, and state in general terms of what factors each number is the product.

11. State the terms of which the complete divisor is composed.

12. The product of the complete divisor by the trial units' figure of the root is composed of what terms of the cube?

13. What is done with this product? What does this operation show?

3. Extract the cube root of 148,877, explaining each step in the operation.

COMPOUND PROPORTION

A compound proportion is an equality of ratios, one or more of which are compound;

as,
$$\begin{matrix} 4:8 = 3:6 \\ 2:10 = 5:25 \end{matrix}$$

1. If 10 men can build 50 rods of wall in a certain time, how many men can build 80 rods of wall in the same time?

STATEMENT. $50:80 = 10:?$

2. If 10 men can build a wall in 12 days, how many men can build it in 16 days?

STATEMENT. $16:12 = 10:?$

The above problems are problems in simple proportion. Each statement is the statement of a simple proportion. Combining the two problems into one, you have the following problem:

3. If 10 men can build 50 rods of wall in 12 days, how many men can build 80 rods of wall in 16 days?

This problem is a problem in compound proportion. It will be noticed that in each of the two statements given above, the third term is 10. The first ratio in the first statement differs from the first ratio in the second statement. Bringing these statements together as the statement of a compound proportion, you have the third term 10, and the first and second terms, a compound ratio.

$$\begin{matrix} 50:80 \\ 16:12 \end{matrix} = 10 ?$$

Multiplying the two ratios together, term by term, you have:

$$(50 \times 16):(80 \times 12) = 10:?$$

Multiplying the means together, and dividing by the extremes, you have:

$$\frac{80 \times 12 \times 10}{50 \times 16} = 12.$$

It will thus be seen that problems in compound proportion result from combining two or more problems in simple proportion.

In solving such problems, it is best to use the fractional form of solution. This gives the last expression above, without the preceding steps.

Notice that in the above problem there are two numbers representing rods, two numbers representing days, and one number representing men, and that what is asked for is the number of men. In getting the statement for multiplication, proceed as in simple proportion, using the third term first with the two numbers representing rods.

Q. If 10 men can build 50 rods of wall, will it take a greater or a less number to build 80 rods of wall in the same time? *Ans.* A greater number, or $\frac{80}{50} \times 10$ men.

Write the expression $\frac{80}{50} \times 10$ men at the right as follows: $\frac{80}{50} \times 10$.

Next deal with the men and days, disregarding everything else.

Q. If 10 men can build a wall in 12 days, will it take more or less men to build it in 16 days? *Ans.* Less men, or $\frac{12}{16} \times 10$ men.

Write this fraction, $\frac{12}{16}$, at the left of the expression above, so that it now stands:

$$\frac{12}{\cancel{16}} \times \frac{\overset{\cancel{16}}{\cancel{80}}}{\underset{\cancel{5}}{\cancel{50}}} \times \cancel{10} = 12.$$

Canceling and performing the indicated operations, you secure the result, 12.

4. If 18 men build 75 rods of fence in 10 days of 8 hours each, how many hours a day must 24 men work to build 144 rods in 12 days?

In this problem, hours a day are called for. You have two numbers, 18 and 24, representing men; two numbers, 75 and 144, representing rods; two numbers, 10 and 12, representing days; and one number, 8, representing hours. Each set of two like numbers constitutes a ratio, and must be compared with the number representing hours.

Q. If 18 men build a fence working 8 hours a day, must 24 men work more or less hours a day to build it in the same number of days? *Ans.* Less, or $\frac{18}{24}$ as many hours. $\frac{18}{24} \times 8.$

Q. If a number of men build 75 rods of fence working 8 hours a day, must they work a greater or less number of hours a day to build 144 rods in the same number of days? *Ans.* A greater number of hours a day, or $\frac{144}{75}$ as many hours. Write $\frac{144}{75}$ at the left of the expression above. $\frac{144}{75} \times \frac{18}{24} \times 8.$

Q. If it requires a number of men 10 days working 8 hours a day to build a fence, will it take a greater or less number of hours a day if they work 12 days? *Ans.* A less number, or $\frac{10}{12}$ as many hours. Prefix this fraction to the others with the multiplication sign. $\frac{10}{12} \times \frac{144}{75} \times \frac{18}{24} \times 8.$

5. If 37 horses in 5 days eat $35 worth of oats, how much will it cost to feed 60 horses on oats for 8 days?

Q. In this problem what is asked for? *Ans.* Dollars.

In the solution of these problems begin the question with the statement given in the problem.

If 37 horses eat $35 worth of oats in a certain time, will it cost more or less to feed 60 horses the same time? *Ans.* More, $\frac{60}{37}$ as much. Write this fraction at the left of 35, with the sign of multiplication between them. $\frac{60}{37} \times 35.$

Q. If a number of horses eat $35 worth of oats in 5 days, will it cost more or less to feed the same number of horses 8 days? *Ans.* More, $\frac{8}{5}$ as much. Write $\frac{8}{5}$ at the left of the other numbers. $\frac{8}{5} \times \frac{60}{37} \times 35.$

6. If 30 masons in 15 days can build a wall 387.5 ft. long, what is the length of a wall that can be built by 48 masons in 20 days?

What is asked for in this problem? What is the number to be multiplied by the fractions to be found?

If 30 masons can build a wall 387.5 ft. long in a given time, will 48 masons build a greater or less length of wall in the same time? What is the resulting fraction?

If a number of masons can build a wall 387.5 ft. long in 15 days, can they build a greater or less length of wall in 20 days? What is the resulting fraction?

7. If it takes 60 men 12 days to make a pavement 160 feet long and 40 feet wide, how long will it take 54 men to make a similar pavement 180 ft. long 48 ft. wide?

8. If 180 men in 6 days of $6\frac{2}{3}$ hours each can dig a ditch 600 yd. long, 3 yd. wide, 2 yd. deep, in how many days can 100 men working $5\frac{1}{3}$ hours a day dig a ditch 540 yd. long 4 yd. wide, and 3 yd. deep?

9. 36 men are engaged to do a piece of work in 48 days. After 12 days, 12 men stop. In how many days can the rest finish the work?

FOREIGN EXCHANGE

Exchange between places in different countries is called **foreign exchange**.

Foreign exchanges are usually transacted by *international money orders, letters of credit, travelers' checks* (see p. 374), and *bills of exchange.*

A **foreign bill of exchange** is a draft of a bank in one country upon a bank in another. It is payable in the money of the country on which it is drawn.

EXCHANGE FOR *New York, U.S.A., Aug. 1, 1914.*
£ 50 . 2 . 6
Three days
after sight of this First *of Exchange (second unpaid)*
Pay to the order of Charles Raynor
Fifty Pounds 2/6 Sterling
Value received and charge the same to account of
To Baring Brothers,
London, England. *Hallgarten & Co.*
No. 527

To guard against loss, foreign bills of exchange are usually issued in duplicate. The duplicate reads "Second of Exchange (first unpaid)" and the number 2 appears instead of 1 in the left margin.

Problems in foreign exchange are solved in the same manner as in domestic exchange, except that the currency of one country must be reduced to that of another.

Exchange on England is quoted by giving the cost of a bill for £1. Thus, 4.86 means that a bill of exchange for £1 costs $4.86; German exchange is quoted by giving the number of cents that 4 marks cost. Thus, a quotation of 95⅝ means that 4 marks cost 95⅝¢; French exchange is quoted by giving the number of francs that can be bought for $1. Thus, 5.17 means that 5.17 francs can be bought for $1.

PROBLEMS

Written

1. What is the cost of a draft on London for £380 15*s.*, exchange at $4.8665?

Solution. £380 15*s.* = £380.75; 380.75 × $4.8665 = $1852.92, cost of draft.

2. Find the cost of a bill of exchange on Berlin for 1750 marks, exchange being at 96¼.

Solution. Since 4 marks cost $.96¼, 1750 marks will cost (1750 ÷ 4) × $.96¼, or $421.09, cost of draft.

3. Find the cost of a bill of exchange on Paris for 2060 francs when exchange is quoted at 5.15.

Solution. Since 5.15 francs cost $1, 2060 francs cost (2060 ÷ 5.15) × $1, or $400, cost of draft.

Find, to the nearest cent, the cost of exchange for:

4. £200 @ $4.87.

5. 5000 francs @ 5.17.

6. 10,000 M. @ 95⅛.

7. £500 12*s.* @ 4.88.

8. How much must a merchant in Canton pay for a draft of $1134, if 1 tael costs $.642?

TABLES FOR REFERENCE

Measure of Length

12 inches = 1 foot
3 feet = 1 yard
$16\frac{1}{2}$ feet = 1 rod
320 rods = 1 mile (statute)

A *nautical mile* (*knot*) = 6080.27 ft., or about 1.15 mi.

A *hand*, used in measuring the height of horses, = 4 in.; a *fathom*, used in measuring the depth of water, = 6 ft.; a *furlong* = $\frac{1}{8}$ mi.

Measure of Surface

144 square inches = 1 square foot
9 square feet = 1 square yard
$30\frac{1}{4}$ square yards = 1 square rod
160 square rods = 1 acre

An acre of land in the form of a square is very nearly 209 ft. on a side.

A *square* of roofing or flooring equals 100 sq. ft.

Measure of Volume

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
128 cubic feet = 1 cord

Surveyors' Linear Measure

7.92 inches = 1 link
25 links = 1 rod
100 links = 1 chain
80 chains = 1 mile

Surveyors' Square Measure

16 square rods = 1 square chain
10 square chains = 1 acre
640 acres = 1 square mile

This chain, called *Gunter's chain*, is 4 rd., or 66 ft., long.

Liquid Measure

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
$31\frac{1}{2}$ gallons = 1 barrel

1 gal. = 231 cu. in.; 1 cu. ft. = about $7\frac{1}{2}$ gal. A gallon of water weighs about $8\frac{1}{3}$ lb.; a cubic foot of water weighs about $62\frac{1}{2}$ lb., or 1000 oz.

Dry Measure

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

1 bu. = 2150.42 cu. in., or about $1\frac{1}{4}$ cu. ft. The heaped bu. = 2747.71 cu. in., or nearly $1\frac{5}{8}$ cu. ft. 1 qt. or $\frac{1}{32}$ bu. = 67.2 cu. in.

Avoirdupois Weight

16 ounces = 1 pound
7000 grains = 1 pound
100 pounds = 1 hundredweight
2000 pounds = 1 ton
2240 pounds = 1 *long* ton
20 cwt. = 1 ton
1 av. oz. = $437\frac{1}{2}$ grains.

* 32 pounds = 1 bu. of oats
48 pounds = 1 bu. of barley
* 56 pounds = 1 bu. of shelled corn, or rye; also 1 firkin of butter
* 60 pounds = 1 bu. of wheat, potatoes, or clover seed
70 pounds = 1 bu. of corn on cob
80 pounds = 1 bu. of coal
196 pounds = 1 bbl. of flour
200 pounds = 1 bbl. of beef or pork

* In most states.

Troy Weight

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound
5760 grains = 1 pound

The unit generally used for weighing diamonds, gems, etc., is the *carat*, which is about 3.2 Troy grains. It is used also to express the fineness of gold. 18 carats fine means $\frac{18}{24}$ pure gold and $\frac{6}{24}$ baser metal.

The Troy pound = 5760 grains
The Troy ounce = 480 grains

Apothecaries' Weight

20 grains = 1 scruple — sc. or ℈
3 scruples = 1 dram — dr. or ʒ
8 drams = 1 ounce — oz. or ℥
12 ounces = 1 lb. or ℔

This table is used only in filling medical prescriptions.

Apothecaries' Liquid Measure

60 minims (m) = 1 fluid dram (fʒ)
8 fluid drams = 1 fluid ounce (f℥)
16 fluid ounces = 1 pint (O.)
8 pints = 1 gal. (Cong.)

This table is used only in filling medical prescriptions.

Counting Table

12 things = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross
20 things = 1 score

Stationers' Table

24 sheets = 1 quire
20 quires = 1 ream

Paper is frequently sold by the pad or bulk of 100, 500, or 1000 sheets, or by the pound.

Time Measure

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
365 days	= 1 year
366 days	= 1 leap year
10 years	= 1 *decade*
100 years	= 1 *century*

Thirty days have September,
April, June, and November.
All the rest have thirty-one,
Save February, which alone
Has twenty-eight, and one day more
We add to it one year in four.

Measure of Angles and Arcs

60 seconds ($''$)	= 1 minute ($'$)
60 minutes	= 1 degree ($^\circ$)
360 degrees	= 4 right angles or 1 circumference

90° of angle = 1 *right angle;* 90° of arc = 1 *quadrant.*

United States Money

10 cents	= 1 dime
10 dimes	= 1 dollar
10 dollars	= 1 eagle

English Money

4 farthings	= 1 penny
12 pence	= 1 shilling
20 shillings	= 1 pound
1 pound (£)	= $4.8665

German Money

1 mark (M.) = $.238

French Money

1 franc (fr.) = $.193

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